NEW SUPERCONDUCTIVE UNDULATOR DESIGNS FOR USE WITH LASER WAKEFIELD ACCELERATORS

G. Fuchert*, A. Bernhard, S. Ehlers, P. Peiffer, D. Wollmann, T. Baumbach,
University of Karlsruhe, Germany
R. Rossmanith, Forschungszentrum Karlsruhe, Germany

Abstract

In addition to large synchrotron radiation and FEL facilities, smaller and cheaper intense UV and X-ray facilities are needed (so-called table-top sources). The electron beam for such a table-top source could be provided by a laser wakefield accelerator. Despite dramatic improvements during the last years on stability and intensity of those devices, one disadvantage remains: the energy spread of the electron beam is in the range of several percent, leading to an energy spread of the photon beam emitted by a conventional undulator. In this paper ideas for new designs of superconductive undulators are presented, which can produce an almost monochromatic photon beam out of an electron beam with a relatively large energy spread.

INTRODUCTION

Current radio frequency (RF) based linear accelerators (linacs) are limited by the achievable acceleration gradient. Therefore, the linacs for the generation of X-ray FEL radiation can be a few kilometers long. While this is feasible for high precision experiments, it is not affordable for institutions like hospitals or companies. However, there is a new type of accelerator under development, the laser wakefield accelerator (LWFA). The electrons are accelerated by plasma waves, excited by a femtosecond laser pulse. This has one main advantage compared to classical RF accelerators. The plasma in a LWFA allows higher field strengths and, therefore, higher acceleration gradients in the range of GeV/cm. Unfortunately, it is not yet possible to stage several LWFA to obtain one long accelerator. This is currently limiting the electron energy to about 1 GeV. Another challenge is the energy spread of the electrons. Not only is it large ($\Delta E/E \approx 1 - 10\,\%$), but it even varies from shot to shot. Furthermore, the charge of one electron bunch is lower than 100 pC [1, 2]. Although significant improvements can be expected, the beam quality of a LWFA will not be able to compete with that of a classical RF accelerator in the near future. Therefore, the special properties of a LWFA, in particular the large energy spread, have to be considered when an undulator is designed for LWFA. A first design study is presented in this paper.

* golo.fuchert@student.kit.edu

MAIN IDEAS FOR NEW DESIGNS

Undulator Radiation and Consequences for New Designs

A monoenergetic electron beam produces a line spectrum when passing through an undulator; the spectrum consists of a fundamental frequency and higher harmonics, which arise from constructive interference of the photons, emitted by a single electron along the trajectory through the undulator. The corresponding wavelengths with the harmonic number n follow from the undulator equation

$$\lambda_n = \frac{\lambda_\mathrm{u}}{2n\gamma^2}\left(1 + \frac{K^2}{2}\right). \qquad (1)$$

λ_u is the period length of the undulator, γ the relativistic gamma, and K is the undulator parameter

$$K \approx 0.934 \cdot \lambda_\mathrm{u}[\mathrm{cm}] B_0[\mathrm{T}]. \qquad (2)$$

For a given λ_u, K depends on the amplitude (B_0) of the y component (perpendicular to the beam plane) of the sinusoidal magnetic field. In order to obtain short wavelengths with the low electron energy produced by a LWFA, it is necessary to build short period undulators [4]. This can be achieved with superconductive or cryogenic permanent magnet undulators. Superconductive undulators, which are the focus of this project, can be tuned by selecting the current and, with the ongoing development of new superconductive materials [5], the magnetic field strength should become significantly larger. It is also apparent from (1) that the large energy spread of the electron beam will inhibit monochromatic synchrotron radiation. That leads to line broadening, prevents coherence and, therefore, decreases the brilliance of the radiation.

Compensation of the Energy Spread

The possibility of generating synchrotron radiation with a LWFA and an undulator has been shown by Schlenvoigt et al. [3], but at first it seems to be impossible to get monochromatic synchrotron radiation using an electron beam with a large energy spread. The idea is now to disperse different electron energies spatially $\gamma \rightarrow \gamma(x)$ and vary one of the variables of K in order to to get a constant λ_n on the left hand side of (1). Mechanically, it seems to be very difficult to vary λ_u to match a varying γ. It should be easier to achieve this matching by a spatial variation of the field strength $B_0 \rightarrow B_y(x)$. The $\gamma(x)$ could be achieved with two dipole magnets of the same field strength, but with reversed field direction. To obtain the $B_y(x)$, the first attempt is a tilted undulator as shown in Fig. 1. The question is if $\gamma(x)$ from the dispersion magnets can match with $B_y(x)$.

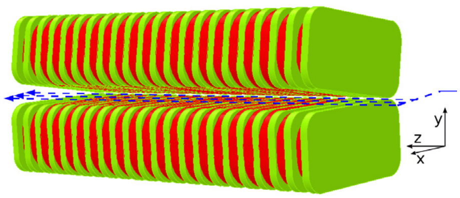

Figure 1: Tilted undulator design. The dashed lines represent trajectories for different electron energies.

Approximation of $\gamma(x)$ and $B_y(x)$

In the presence of the uniform magnetic field of a dipole magnet, a relativistic electron undergoes circular motion with the Larmor radius

$$r_{\mathrm{L}} = \frac{\gamma m_0 c}{eB} . \tag{3}$$

Therefore, different electron energies have different Larmor radii and will leave the dipole magnet with different deflection angles. After the second dipole magnet (with reversed field direction) all electron trajectories are parallel again, but different energies are now separated. The equations of motion for a given setup yield for small deviations

$$\gamma(x) \approx const. \cdot \frac{1}{x} . \tag{4}$$

The constant depends on the dipole strength and the distance between the two dipole magnets.

The field of the tilted undulator is approximated by determining the field separately for the upper and lower half. This can be treated as a magnetostatic problem in vacuum and, therefore, a scalar magnetic potential $\phi_{\mathrm{magn.}}$ with $\vec{B} = -\vec{\nabla}\phi_{\mathrm{magn.}}$ can be found. This can be shown using the time-independent Maxwell's equations in vacuum.

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{5}$$
$$\vec{\nabla} \times \vec{B} = 0 \tag{6}$$

(6) can be written as $\mathrm{rot}(\mathrm{grad}\phi = 0)$, leading to the scalar potential mentioned above. With this, (5) becomes Laplace's equation.

$$\vec{\nabla} \cdot \vec{\nabla}\phi_{\mathrm{magn.}} = \Delta\phi_{\mathrm{magn.}} = 0 \tag{7}$$

For the planar undulator geometry it is solved by the hyperbolic sine and cosine. From $\lim_{y\to\infty} B_y = 0$ follows

$$\phi_{\mathrm{magn.}} = const. \cdot \sin(k_{\mathrm{u}} z)(\cosh(k_{\mathrm{u}} y) \pm \sinh(k_{\mathrm{u}} y)) . \tag{8}$$

The plus/minus sign in (8) is for the upper/lower undulator half. $\tilde{B}$ is the a constant, mainly depending on the current of the superconductive wires and $k_{\mathrm{u}} = 2\pi/\lambda_{\mathrm{u}}$. Now, before adding both solutions, one is rotated by $\alpha/2$ and the other by $-\alpha/2$, leading to an approximate field for a tilted undulator with tilt angle α. Important is the y component of the magnetic field in the beam plane (i.e. at $y = 0$)

$$B_y = \tilde{B} \cdot \cos(\alpha) \cdot \exp(-k_{\mathrm{u}} x \sin(\alpha)) . \tag{9}$$

Not only is the matching important, but also the beam size should be at most a few millimeters. The latter requires a large $\partial B_y(x)/\partial x$ so that every γ matches a B_y in a small width. These questions can only be answered for specified beam parameters. As a realistic example, the electron energy $E_0 = 0.5$ GeV is chosen with a total energy spread of $10\,\%$ ($0.45 \leq E_e[\mathrm{GeV}] \leq 0.55$). For this, the tilt angle α has to be quite large (about 45°) in order to obtain a small beam size. The matching of $\gamma(x)$ and $B_y(x)$ is possible, but, because of the fast decay of the exponential function in (9), the beam position has to be at very small x values (millimeters). This is not feasible, due to the intersection of the two undulator halves at $x = 0$. To build an undulator without this intersection, but still not having problems with unavoidable field errors near the boundaries, the beam position would have to be far away from $x = 0$ (at least a few centimeters). This implies that the tilted undulator geometry cannot sufficiently compensate the energy spread.

MORE ADVANCED GEOMETRIES

The tilted undulator was modeled with infinite pole faces, crossing at $x = 0$. This intersection cannot be achieved in a real tilted undulator. But as it has been discussed, the beam has to be near the crossing point. A more advanced geometry needs to be symmetrical about the y-axis without intersecting at $x = 0$. In the cross section of an infinite tilted undulator, the equipotential lines of the pole faces can be described by two lines, one with a positive slope, and the other with a negative slope. Their graph coincides with the asymptotes of a hyperbola (see Fig. 2). A hyperbolic geometry would fulfill the requirement stated above.

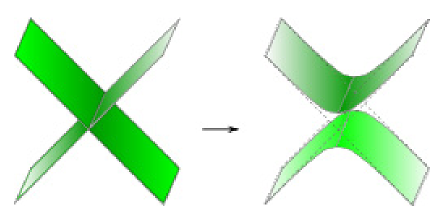

Figure 2: Transition from tilted undulator geometry to hyperbolic pole faces with crossing lines as asymptotes.

Approximate Field for the Hyperbolic Geometry

One possibility, and probably the most precise one, is to calculate the field numerically. However, with an analytical approximation it is easier to study how the eccentricity of the hyperbola affects the achievable beam size. This can be done in elliptic cylindrical coordinates [6], where the coordinate lines of the so-called u coordinate describe confocal elliptic cylinders and the coordinate lines of the orthogonal v coordinate describe confocal hyperbolic cylinders. Hence, two of these hyperbolic cylinders can be chosen to represent the pole faces of the undulator (one for the upper and one for the lower half). In these coordinates, Laplace's equation is separable and leads to Mathieu's differential equation (See [7] for a detailed discussion). In general, it is not possible to express the solutions in a convenient analytical form, but infinite series are needed. However, for this case and for small x values ($x \ll \sinh[1]$) an approximate solution in the beam plane can be found to be

$$B_y(x) = \tilde{B} \cdot \sin(k_u z) \frac{\exp(-q \cdot \mathrm{asinh}(x/a)^2)}{\sqrt{(x/a)^2 + 1}} . \quad (10)$$

$2a$ is the distance between the foci of the two hyperbolas, in this case determined by the gap height of the undulator and the eccentricity of the hyperbola; and q is a eigenvalue from Mathieu's differential equation. The slope of $B_y(x)$ is strongly affected by the geometry parameter a. Further analysis shows that the matching with $\gamma(x)$ is possible, but to achieve beam sizes in the order of millimeters, very small a-values are needed, corresponding to pole faces with large slopes. This leads to two problems. First, it would be very complicated to align the two undulator halves. A poor alignment would cause field errors and, therefore, the beam quality would decrease. Second, the bending radius of the superconductive wires at the extrema of the pole faces (at $x = 0$) is too small for the NbTi wires, used for today's superconductive undulators. That means that a hyperbolic undulator cannot be manufactured without the development of new superconductive materials.

Cylindrical Pole Face Geometry

A new pole geometry has to be chosen, considering that the minimum bending radius of the superconductive wires is an issue. A cylindrical pole shape (Fig. 3) seems to be most promising. The reason is that a cylinder can be chosen with a bending radius matched to the minimum bending radius of the wires. This provides the fastest decay of $B_y(x)$ without going below this minimal radius. Another advantage is that it is far easier to align the two undulator halves due to the rotation symmetry of the cylinder.

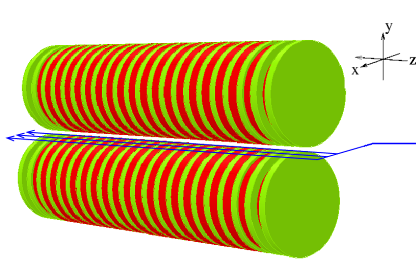

Figure 3: Undulator with cylindrical pole face geometry.

The solution of Laplace's equation for one half of a cylindrical undulator are the modified Bessel functions of the second kind K_n (MacDonald functions). The approximate field of a cylindrical undulator in the beam plane is

$$B_y(x) = \tilde{B} \cdot \sin\left(\frac{2\pi}{\lambda_\mathrm{u}} z\right) \left[\mathrm{K}_1(\mathrm{r}_\mathrm{l}(x)) + \mathrm{K}_1(\mathrm{r}_\mathrm{u}(x))\right] . \quad (11)$$

The functions $\mathrm{r}_{\mathrm{l,u}}(x) = \mathrm{r}_{\mathrm{l,u}}(x, y = 0)$ are the transformations from the Cartesian coordinate system to the cylindrical coordinates of the upper and lower undulator half. They depend on the radius of the cylinders r_cyl because it determines the location of the origins of the two cylindrical coordinate systems. For a given undulator, the beam size depends on the energy spread. For $r_\mathrm{cyl} = 1.3\,\mathrm{cm}$ ($1\,\mathrm{cm}$ minimum bending radius of NbTi plus the estimated thickness of the superconducting winding for a sufficient current), the achievable beam has a size between 1 and $2\,\mathrm{mm}$. Again, this could be improved with new superconductive materials, allowing higher field strengths (leading to larger $\partial B_y(x)/\partial(x)$) or smaller bending radii.

Properties of the Cylindrical Undulator

The cylindrical geometry seems to be both easy to align and able to provide reasonable beam sizes. In the following the match between $\gamma(x)$ and $B_y(x)$ is calculated. With the undulator equation (1) it is possible to translate $B_y(x)$ into $\gamma_\mathrm{undulator}(x)$. This describes what γ the undulator expects at a certain position x, in order to emit monochromatic radiation. Figure 4 indicates a good match of $\gamma(x)$ with $\gamma_\mathrm{undulator}(x)$, however, it is not perfect. The

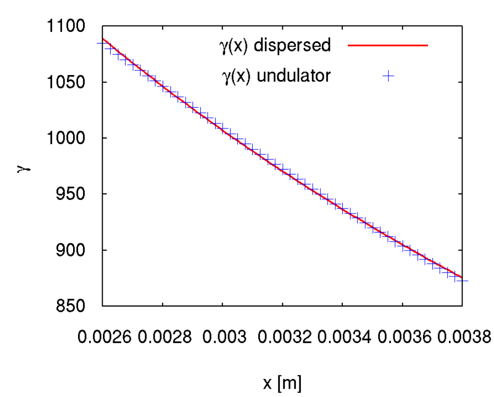

Figure 4: Matching of $\gamma_\mathrm{electrons}(x)$ and $\gamma_\mathrm{undulator}(x)$.

energy spread for the example energy range used before ($0.45 \leq E_e[\mathrm{GeV}] \leq 0.55$) is reduced to $1/5$ of the initial energy spread. Since the mismatch is significantly larger for lower energies, a filtering of the part with the lowest energy photons could reduce the energy spread to $1/10$ but decreases the radiation intensity. One should keep in mind that all this holds just for this example. The better the beam quality of LWFAs gets (i. e. the energy spread), the smaller the beam size and the better the matching of $\gamma(x)$ and $B_y(x)$.

Correction of the Electron Path

The spatial varying field $B_y(x)$ leads to a bend in the electron trajectories. Figure 5 shows the simulation of two electrons of different energy (0.45 GeV and 0.55 GeV) passing through a cylindrical undulator.

A simple dipole magnet cannot correct the trajectories for all energies because of the different Larmor radii. From the simulated trajectories follows the required correction field strength for every electron energy $B_{\mathrm{cor.}}(\gamma) = B_{\mathrm{cor.}}(x)$. Simulations show that, by adding the field of a dipole and a sextupole magnet as correction field, the trajectories can be straightened over the entire energy range (shown in Fig. 5). However, the required field values

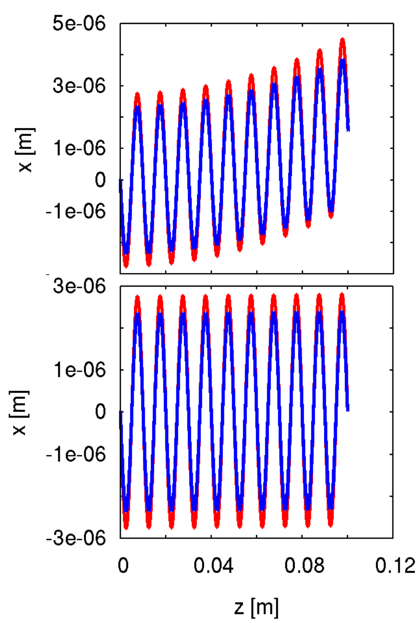

Figure 5: Electron trajectories for 0.45 GeV (blue) and 0.55 GeV (red) in a cylindrical undulator. Top: uncompensated, bottom: compensated.

are in the the same order of magnitude as the expected field errors of the undulator ($\sim 1/10\,\mathrm{mT}$). That does not need to be a problem because the field errors are usually distributed randomly. Whether this simple correction scheme is sufficient or not has to be shown experimentally. For the realization, a special magnetic setup, providing almost arbitrary multipole fields over the whole undulator length, is commercially available [8].

FUTURE AND ONGOING PROJECTS

The general construction of a superconductive undulator with a short period and a high magnetic field for a LWFA is the topic of a collaboration between the LMU Munich and the University of Karlsruhe. In this project the focus is on the development of new superconductive materials. Nb_3Sn (in cooperation with CERN) and high temperature superconductors are investigated.

An additional collaboration with the Institute for Optics and Quantum Electronics at the Friedrich-Schiller-University Jena, focuses on advanced undulator geometries.

For 2011, first experiments are planned at the JETI-Laser at the University of Jena. From today's point of view the experimental setup will include the LWFA, the dispersion section of dipole magnets, a cylindrical undulator, and a magnet setup for the correction field. The aim is to reduce the energy spread of the X-ray radiation to $1/10$, compared to the electron beam.

Additionally, further simulations are needed to decide if the achievable degree of monochromaticity is yet sufficient for coherent synchrotron radiation and compensates the large emittance due to the beam size.

SUMMARY

It has been shown theoretically that, by using new undulator designs, the energy spread of the synchrotron radiation can be reduced. This is an important step towards the generation of monochromatic radiation with LWFA by superconductive undulators. An experimental verification of this concept is in preparation.

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