Towards Sub-Ångström Coherent Light Sources: The Quantum FEL

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Outline

1. Introduction
2. Classical FEL and SASE
3. Quantum FEL
4. Harmonic Generation in a Quantum FEL
5. Summary

1. Introduction

We consider classical and quantum regimes of SASE-FEL operation

The parameter we use to identify the different regimes is the "quantum FEL parameter"

$$\bar{\rho} = \rho\left(\frac{mc\gamma}{\hbar k}\right) = \frac{\sigma(P)}{\hbar k}$$

where $$\rho = \frac{1}{2\gamma}\left(\frac{I}{I_A}\right)^{1/3}\left(\frac{\lambda_L a_W}{4\pi\sigma_{Beam}}\right)^{2/3}$$

$\bar{\rho} >> 1$: Classical regime

$\bar{\rho} < 1$: Quantum effects

2. Classical FEL and SASE

In usual classical FEL theory, photon recoil momentum is neglected and electron-light momentum exchange is continuous.

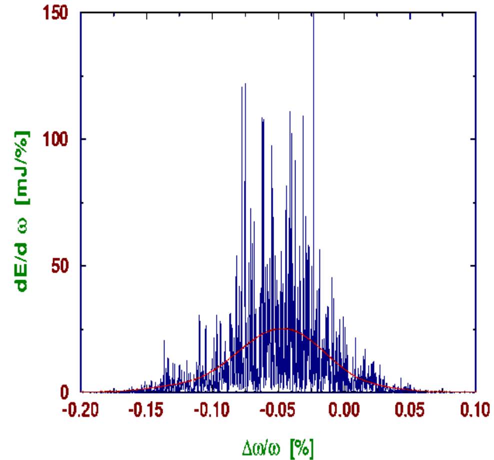
Classical induced momentum spread ($\gamma_R mc\rho$) >> one-photon recoil momentum($\hbar k$)

i.e. $\bar{\rho} >> 1$ where $\bar{\rho} = \frac{mc\gamma_R \rho}{\hbar k}$

Classical SASE-FELs produce VUV (DESY) and X-ray (LCLS) radiation :

High power

Broad spectrum / poor temporal coherence



3. Quantum FEL (QFEL)

We now consider the case where

Classical induced momentum spread ($\gamma_R mc\rho$) < one-photon recoil momentum($\hbar k$)

i.e. $\bar{\rho} < 1$ where $\bar{\rho} = \frac{mc\gamma_R \rho}{\hbar k}$

Electron-radiation momentum exchange is now discrete i.e.

$$\Delta P = n\hbar k$$

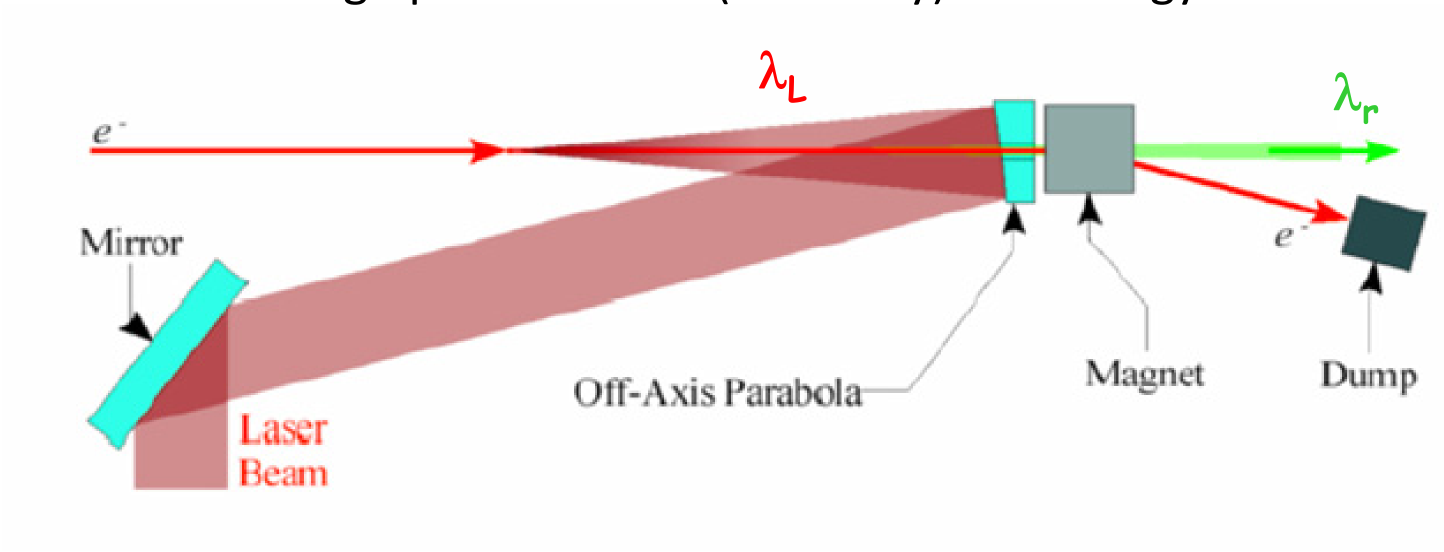
so a quantum model of the electron-radiation interaction is required.

SASE-QFEL may produce radiation with lower power than classical SASE-FELs, but better temporal coherence, even at sub-A wavelengths.

3. Quantum FEL (QFEL)

Conceptual design of a QFEL : Similar to (COLLECTIVE) Compton back-scattering

High power laser + (relatively) low energy e-beam



$$\lambda_r = \frac{\lambda_L}{4\gamma^2}\left(1 + a_0^2\right) \quad \lambda_L = 1\mu m$$

If $\gamma \cong 200$ ($E \cong 100$ MeV) $\Rightarrow \lambda_r \cong 0.3$ Å !

$$a_0 = 2.4\sqrt{P_L(TW)}\,\frac{\lambda_L}{R}$$

$$P_L \approx 100TW \;, \lambda_L = 1\,\mu m, R = 10\,\mu m \Rightarrow a_0 \approx 2$$

3. Quantum FEL (QFEL) - Model

Procedure :

Describe N particle system as a Quantum Mechanical ensemble

Write a Schrödinger-like equation for macroscopic wavefunction: Ψ

Details in :

G. Preparata, Phys. Rev. A **38**, 233(1988)

R.Bonifacio, N.Piovella, G.Robb, A. Schiavi, PRST-AB **9**, 090701 (2006)

3. Quantum FEL (QFEL) – 1D Model

Using scaled variables :

$$\bar{z} = \frac{z}{L_g} \qquad z_1 = \frac{z - v_z t}{L_c} \qquad \theta = \frac{2\pi}{\lambda}(z - v_z t) \qquad L_g = \frac{\lambda_L}{8\pi\rho},\; L_c = \frac{\lambda_r}{4\pi\rho}$$

Electron dynamical equations

$$\frac{d\theta_j}{d\bar{z}} = \bar{p}_j$$

$$\frac{d\bar{p}_j}{d\bar{z}} = -(A e^{i\theta_j} + c.c.)$$

Single electron Hamiltonian

$$H = \frac{p^2}{2\bar{\rho}} - i\bar{\rho}\left(A e^{i\theta} - \text{c.c.}\right) \qquad [\theta, p] = i \qquad p = -i\frac{\partial}{\partial\theta}$$

Maxwell-Schrodinger equations for electron wavefunction Ψ and classical field A

$$i\frac{\partial\Psi}{\partial\bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2\Psi}{\partial\theta^2} - i\bar{\rho}\left(A e^{i\theta} - \text{c.c.}\right)\Psi$$

$$\frac{\partial A}{\partial\bar{z}} + \frac{\partial A}{\partial z_1} = \int_0^{2\pi} |\Psi|^2 e^{-i\theta}\, d\theta + i\delta A$$

bunching

3. Quantum FEL (QFEL) –1D Model

Assuming electron wavefunction is periodic in θ :

$$\Psi(\theta, \bar{z}, z_1) \propto \sum_{n=-\infty}^{\infty} c_n(\bar{z}, z_1) e^{in\theta}$$

$|c_n|^2 = p_n$ = Probability of electron having momentum $n(\hbar k)$

Only discrete values of momentum are possible : $p_z = n\,(\hbar k)$, n=0,±1,..

M-S equations in terms of momentum amplitudes

$$\frac{\partial c_n}{\partial \bar{z}} = -i\frac{n^2}{2\bar{\rho}} c_n - \bar{\rho}\left(A c_{n-1} - A^* c_{n+1}\right)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial \bar{z}_1} = \sum_{n=-\infty}^{\infty} c_n c^*_{n-1} + i\delta A$$

bunching

3. Quantum FEL (QFEL) –Linear Analysis

Linearising and looking for solutions : $A \propto e^{i(\lambda \bar{z} + \omega z_1)}$

$$(\lambda - \Delta)\left(\lambda^2 - \frac{1}{4\bar{\rho}^2}\right) + 1 = 0$$

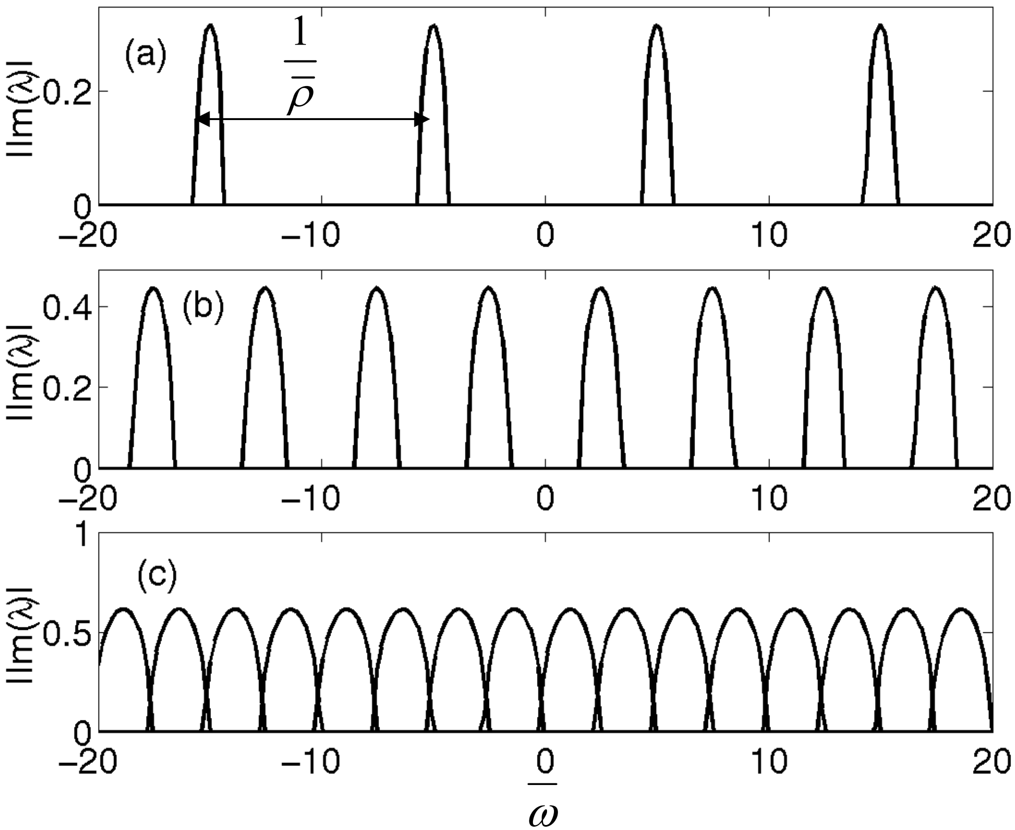
Quantum term

$$\left(\Delta = \frac{n}{2\bar{\rho}} - \bar{\omega}\right) \qquad \left(\bar{\omega} = \frac{\omega - \omega_{sp}}{2\rho\omega_{sp}}\right)$$

$\bar{\rho} = 0.1$

$\bar{\rho} = 0.2$

$\bar{\rho} = 0.4$



$$\bar{\omega}_n = \frac{1}{2\bar{\rho}}(2n - 1)$$

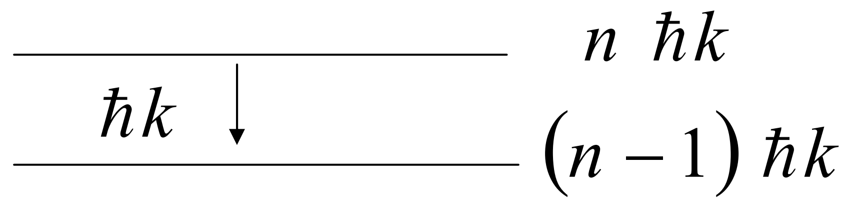
Spacing = $\frac{1}{\bar{\rho}}$

Width= $4\sqrt{\bar{\rho}}$

Continuous limit : $4\sqrt{\bar{\rho}} > \frac{1}{\bar{\rho}}$ i.e. $\bar{\rho} > 0.4$

3. QFEL Physics

Momentum-energy levels:
($p_z=n\hbar k$, $E_n \propto p_z^2 \propto n^2$)



$$\varpi_n \propto E_n - E_{n-1} \propto \frac{1}{\bar{\rho}}\left(n - \frac{1}{2}\right)$$

Transition frequencies equally spaced by $1/\bar{\rho}$ **with width** $4\sqrt{\bar{\rho}}$

Increasing $\bar{\rho}$ **the lines overlap for** $\bar{\rho} > 0.4$

CLASSICAL REGIME: $\bar{\rho} >> 1$

→ Many transitions
→ broad spectrum

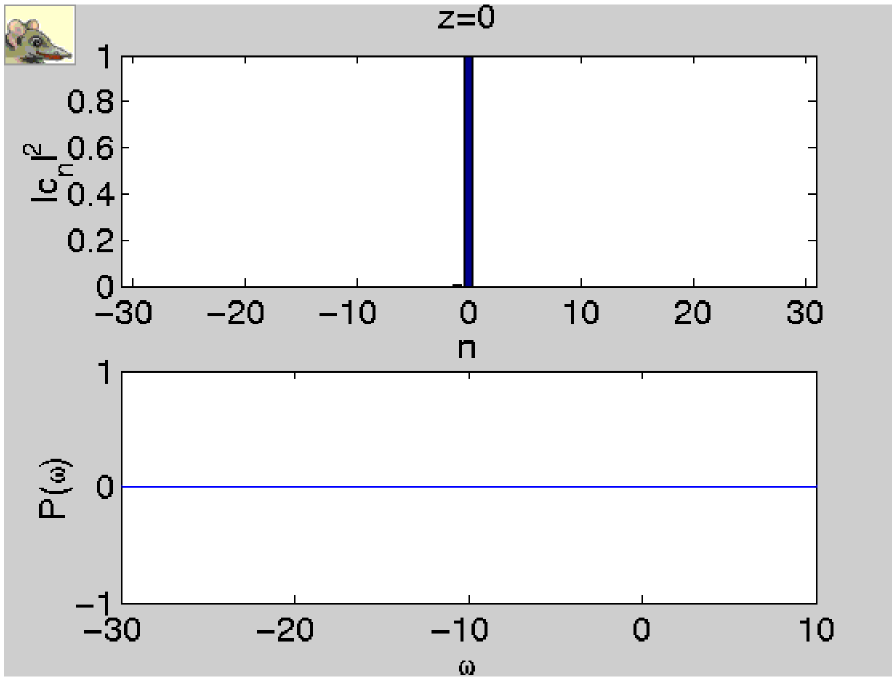
QUANTUM REGIME: $\bar{\rho} \leq 1$

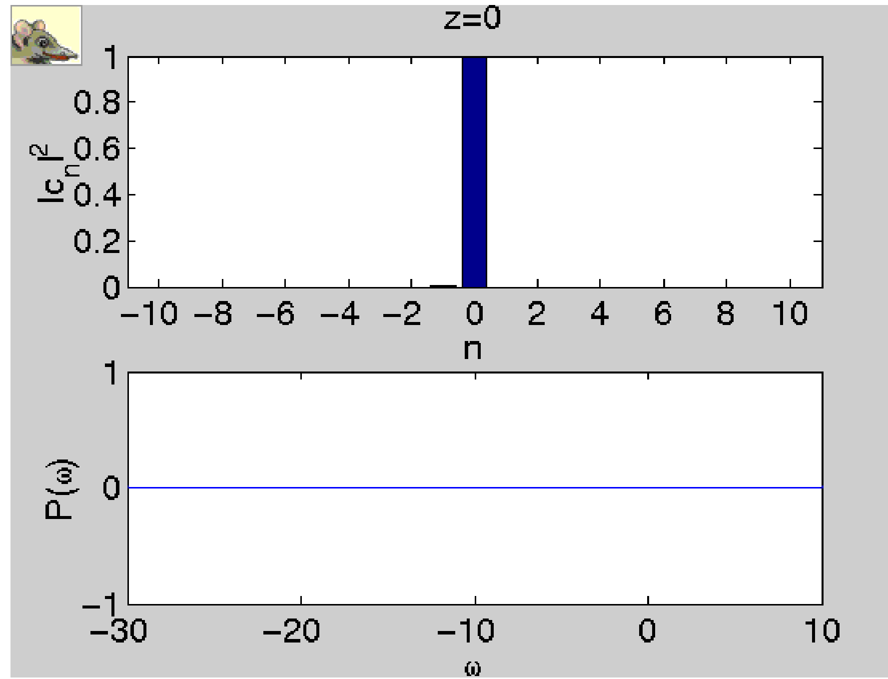
→ a single transition
→ narrow line spectrum

3. QFEL Physics – Momentum distribution evolution

CLASSICAL REGIME: $\bar{\rho}=5$

QUANTUM REGIME: $\bar{\rho}=0.1$



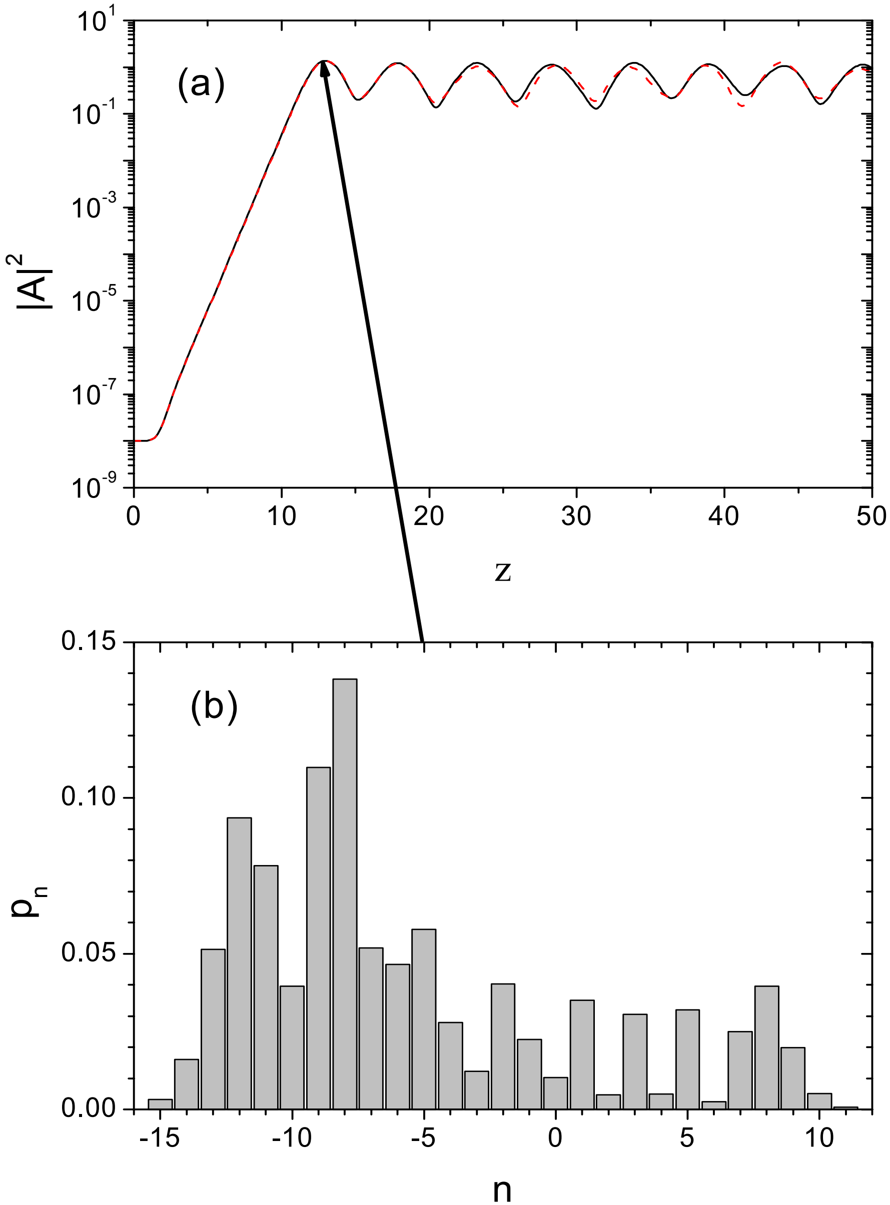


Classical regime:
both n<0 and n>0 occupied

Quantum regime:
only n<0 occupied

steady-state evolution: $\left(\frac{\partial A}{\partial z_1} = 0 \right)$

$\bar{\rho}$=10, no propagation



classical limit
is recovered for

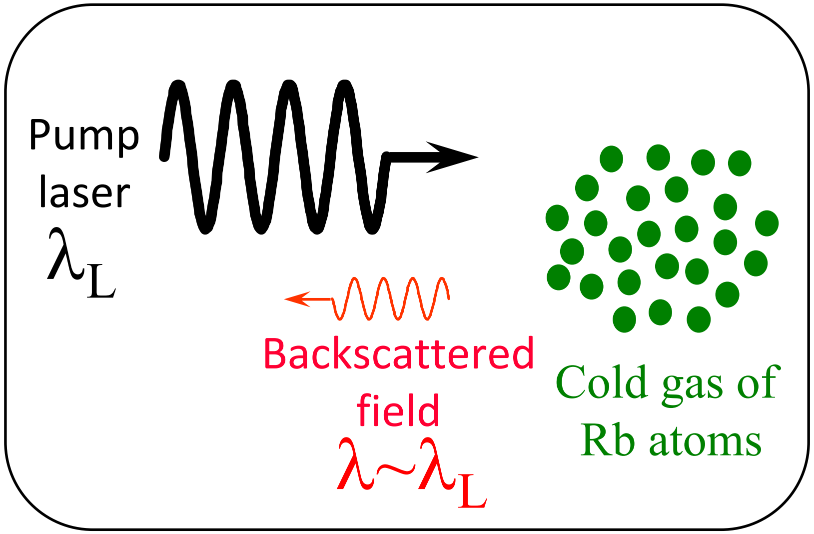
$$\bar{\rho} >> 1$$

many momentum states occupied,
both with n>0 **and** n<0

Evolution of field, <p> etc.
is identical to that of a classical particle simulation

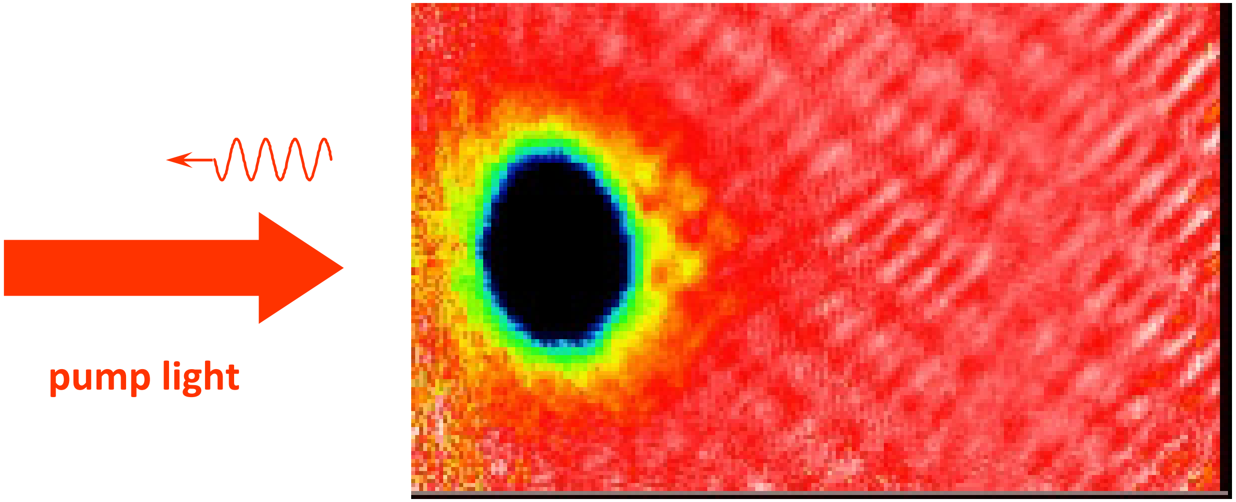
3. QFEL Physics – Evidence of quantum dynamics

Behaviour similar to quantum regime of QFEL observed in experiments involving Backscattering from cold atomic gases (Collective Rayleigh backscattering or Collective Recoil Lasing (CRL))



QFEL and CRL described by same theoretical model

Main difference from QFEL – negligible Doppler upshift of scattered field

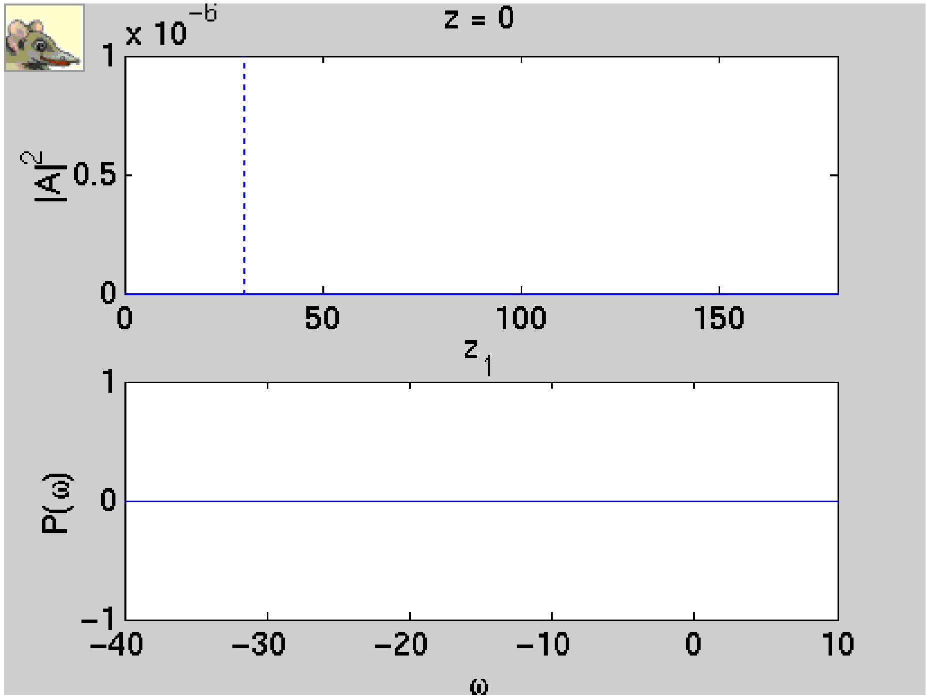


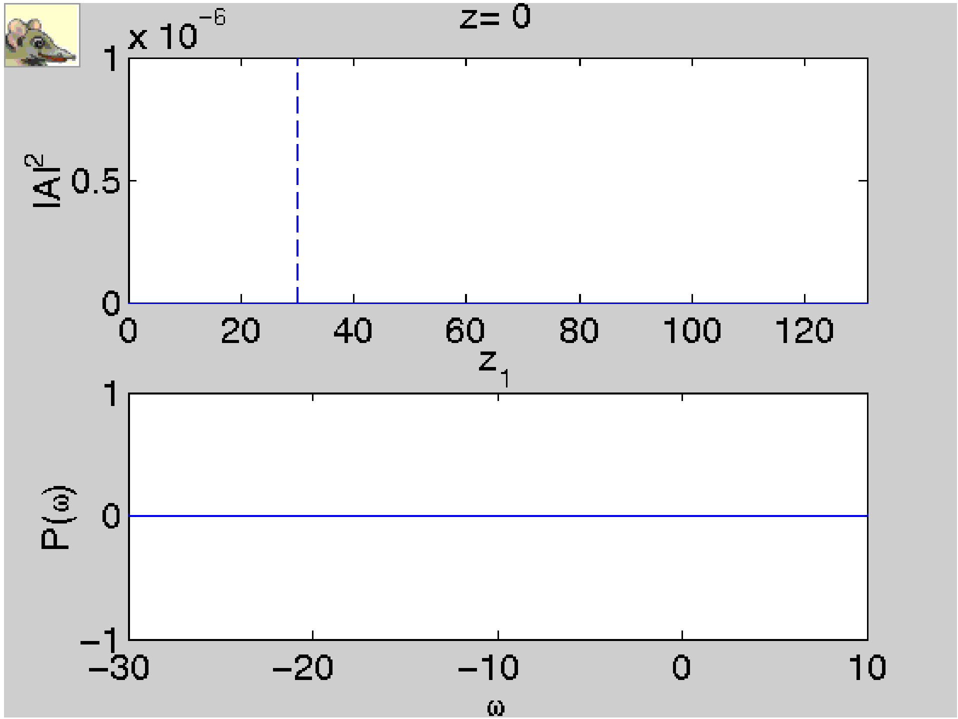
See Bonifacio et al., Optics Comm. **233**, 155(2004) and Fallani et al., Phys. Rev. A 71, 033612 (2005)

3. QFEL Physics - Quantum "Purification" of SASE spectrum

quantum regime $(\bar{\rho} = 0.05)$

classical regime $(\bar{\rho} = 5)$



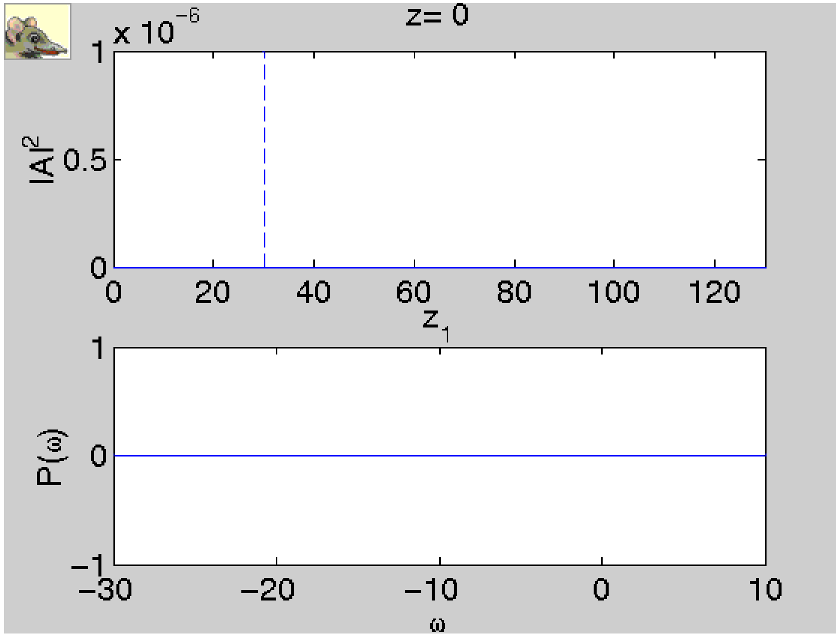


$L / L_c = 30$

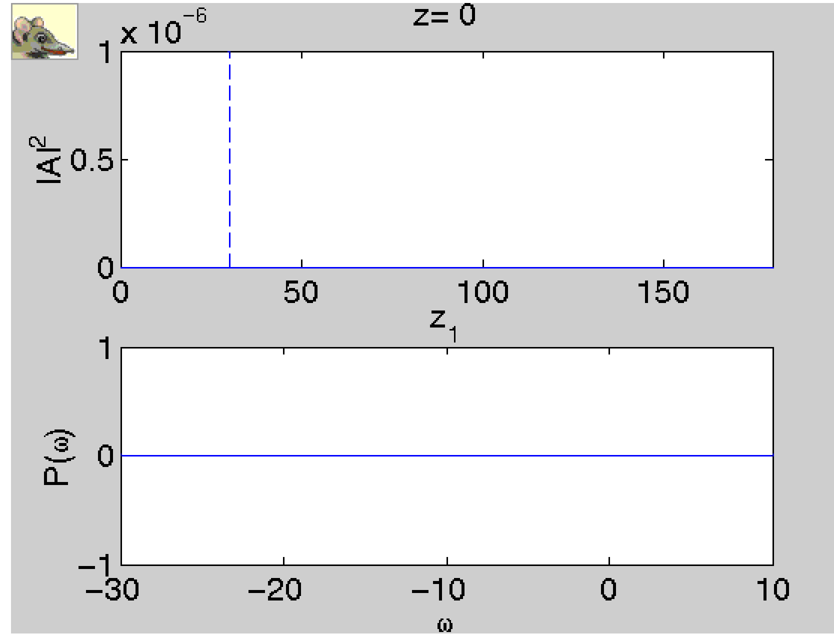
R.Bonifacio, N.Piovella, G.Robb, NIMA **543**, 645 (2005)

$$\overline{\omega}_n = (2n-1)/2\overline{\rho} \quad [n = 0,-1,..]$$

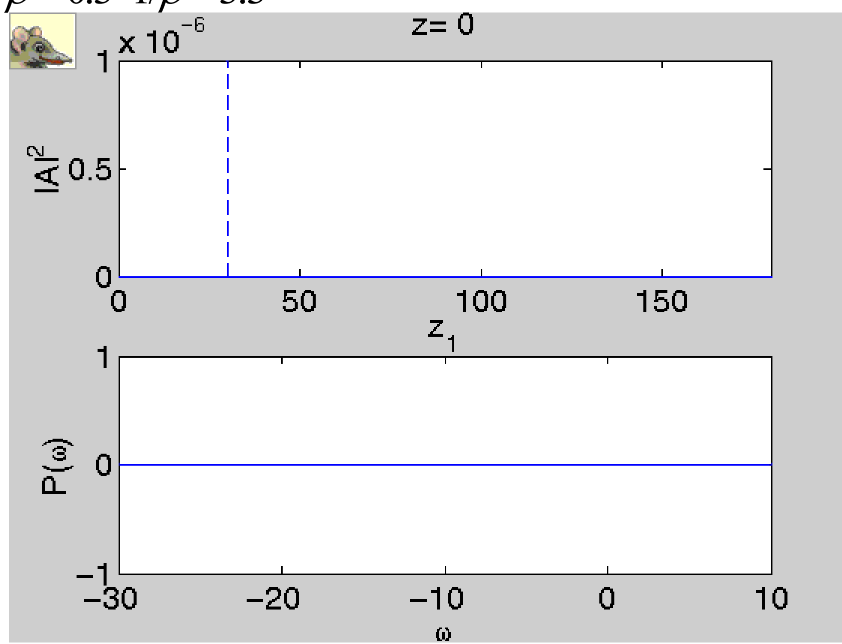
$\bar{\rho} = 0.1 \;\; 1/\bar{\rho} = 10$



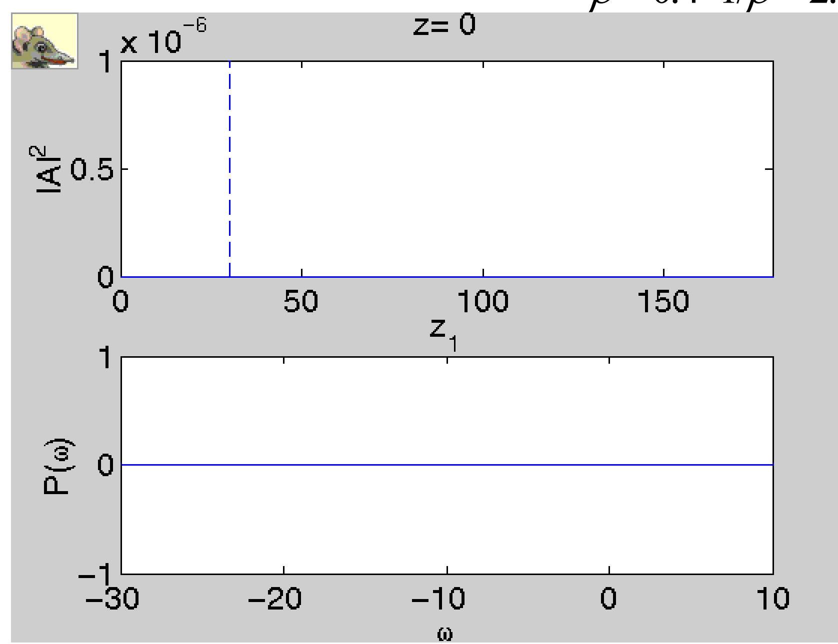
$\bar{\rho} = 0.2 \;\; 1/\bar{\rho} = 5$



$\bar{\rho} = 0.3 \;\; 1/\bar{\rho} = 3.3$



$\bar{\rho} = 0.4 \;\; 1/\bar{\rho} = 2.5$



3. QFEL Requirements

Writing conditions for gain in terms of $\lambda_r, \lambda_L, \bar{\rho}$:

Energy spread < gain bandwidth:

$$\frac{\sigma(E)}{E} < 5 \cdot 10^{-4} \frac{\bar{\rho}^{3/2}}{\sqrt{\lambda_L \lambda_r (1 + a_0^2)}}$$

$\lambda_r \left(\mathring{A}\right), \lambda_L (\mu m)$

Beam current :

$$I(A) \approx 300 \frac{\sigma^2 \bar{\rho}^3}{\lambda_r^3 \lambda_L^2 a_0^2}$$

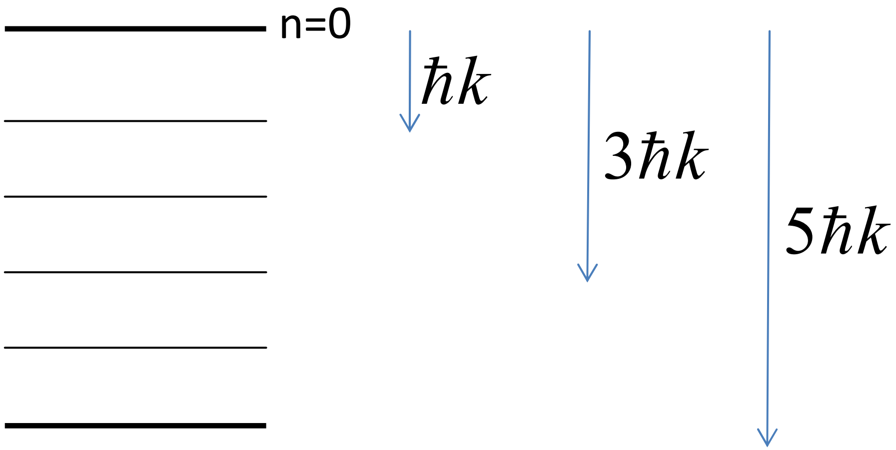
$\sigma(\mu m)$:e-beam radius

In order to generate Å or sub- Å wavelengths with $\lambda_L \approx 1 \mu m$ energy spread requirement becomes challenging for $\rho < 0.4$.

Is there a way of a reaching quantum regime without having to use $\bar{\rho} < 0.4$?

Bonifacio, Piovella, Cola, Volpe NIMA **577**, 745 (2007)

4. Quantum Harmonic Generation



Possible frequencies

$$h\omega \; (h = 1,3,5,..)$$

Larger momentum level separation for transitions involving harmonics ➡ quantum regime easier to attain?

Need to extend QFEL model to include harmonics

[G Robb NIMA A **593**, 87 (2008)]

4. Quantum Harmonic Generation – Model

Consider radiation field consisting of
fundamental + odd harmonics (h=1,3,5,...)

Following the same procedure as previously :

Maxwell-Schrodinger equations for electron wavefunction Ψ and radiation field A

$$i\frac{\partial\Psi(\theta,\bar{z})}{\partial\bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2\Psi}{\partial\theta^2} - i\bar{\rho}\sum_h \frac{F_h(\xi)}{h}\left(A_h e^{ih\theta_j} - \text{c.c.}\right)\Psi$$

$$\frac{\partial A_h}{\partial\bar{z}} + \frac{\partial A_h}{\partial z_1} = F_h(\xi)\int_0^{2\pi}|\Psi|^2 e^{-ih\theta}\, d\theta + ih\delta A_h$$

where $\xi = \frac{a_0^{\,2}}{2\left(1+a_0^{\,2}\right)}$ $\quad F_h(\xi) = (-1)^{\frac{h-1}{2}}\left(J_{\frac{h-1}{2}}(h\xi) - J_{\frac{h+1}{2}}(h\xi)\right)$

M-S equations in terms of momentum amplitudes

$$\frac{\partial c_n}{\partial\bar{z}} = -i\frac{n^2}{2\bar{\rho}}c_n - \bar{\rho}\sum_h \frac{F_h(\xi)}{h}\left(A_h c_{n-h} - A^*{}_h c_{n+h}\right)$$

$$\frac{\partial A_h}{\partial\bar{z}} + \frac{\partial A_h}{\partial\bar{z}} = F_h(\xi)\sum_{n=-\infty}^{\infty} c_n c^*{}_{n-h} + ih\,\delta A_h$$

4. Quantum Harmonic Generation

Repeating linear analysis for harmonics :

Frequency separation between gain lines: $\Delta = \dfrac{h}{\bar{\rho}}$

Gain bandwidth of each line : $\sigma = \dfrac{4\sqrt{\bar{\rho}}}{h}$

Discrete emission lines if width (σ) < separation (Δ) i.e. $\bar{\rho} \leq 0.4 h^{4/3}$

$h = 1 \Rightarrow \bar{\rho} < 0.4$ $\qquad h = 3 \Rightarrow \bar{\rho} < 1.7$ $\qquad h = 5 \Rightarrow \bar{\rho} < 3.4$

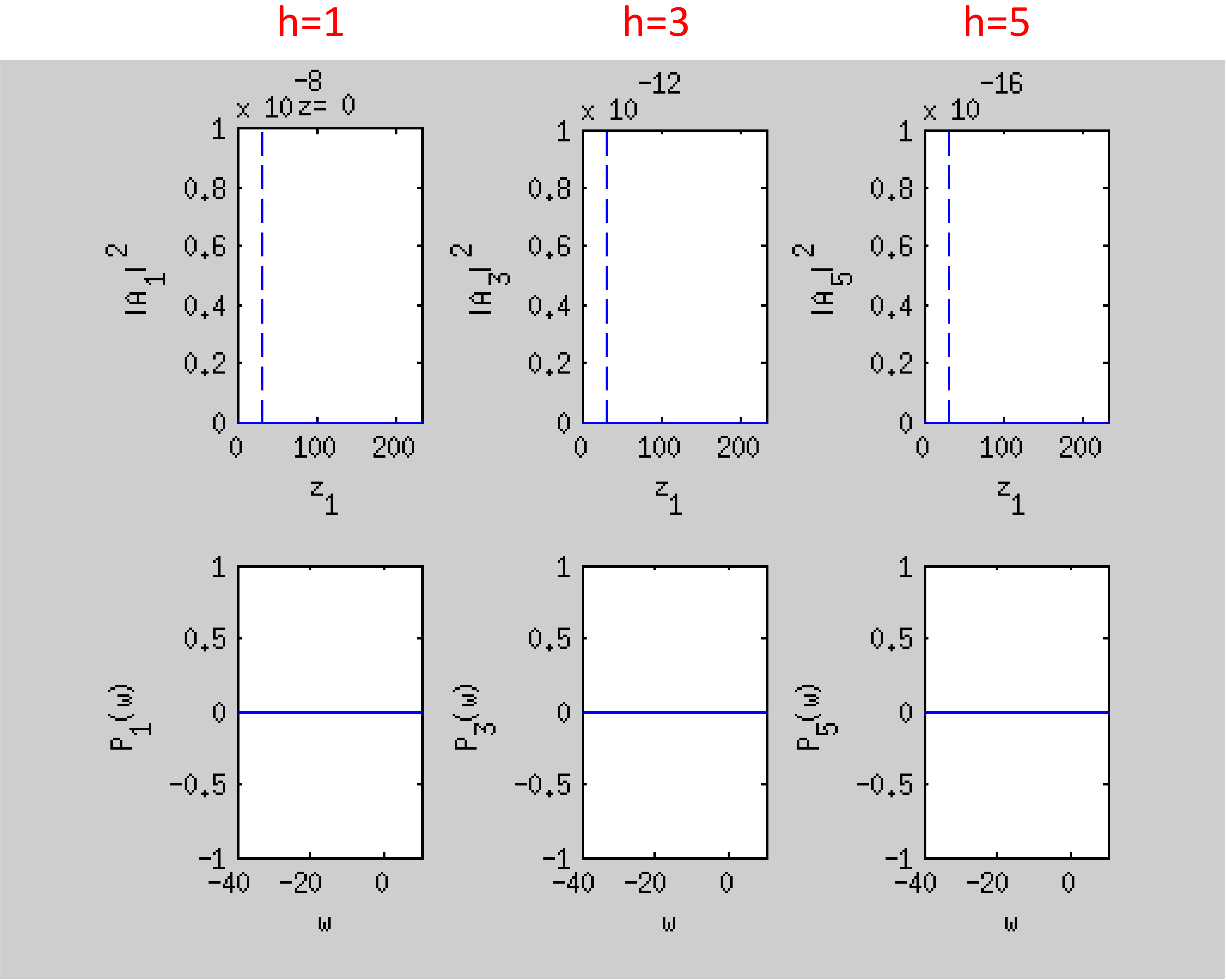
Possible classical behaviour for fundamental
BUT quantum for harmonics

e.g. when $\bar{\rho} = 1$:

h=1 - classical
h=3 – classical/quantum
h=5 – quantum

$\overline{\rho} = 1$

$a_0=2$



Parameters for QFEL

Electron beam

Q (pC)	1
τ (fs)	1.3
I (kA)	0.77
ε_n (mm mrad)	0.03
E (MeV)	100
σ (μm)	0.5
$\Delta E/E$	$4x10^{-4}$

Laser beam

λ_L (μm)	1
P_L (TW)	100
a_w	2
τ (ps)	3.4
R (μm)	12.6
L_{int} (mm)	1

QFEL beam (fundamental)

λ_r (A)	0.3
P_r (MW)	30
$\Delta\omega/\omega$	$7x\ 10^{-5}$
N_{phot}	$6x\ 10^6$
τ (fs)	1

Fundamental at 0.3 Å will be in classical regime

5th harmonic at 0.06 Å will be in quantum regime – coherent γ-rays

These parameters satisfy the condition to neglect diffraction

$$Z_L = \frac{4\pi R^2}{\lambda_L} > L_{\text{int}}$$

This restriction can be relaxed using a plasma channel (guiding) : Dino Jaroszynski

5. Conclusion

Quantum FEL - promising for extending coherent sources to sub-Å wavelengths

CLASSICAL SASE

needs:

GeV Linac

Long undulator (100 m)

yields:

High Power

Broad spectrum

QUANTUM SASE

needs:

100 MeV Linac

Laser undulator (λ~1μm)

Powerful laser (~100TW)

yields:

Lower power but better coherence

Narrow line spectrum

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