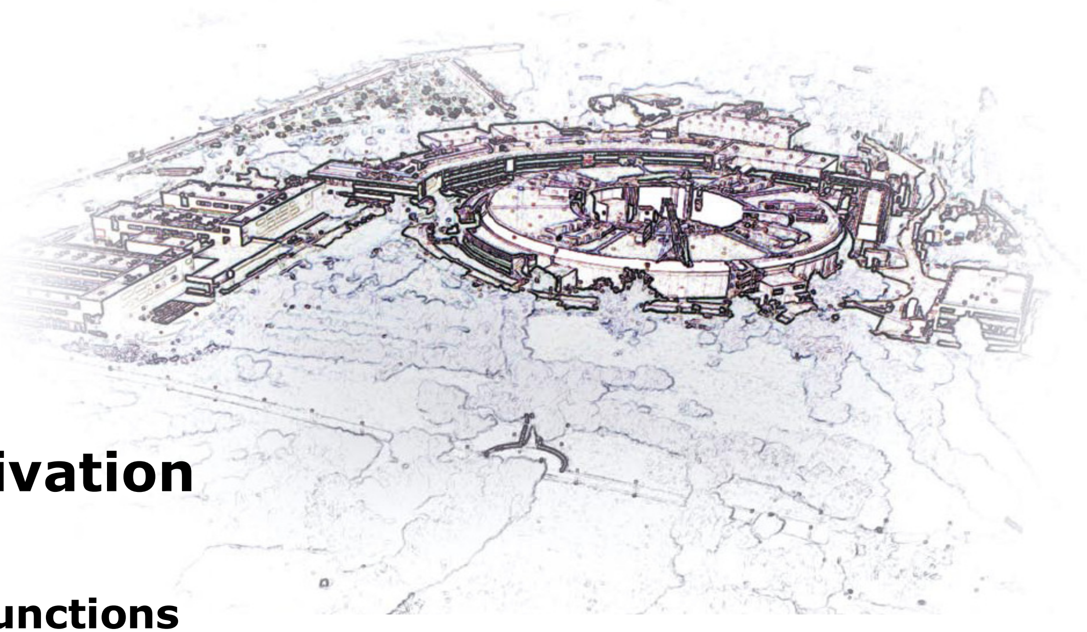
Impact of a chirp and curvature in the electron energy distribution on the seeded Harmonic Generation FEL

Alberto Lutman,

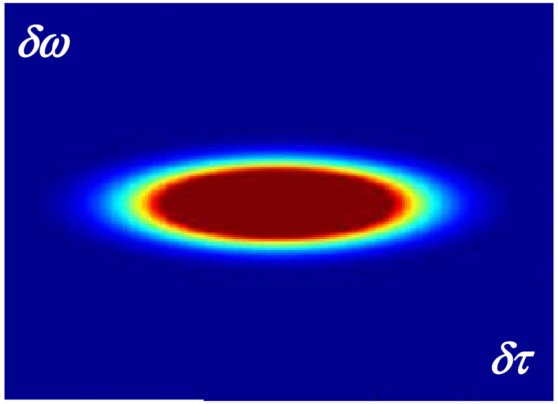
G. Penco, P. Craievich, J. Wu, R. Vescovo

Summary

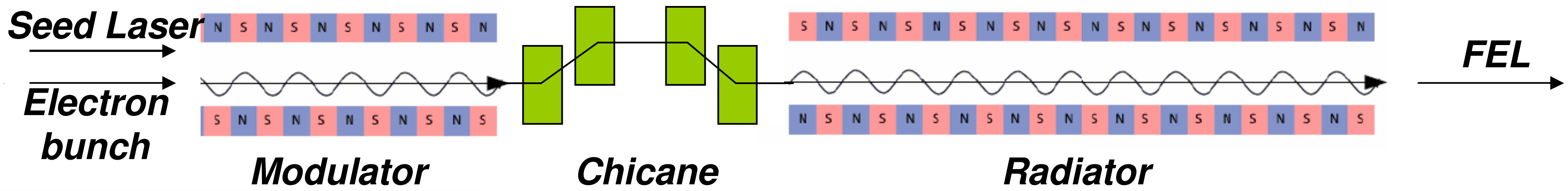


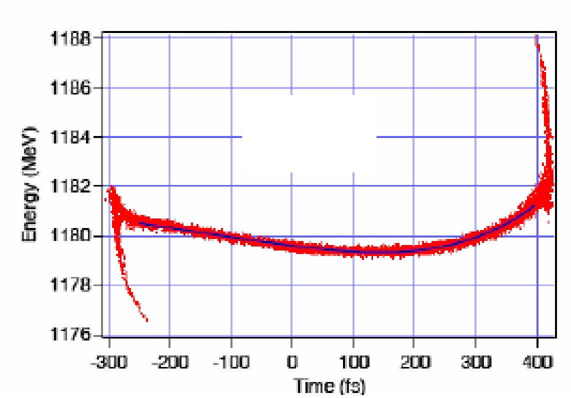
- **Model description**
- **Mathematical derivation**
  - **Seeded FEL Green functions**
  - **Evaluation of the bunching**
- **An example for FERMI case**

Model Description

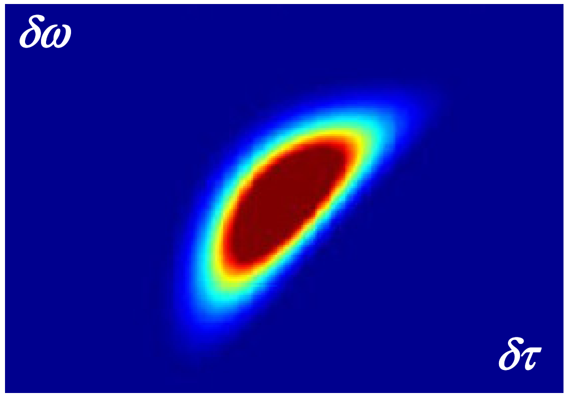


Phase space plot of the Seed Laser using the Wigner function



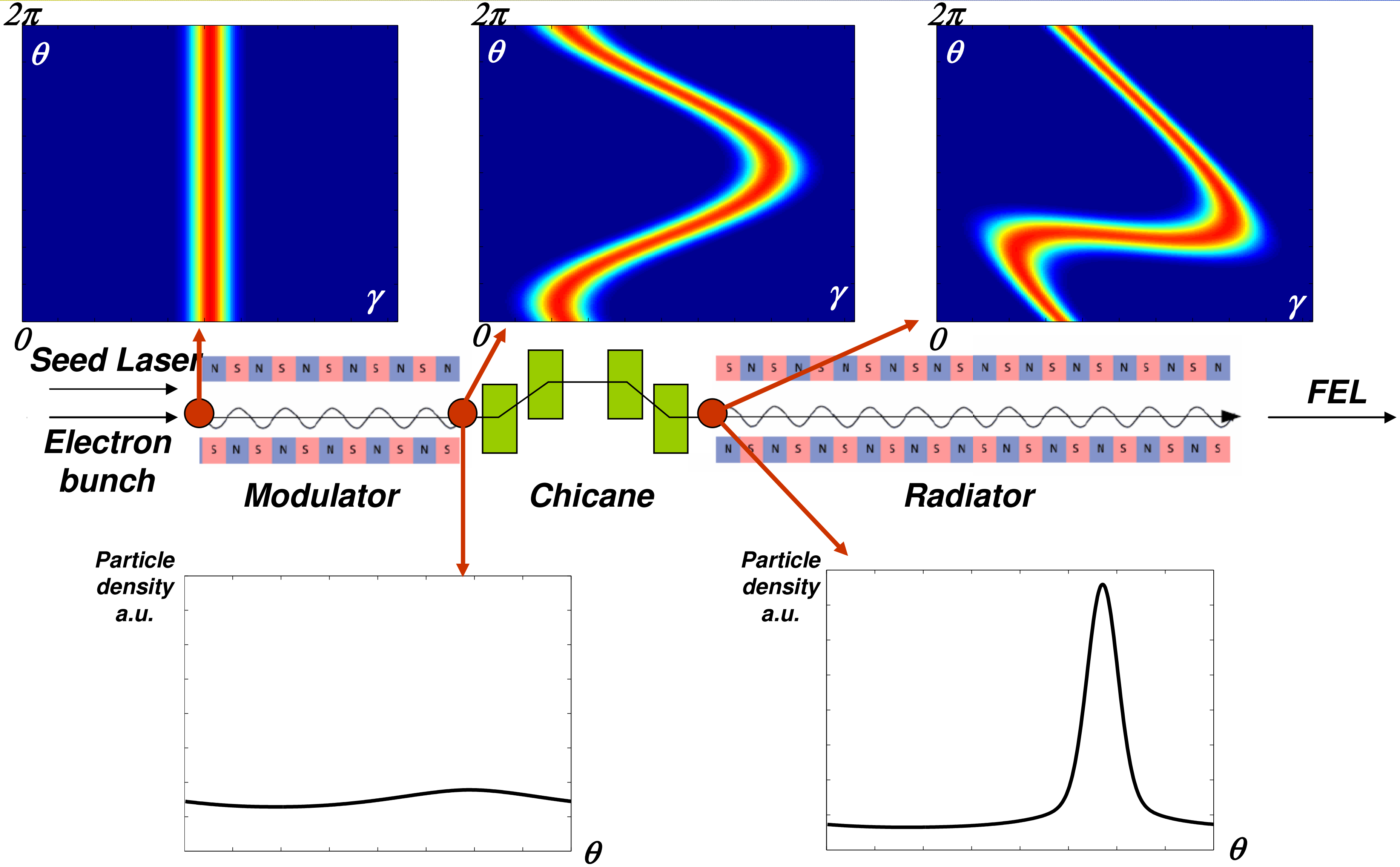


Electron bunch with energy chirp and curvature



Phase space plot of the FEL Pulse using the Wigner function

Model Description



Mathematical derivation of the Green Functions

We start from:

S. Krinsky and Z. Huang derived the Green function for linear enegy chirped FEL PRST-AB Vol 6. 050702 (2003)

Vlasov-Maxwell system of equations

$$\begin{cases} \dfrac{\partial \psi}{\partial Z} + p\dfrac{\partial \psi}{\partial \theta} - \dfrac{2D_2}{\gamma_0^2}\left(Ae^{i\theta} + A^* e^{-i\theta}\right)\dfrac{\partial \psi_0}{\partial p} = 0 \\ \left(\dfrac{\partial}{\partial Z} + \dfrac{\partial}{\partial \theta}\right)A(\theta, Z) = \dfrac{2D_1}{\gamma_0} e^{-i\theta}\int dp\, \psi(\theta, p, Z) \end{cases}$$

$\psi(\theta, p, Z)$ ***Electron distribution function***

$A(\theta, Z)$ ***Electric field envelope***

$$p = 2\frac{\gamma - \gamma_0}{\gamma_0} \qquad \theta = (k_0 + k_w)z - \omega_0 t \qquad Z = k_w z$$

Mathematical derivation of the Green Functions

We find $\psi(\theta, p, Z)$ ***introducing a small perturbation*** ψ_1

$$\psi(\theta, p, Z) = \psi_0 + \psi_1$$

$$\psi_0(\theta, p, Z) = \delta\left(p + \mu\theta_0 + \frac{1}{2}\nu\theta_0^2 \right) \qquad , \theta_0 = \theta - pZ$$

$$\mu = \frac{2}{\gamma_0\omega_0}\frac{\partial\gamma}{\partial t}$$ ***Linear chirp parameter***

$$\nu = -\frac{2}{\gamma_0\omega_0^2}\frac{\partial^2\gamma}{\partial t^2}$$ ***Curvature parameter***

$$\psi_1 = \frac{2D_2}{\gamma_0^2} e^{i\theta_0} \frac{\partial\psi_0(\vartheta_0)}{\partial p} \int_0^Z e^{ip(Z_1 - Z)} A(\theta - p(Z - Z_1), Z_1) + e^{i\theta_0} F(\theta_0, p)$$

Interaction with the fields

Electrons Distribution Initial Condition

Mathematical derivation of the Green Functions

Initial Condition $e^{i\theta_0} F(\theta_0, p)$

We choose

$$F \propto \delta\left(p + \mu\theta_0 + \frac{1}{2}\nu\theta_0^2\right)$$

Initial electron density modulation

Maxwell equation is solved in the Laplace domain

$$f(\theta, s) = \int_0^Z A(\theta, Z) e^{-sZ}\, dZ$$

$$\frac{\partial}{\partial Z} A(\theta, Z) \xrightarrow{Laplace\ Transform} s\, f(\theta, s) - A(\theta, 0)$$

Seed Initial Condition

Mathematical derivation of the Green Functions

FEL radiation along the undulator

$$A(\hat{s},\hat{z}) = \int_0^{\hat{z}/2} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{d\hat{p}}{\pi i} e^{\hat{p}(\hat{z}-2\hat{\xi})+4i\hat{\beta}\frac{\arctan\left(\frac{\hat{\alpha}-\hat{s}\hat{\beta}}{\sqrt{2i\hat{p}\hat{\beta}-\hat{\alpha}^2}}\right)-\arctan\left(\frac{\hat{\alpha}+\hat{\beta}(\hat{\xi}-\hat{s})}{\sqrt{2i\hat{p}\hat{\beta}-\hat{\alpha}^2}}\right)}{\left(2i\hat{p}\hat{\beta}-\hat{\alpha}^2\right)^{3/2}}-4i\frac{\frac{\hat{\alpha}-s\hat{\beta}}{2i\hat{p}-2\hat{s}\hat{\alpha}+s^2\hat{\beta}}+\frac{\hat{\alpha}+\hat{\beta}(\hat{\xi}-\hat{s})}{(\hat{s}-\hat{\xi})(2\hat{\alpha}+\hat{\beta}(\hat{\xi}-\hat{s}))-2i\hat{p}}}{\hat{\alpha}^2-2i\hat{p}\hat{\beta}}}$$

$$\times\left[A\left(\hat{s}-\hat{\xi},0\right)+\frac{D_1/(\rho\gamma_0)\,F\left(\hat{s}-\hat{\xi}\right)}{2\hat{p}+i(\hat{s}-\hat{\xi})(2\hat{\alpha}-\beta(\hat{s}-\hat{\xi}))}\right]d\hat{\xi} = \int_0^{\hat{z}/2} G_{seed}\,A\left(\hat{s}-\hat{\xi},0\right)+G_{bun}\,F\left(\hat{s}-\hat{\xi}\right)d\hat{\xi}$$

Seed ***Initial Bunching***

We calculate the inverse Laplace transform to find the Green functions

$$\hat{z}=2\rho Z \quad \hat{s}=\rho\theta \quad \hat{\alpha}=-\frac{\mu}{2\rho^2} \quad \hat{\beta}=-\frac{\nu}{2\rho^3} \quad \hat{p}=\frac{s}{2\rho}$$

ρ : Pierce Parameter

Mathematical derivation of the Green Functions

Green functions are found by analytical inverse Laplace transforming without using approximations.

Green function for Seeded FEL with linear chirp and curvature:

$$G_{Seed}=\sum_{j=1}^{\infty}\sum_{W_l=0}^{J-\sum_{k=1}^{l-1}W_k}\frac{\left(\hat{z}-2\hat{\xi}\right)^{\sum_{h=1}^{\infty}hW_h+2J-1}}{\left(\sum_{h=1}^{\infty}hW_h+2J-1\right)!}\frac{\left(2i\hat{\xi}\right)^{J-\sum_{h=1}^{\infty}W_h}}{\left(J-\sum_{h=1}^{\infty}W_h\right)!}\frac{T(l)^{W_l}}{W_l!}+\delta(\hat{\xi}-\hat{z}/2)$$

Green function for Bunching with linear chirp and curvature:

$$G_{bun}=\sum_{H=0}^{\infty}\sum_{j=1}^{\infty}\sum_{W_l=0}^{J-\sum_{k=1}^{l-1}W_k}\frac{\left(\hat{z}-2\hat{\xi}\right)^{\sum_{h=1}^{\infty}hW_h+2J+H+1}}{\left(\sum_{h=1}^{\infty}hW_h+2J+H+1\right)!}\frac{T(l)^{W_l}}{W_l!}R(H)\delta_{(J,\sum_{h=0}^{\infty}W_h)}$$

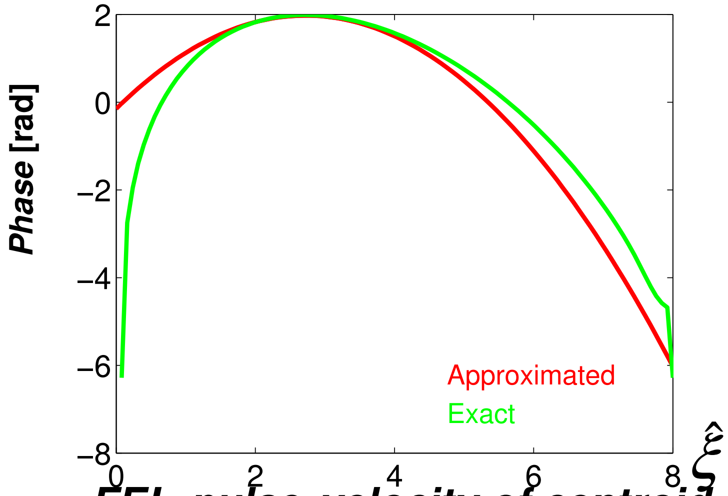
$$T(m)=\sum_{n=0}^{m}\frac{i^{2n+m+1}(m+1)!(\hat{s}^{2m-n+1}-(\hat{s}-\hat{\xi})^{2m-n+1})\hat{\alpha}^n\hat{\beta}^{m-n}}{(2m-n+1)!2^{m-n-1}\,n!\,(m-n)!}$$

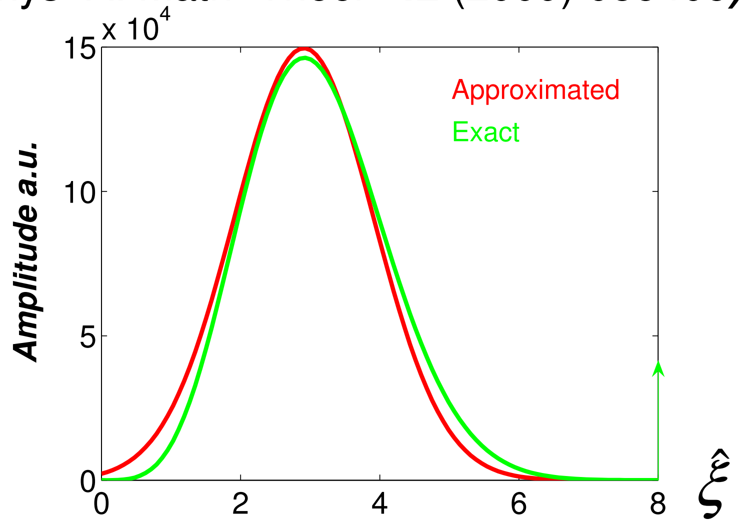
$$R(H)=\frac{D_1}{\rho\gamma_0}\frac{i^{3H+1}}{2^H}(\hat{s}-\hat{\xi})^H(2\hat{\alpha}-\hat{\beta}(\hat{s}-\hat{\xi}))^H$$

Are expressed as polynomials

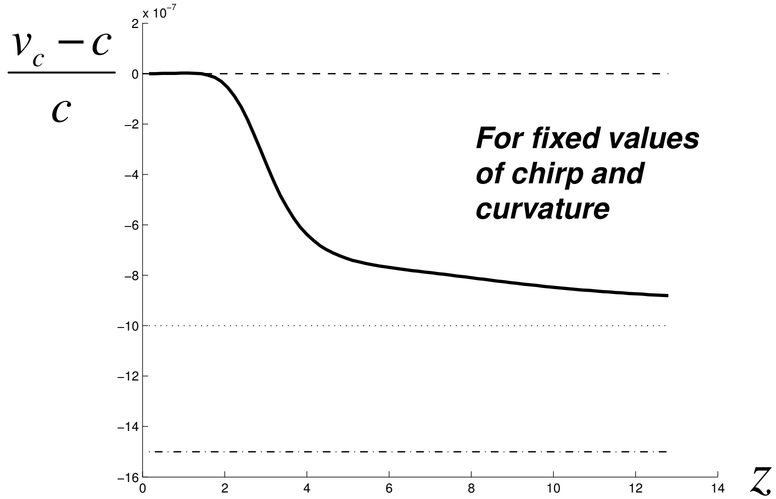
Seeded FEL Green Function

Comparison between the "exact" Green function and the saddle point approximated formula. (A.A.Lutman et al. J. Phys. A: Math. Theor. **42** (2009) 085405***)***

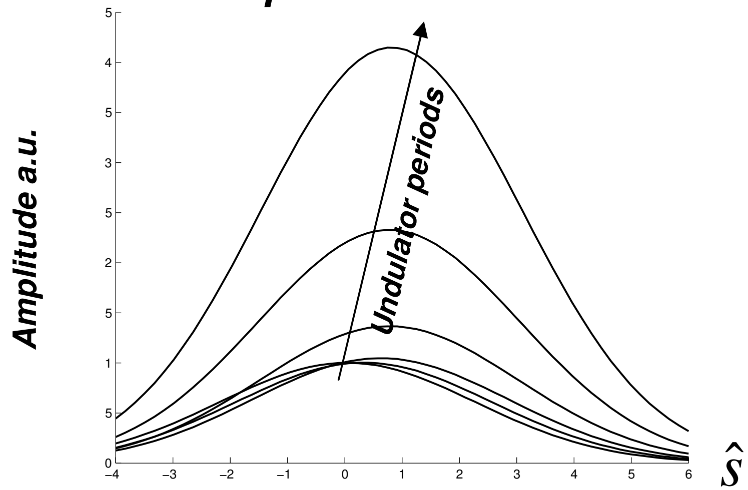




FEL pulse velocity of centroid

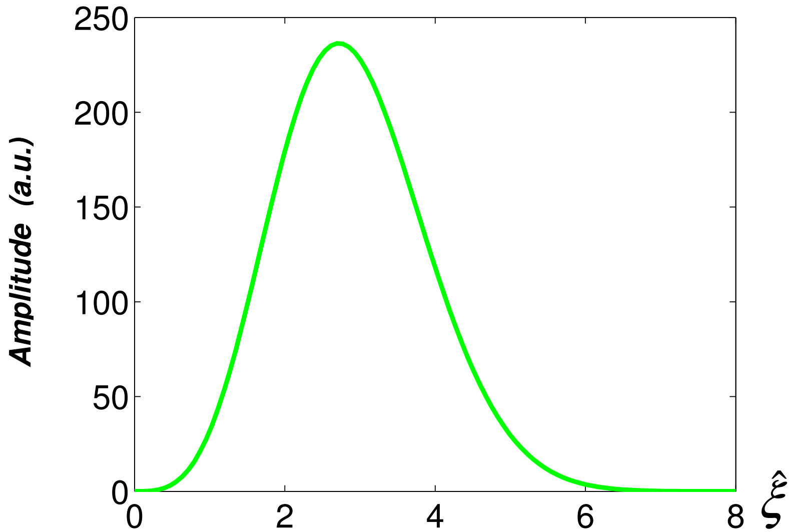


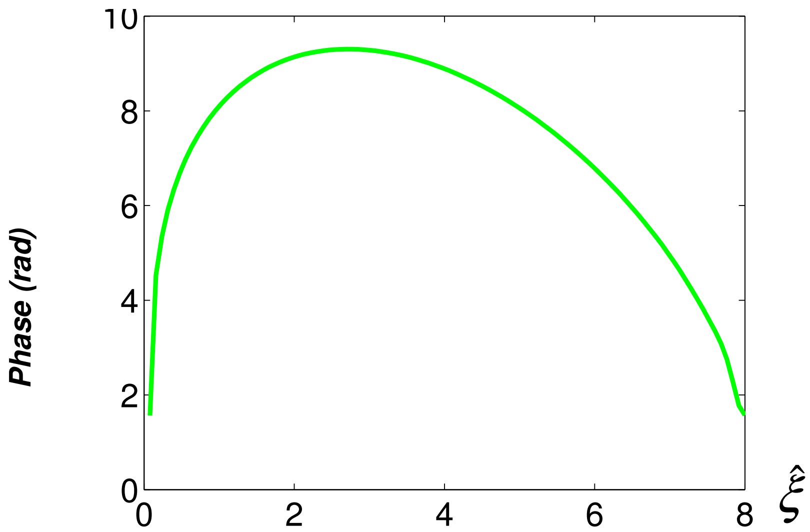
FEL pulse evolution



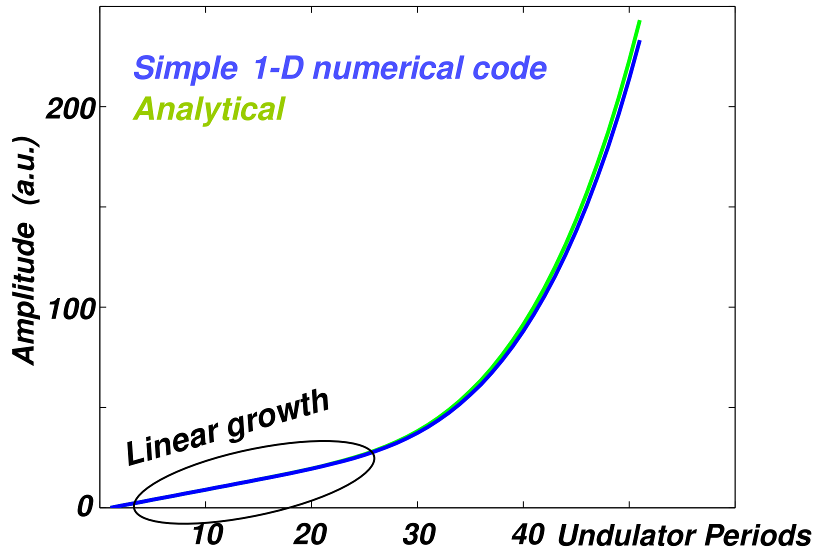
Bunching Green function

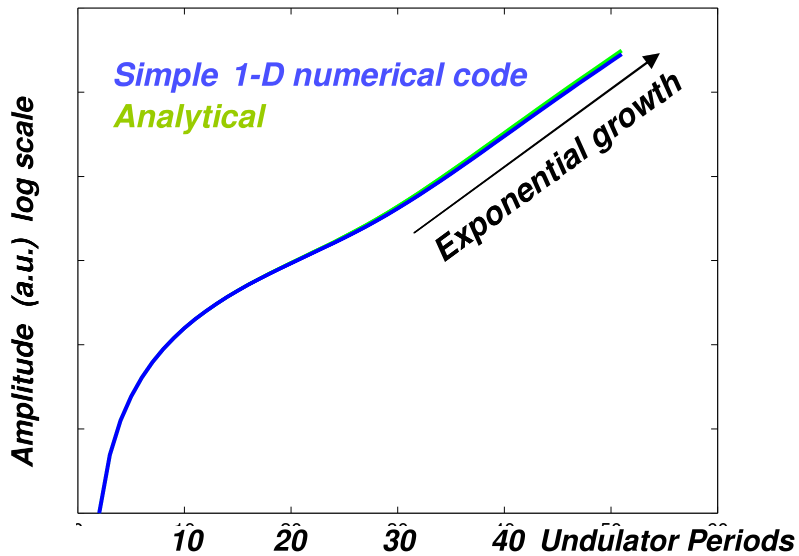
Green function plots





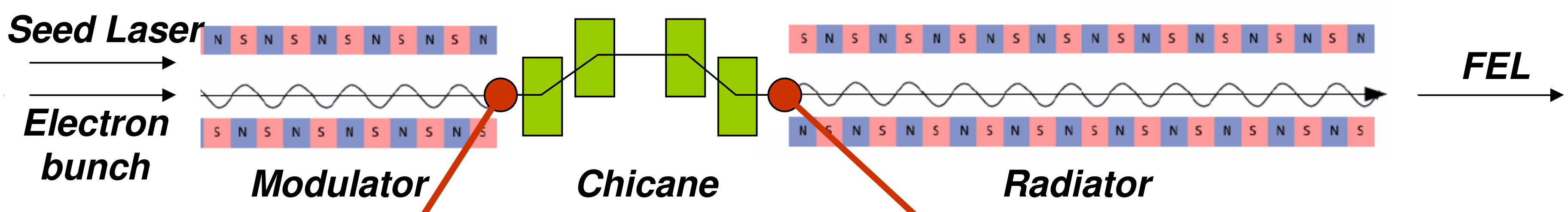
Peak electrc field envelope

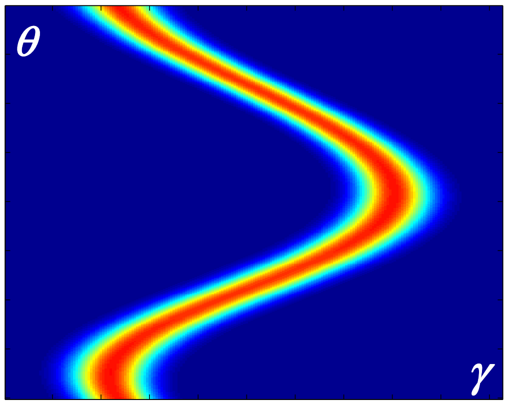




Bunching at radiator entrance

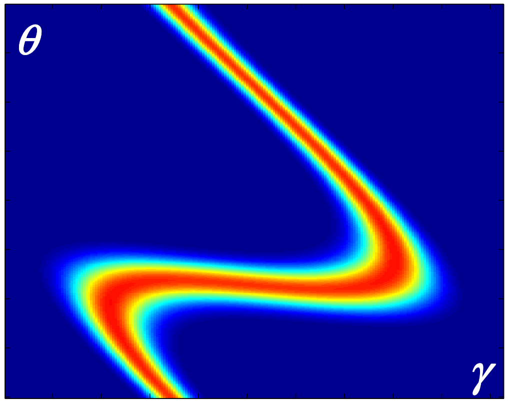
Bunching Amplitude





We use the formula:

$$B_n = 2e^{-\frac{1}{2}n^2\sigma_\gamma^2\left(\frac{d\theta}{d\gamma}\right)^2} J_n\left(n\Delta\gamma\left(\frac{d\theta}{d\gamma}\right)\right)$$



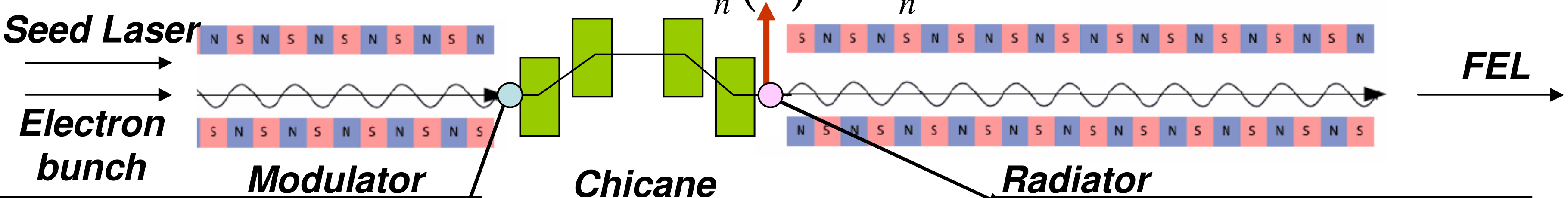
L.H.Yu Physical Review A, Vol 44, N° 8 (1991)

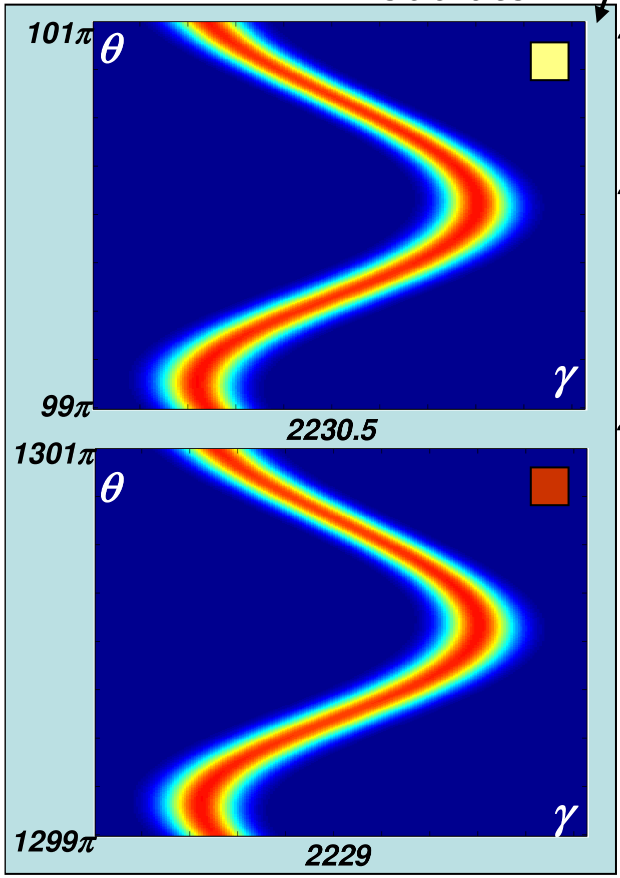
This formula does not take into account phase dependent energy spread induced in the non zero length modulator.

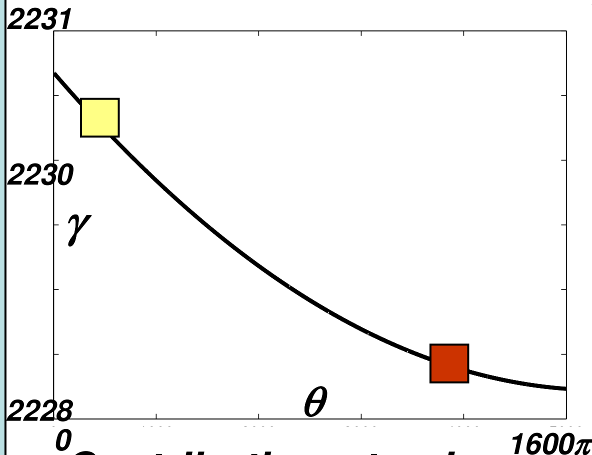
Bunching at radiator entrance

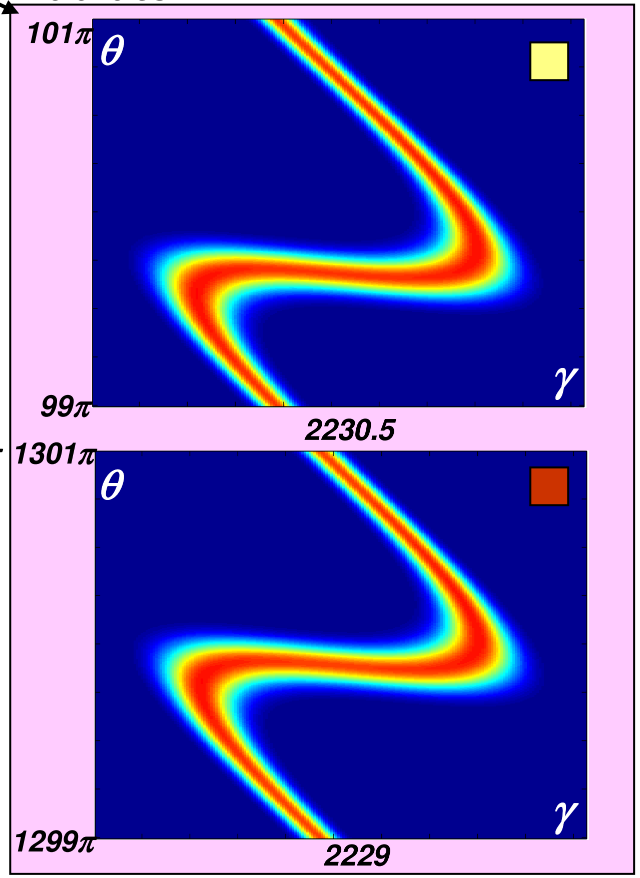
Bunching Phase

$$B_n(\theta) = B_n\, e^{i\,n\,(\Delta\theta_m + \Delta\theta_c)}$$









Contributions to phase:

Modulator

$$\Delta\theta_M = 4\pi \frac{\gamma - \gamma_0}{\gamma_0} N_w$$

Chicane

$$\Delta\theta_C = (k_0 + k_w)\frac{\gamma - \gamma_0}{\gamma_0} R_{56}$$

An example for FERMI case

Bunching Amplitude with respect to the fundamental

Energy	R56	Energy spread	Peak to peak modulation
1140 MeV	30 μm	150 keV	3 MeV

Bunching	1	10	20
B_n	0.45	0.14	0.023

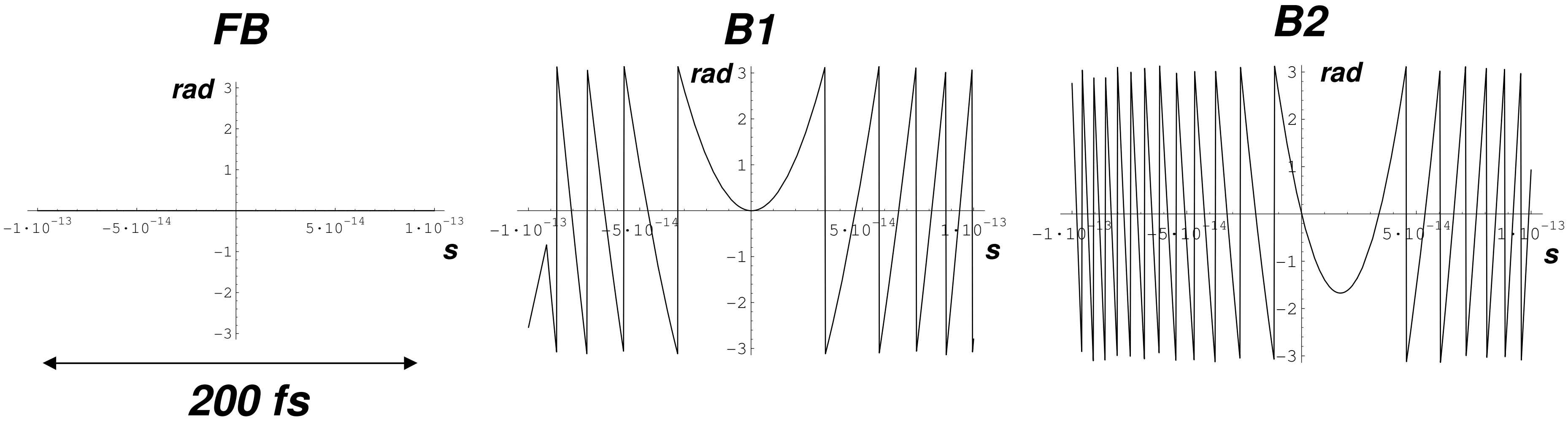
An example for FERMI case

Bunching Phase at radiator entrance

	Energy	Linear Chirp	Curvature
Flat Bunch (FB)	1140 MeV	0 MeV/fs	0 MeV/fs^2
Bunch 1 (B1)*	1140 MeV	1.4 10^{-5} MeV/fs	3.2 10^{-6} MeV/fs^2
Bunch 2 (B2)*	1140 MeV	2.2 10^{-3} MeV/fs	6.5 10^{-6} MeV/fs^2

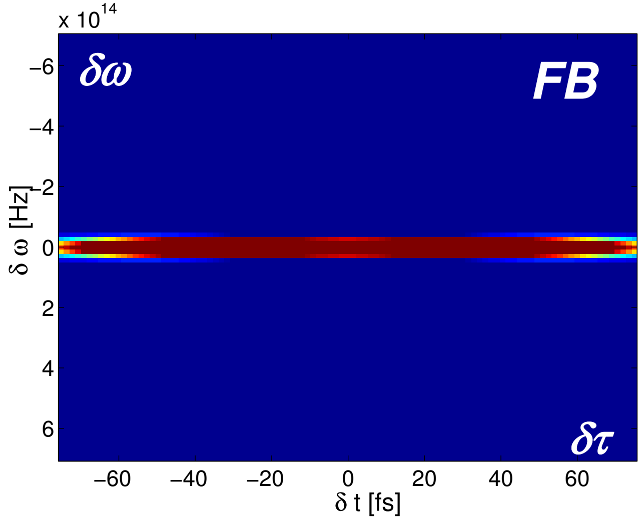
****** bunch configurations generated with LiTrack***

Harmonic Number: 10



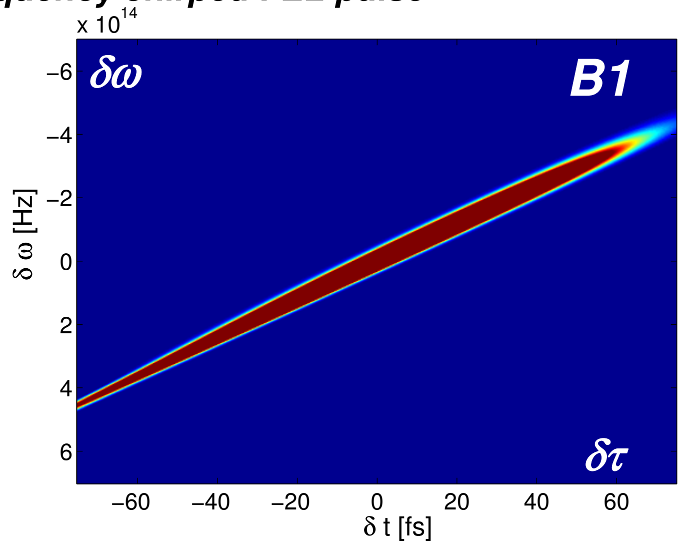
An example for FERMI case

Electric Field Envelope Wigner Function after 400 radiator periods

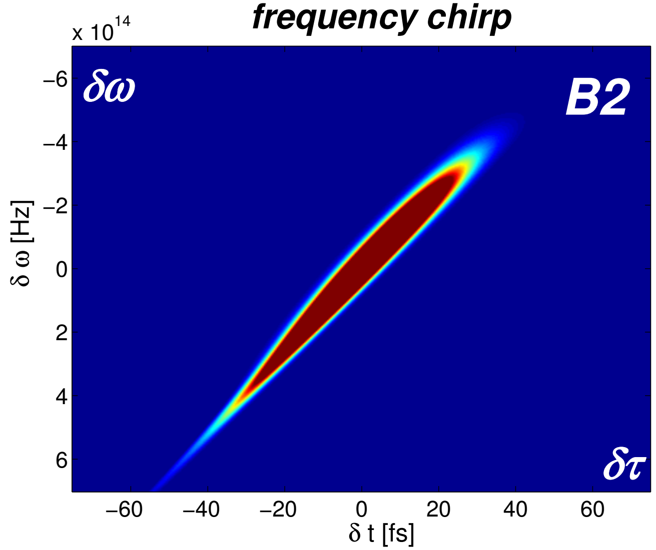


The Flat bunch presents the shortest bandwidth

The curvature on the electrons energy distribution gives a frequency chirped FEL pulse



The bunch with larger curvature yields a shorter pulse with strong frequency chirp



Conclusion and work extensions

Results:

+ Green function expression for FEL with energy chirp and curvature on the electrons starting from:

- ***Seed***
- ***Bunching***

+ Evaluation of the bunching at the radiator entrance in both amplitude and phase.

Work extensions:

+ Estimate the effect of resistive wall and roughness wakefields in the radiator

+ Full treatment with uncorrelated energy spread