Deep saturation dynamics in a Free Electron Laser

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After saturation, what's next ?

The model: the Colson-Bonifacio equations
(1-D [no transverse dynamics],
monochromatic, no slippage or space-charge)

$$\dot{\theta}_j = p_j$$
$$\dot{p}_j = -2\sqrt{I}\cos(\theta_j - \phi)$$
$$\dot{\phi} = \frac{1}{\sqrt{I}}\sum_j \sin(\theta_j - \phi)$$
$$\dot{I} = 2\sqrt{I}\sum_j \cos(\theta_j - \phi)$$

θ_j : **position** of part. j

p_j : normalized **momentum**

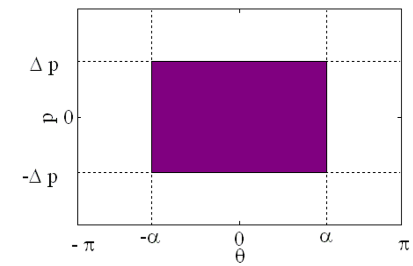
I : wave **intensity**

θ_j : wave ponderomotive **phase**

W. Colson, Phys. Lett. A **59**, 187 (1976)

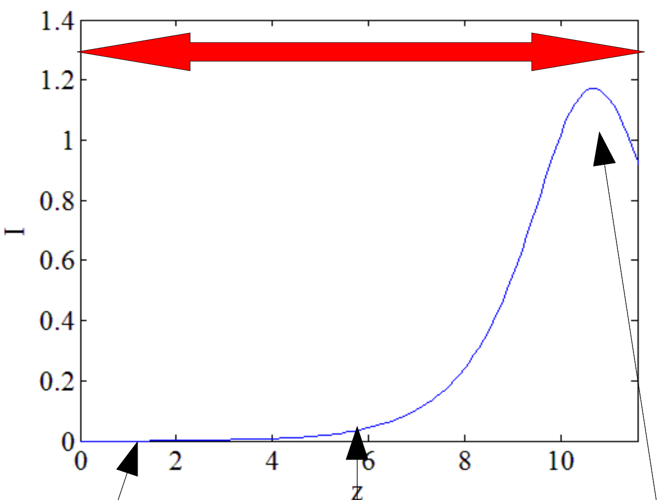
R. Bonifacio, F. Casagrande and C. Pellegrini, Opt. Commun. **61**, 55 (1987).

Initial conditions: water-bag for the particles, no wave



$$\epsilon = \frac{\Delta p^2}{6}$$
$$b_0 = \frac{\sin\alpha}{\alpha}$$

Evolution of the FEL intensity

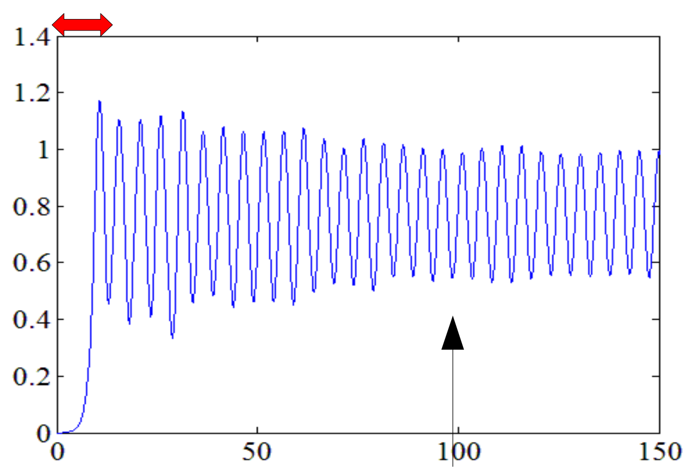


„Real“ FEL

Lethargy

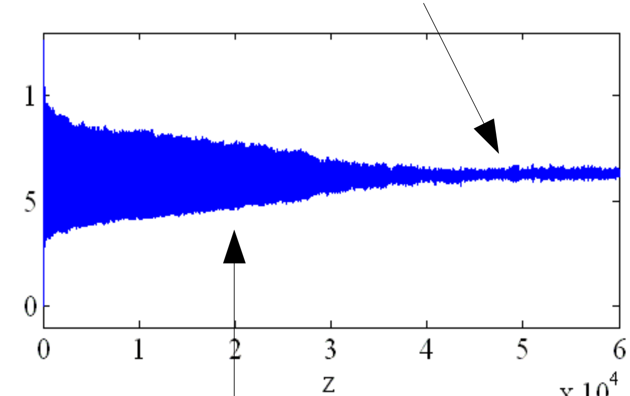
Linear regime

Saturation



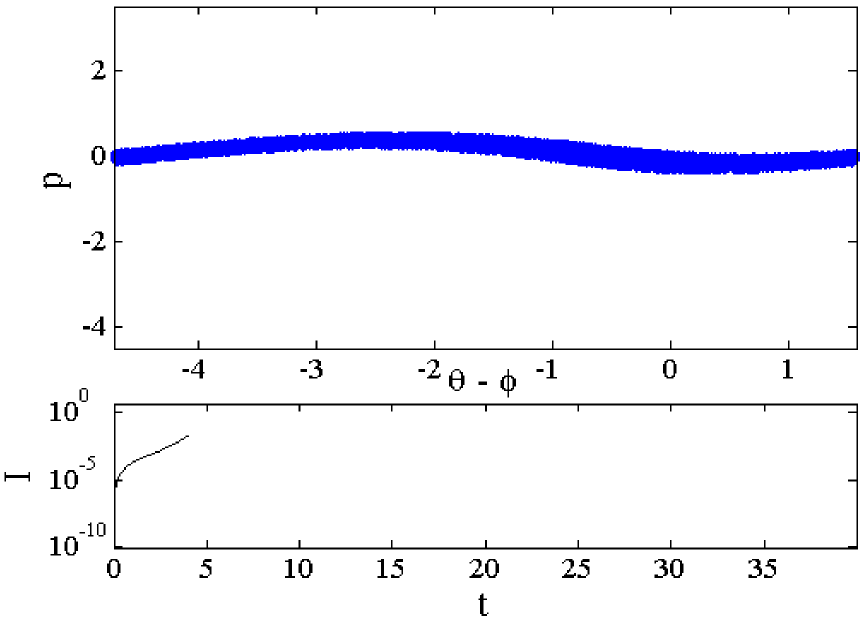
Quasi-Stationary States (QSS)

Thermodynamic equilibrium

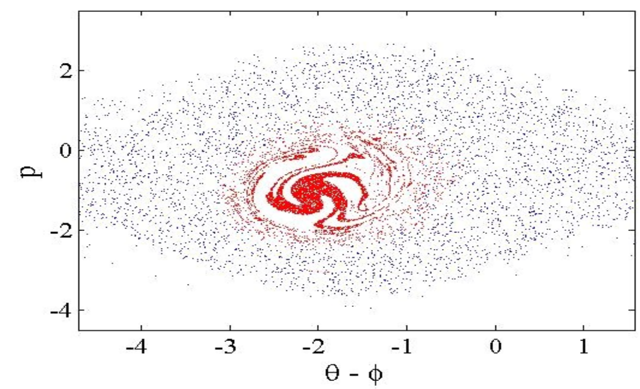


Slow relaxation

Particles dynamics

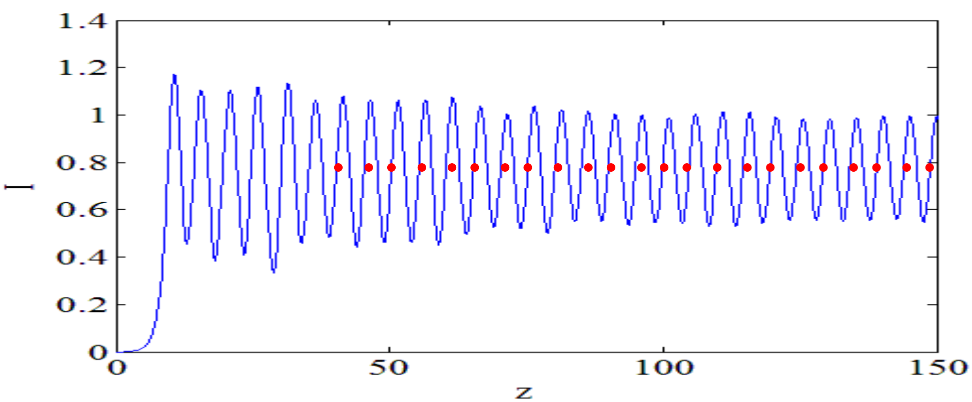


- Collective motion of a cluster of particles (synchrotron motion)
- Oscillation of the wave intensity

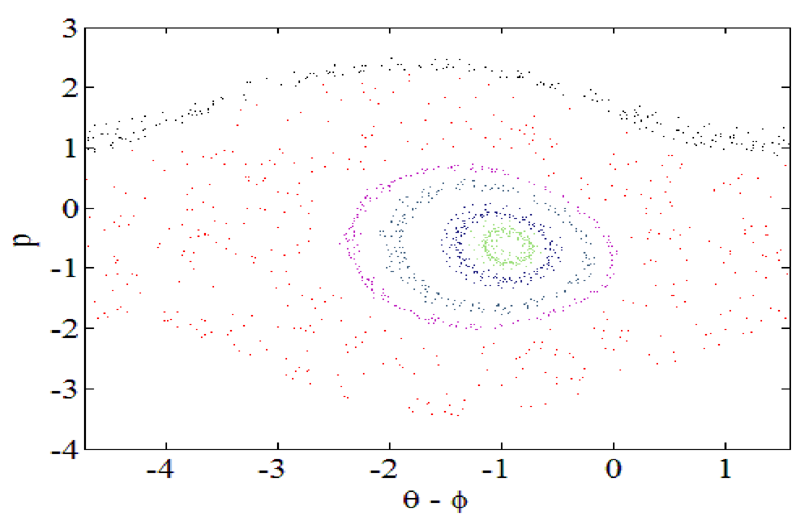


Snapshot of the particles phase-space

„Poincaré section“ technique

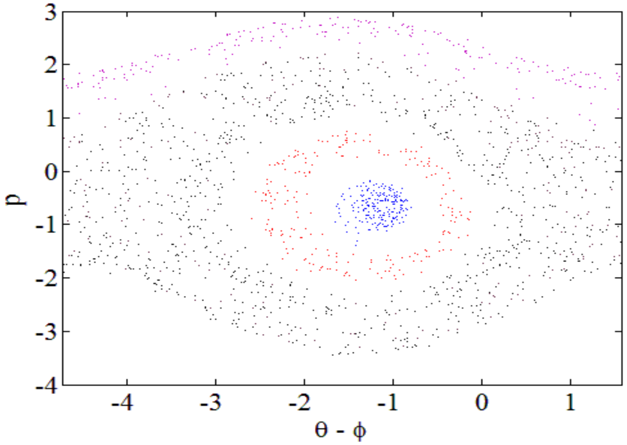


Each time the intensity crosses its average value, the position and momentum of particles is recorded

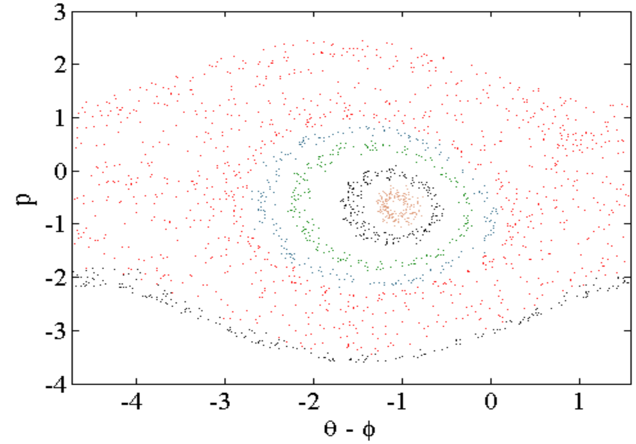


The Poincaré section reveals that **most particles** actually nearly evolve **on a 1-D structure** of the *(θ,p)* space

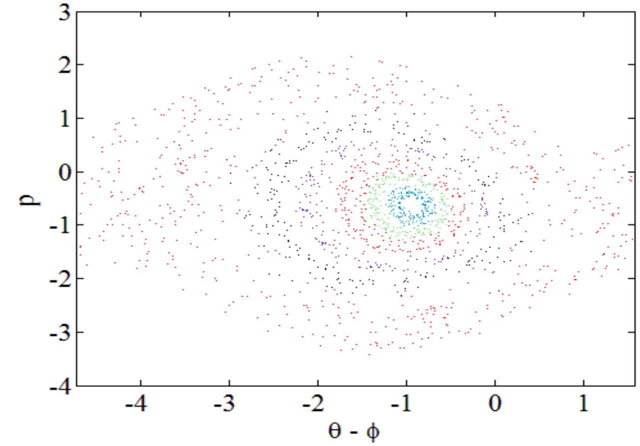
Convergence to a low-dimensional dynamics



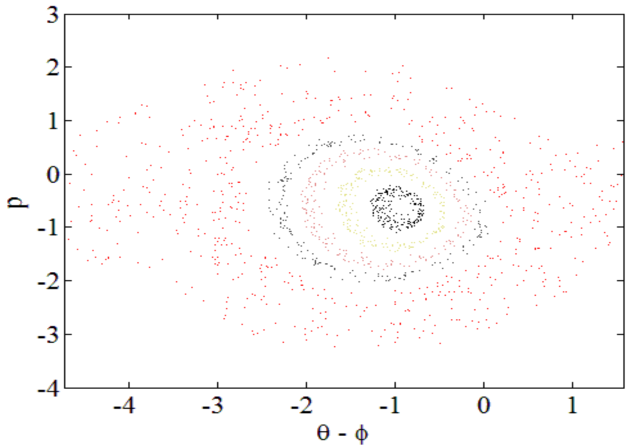
N = 1000



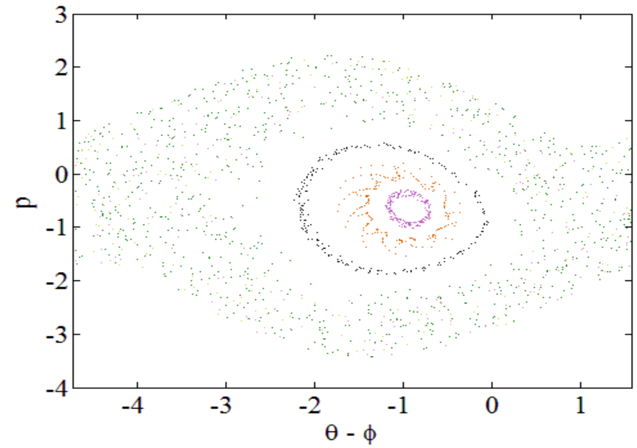
N = 3000



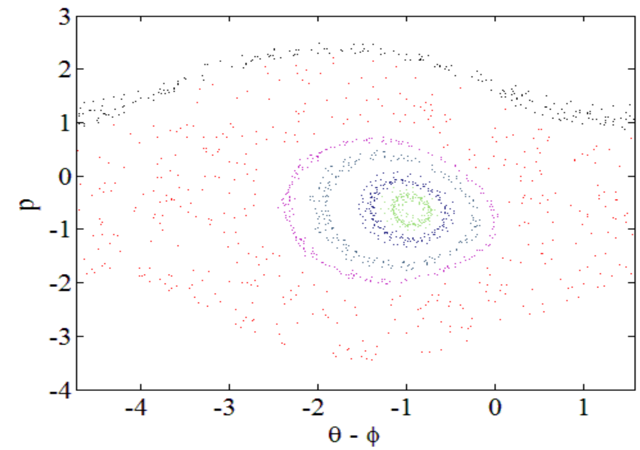
N = 10000



N = 30000



N = 100000



N = 300000

The more particles there are,

the closer to a (quasi-)periodic orbit the particle trajectory is,

the closer to a low-dimensional dynamics the system is.

Vlasov dynamics

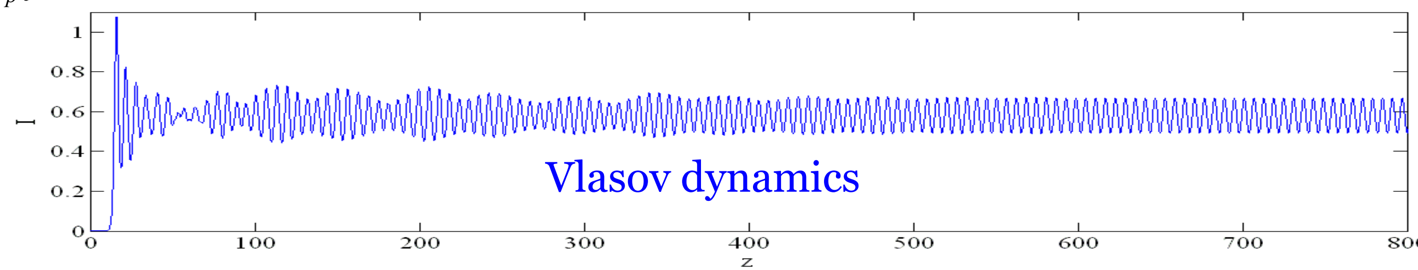
Vlasov equation: the particles are described by a density function

In the $N \to \infty$ limit, the N-body system becomes equivalent to the Vlasov one

$$\dot{f}(\theta,p) = -p\,\partial_\theta f + 2\sqrt{I}\cos(\theta-\phi)\,\partial_p f$$

$$\dot{\phi} = \frac{1}{\sqrt{I}} \iint d\theta\, dp\, f \sin(\theta-\phi)$$

$$\dot{I} = 2\sqrt{I} \iint d\theta\, dp\, f \cos(\theta-\phi)$$

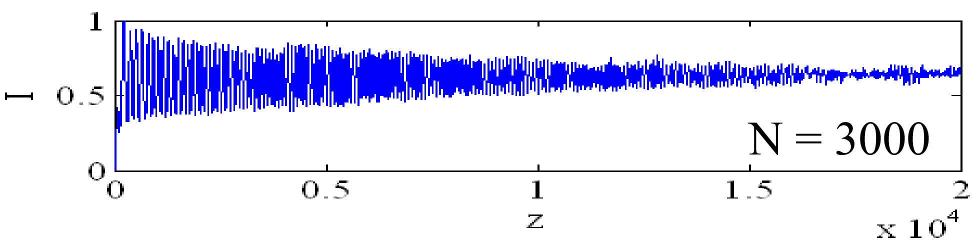


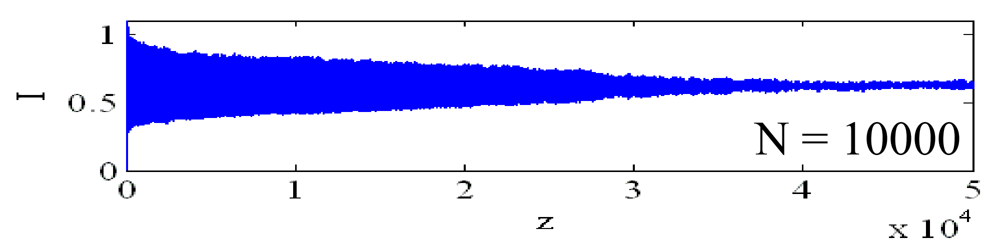
The Vlasov eq. has some stationary solution, in which the system is trapped forever

The *Quasi-Stationary States* occur when the N-body system starts close to these solutions, and then sticks to them

The more particles, the closer to Vlasov dynamics, the longer the sytem will stick to the Vlasov stationary solution

Typically, the lifetime of QSS scales as N^{α}, with $\alpha > 0$





V. Latora, A. Rapisarda, C. Tsallis, *Non-Gaussian equilibrium in a long-range Hamiltonian system*, Phys. Rev. E **64**, 056134 (2001).

Y. Y. Yamaguchi *et al.*, *Stability criteria of the Vlasov equation and quasi-stationary states of the HMF model*, Physica A **337**, 36 (2004).

Lynden-Bell (LB) statistical approach: A prediction of Vlasov stationary states

A principle of maximization of entropy, under constraints of mass, energy and momentum conservation

$$s(\bar f) = -\int dp\, d\theta \left[\frac{\bar f}{f_0}\ln\frac{\bar f}{f_0} + \left(1-\frac{\bar f}{f_0}\right)\ln\left(1-\frac{\bar f}{f_0}\right)\right]$$

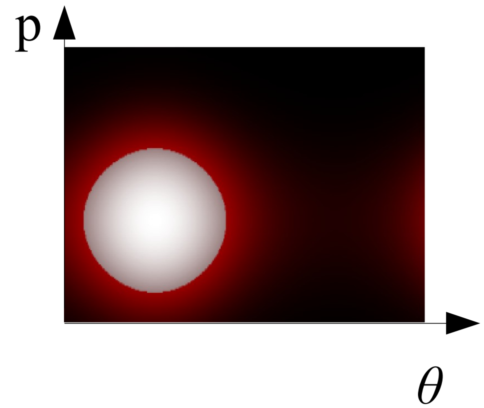
f_0 : initial distribution

$I_0 = 0$

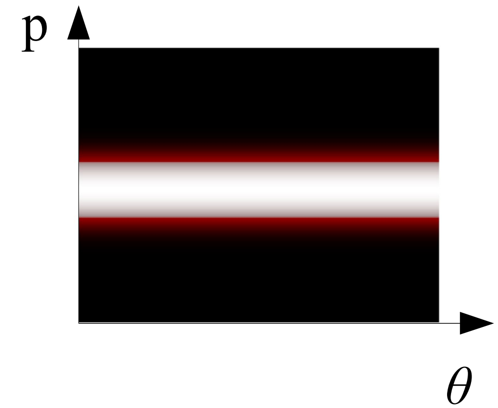
$\bar f$: LB solution

$\bar I$: saturated intensity

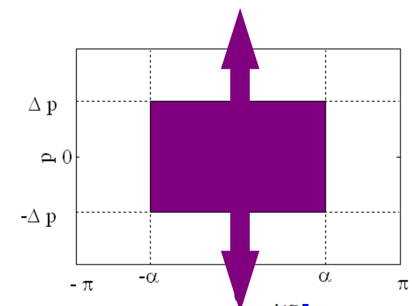
Two resulting solutions $(\bar I, \bar f)$



LBA, with $\bar I > 0$

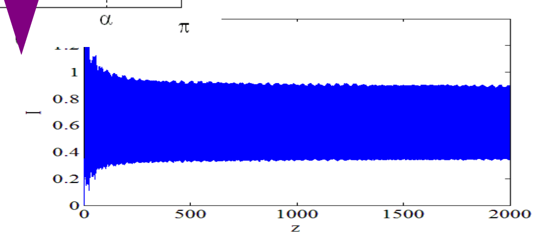


LBO, with $\bar I = 0$

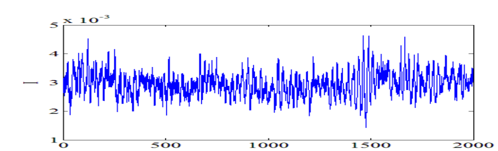


FEL dynamics:

- For a low energy spread, the wave grows and the particles reach a state close to LBA configuration



- For a large energy spread, nearly no growth of the wave the electrons stay homogeneously distributed



D. Lynden-Bell, *Statistical mechanics of violent relaxation in stellar systems*, Mon. Not. R. Astr. Soc. **136**, 101 (1967).

Phase transition: from high-gain to no-gain

Expected scenario (observed for a similar model):

- The system chooses the solution with the largest entropy
- From high-gain to no-gain, the phase transition occurs when

the LBO entropy gets larger than the LBA one ($S_{LBO} > S_{LBA}$)

A. Antoniazzi *et al.*, Nonequilibrium Tricritical Point in a System with Long-Range Interactions, Phys. Rev. Lett. **99**, 040601 (2007).

However, in the FEL:

- The situation is more tricky: LBA entropy always larger than LBO one
- There is a „dynamical“ switch from one solution to the other which has still to be explained

P. de Buyl *et al.*, *Out-of-equilibrium mean-field dynamics of a model for wave-particle interaction*, Phys. Rev. ST Accel. Beams **12**, 060704 (2009).

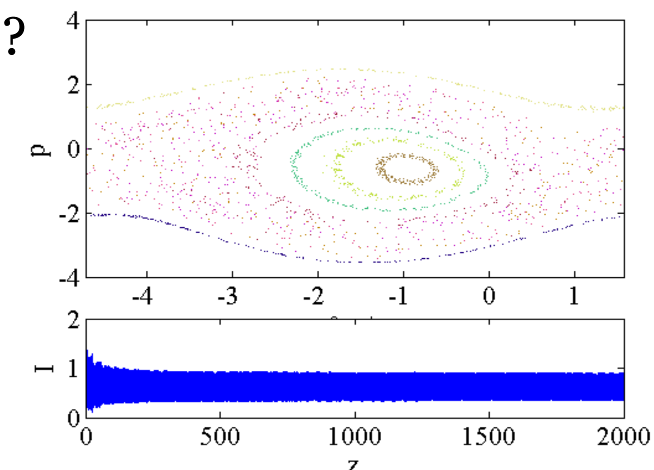
Conclusion & Perspectives

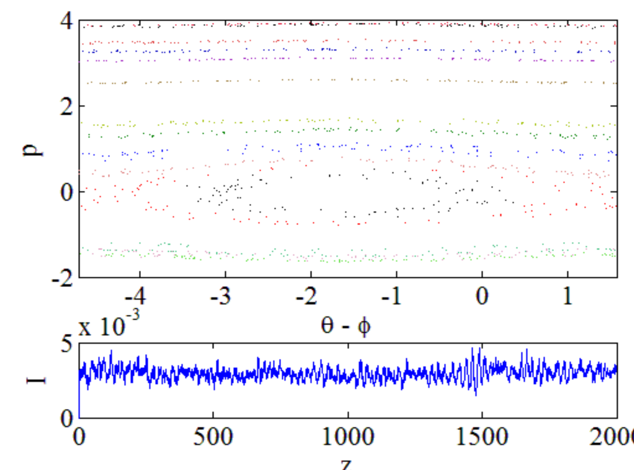
• What is the phase transition mechanism in the FEL ?

→ Change of stability of the Vlasov stationary solutions ? (in progress)

→ Adequacy of this entropy to investigate the transition ?

• As for the dynamics, which bifurcations occur in the particle phase-space during the phase transition ?





• The LB statistical approach does not provide any information on dynamics, only on statistical values: Is it possible to combine this method with the Poincaré section technique ?

•

• Toward a more realistic FEL by including extra effects ?