THEORY OF EDGE RADIATION. PART II: ADVANCED APPLICATIONS AND IMPACT ON XFEL SETUPS

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Abstract

In this paper we exploit a formalism to describe Edge Radiation, which relies on Fourier Optics techniques, described in another contribution to this conference. First, we apply our method to develop an analytical model to describe Edge Radiation in the presence of a vacuum chamber. Such model is based on the solution of the field equation with a tensor Green's function technique. In particular, explicit calculations for a circular vacuum chamber are reported. Second, we consider the use of Edge Radiation as a tool for electron beam diagnostics. We discuss coherent Edge Radiation, extraction of Edge Radiation by a mirror, and other issues becoming important at high electron energy and long radiation wavelength. Based on this work we also study the impact of Edge Radiation on XFEL setups and we discuss recent results. **These proceedings are based on the article [1], to which we address the interested reader for further information and references.**

INTRODUCTION

It is the purpose of this article to conclude the discussion begun in [2] about the principles of production and properties for all applications of Edge Radiation (ER). In particular, in this paper we analyze a few advanced applications of our formalism.

In [2], a theory of near-field ER based on Fourier Optics (FO) techniques was developed within the framework of the more general Synchrotron Radiation (SR) theory, relying on the paraxial approximation. As has been demonstrated in that reference, any ER setup can be characterized, starting from the far-zone field, in terms of virtual sources. These sources exhibit a plane wavefront, and can be pictured as waists of laser-like beams. Using this kind of description one can use Fourier Optics (FO) techniques to characterize the ER field at any distance, thus providing a tool for designing and analyzing ER setups. Our previous study [2] made consistent use of both dimensional analysis and comparisons with outcomes from numerical simulation with the help of the computer code SRW by Oleg Chubar.

There are, however, situations when existing computer codes cannot predict the radiation characteristics. One of these is the case when perturbations of the long-wavelength radiation by vacuum chambers are present, which may potentially affect the performance of ER setups. Since the diffraction size of THz radiation exceeds the vacuum chamber dimensions, characterization of far-infrared ER must be performed accounting for the presence of a waveguide. In order to deal with this situation we developed a theory of ER in a waveguide. The task to be solved differs from the unbounded-space case in the formulation of boundary conditions only. The paraxial approximation applies as in the unbounded-space case. However, on perfectly conducting walls the electric field must be orthogonal to the vacuum chamber surface. As in the unbounded-space case, one can use a Green's function approach to solve the field equations. The presence of different boundary conditions complicates the solution of the paraxial equation for the field, which can nevertheless be explicitly found by accounting for the tensorial nature of the Green's function. Here we take advantage of a mode expansion approach to calculate ER emission in the metallic waveguide structure. We solve the field equations with a tensor Green's function technique, and we extract figure of merits describing in a simple way the influence of the vacuum chamber on the radiation pulse as a function of the problem parameters. We put particular emphasis on a vacuum chamber with circular cross-section, which is natural for future linac-based sources.

Finally, we address the problem electron beam characterization for linac-based sources and laser-plasma accelerators. One possibility to perform electron-beam diagnostics in these kind of facilities is to use coherent ER. This is an attractive tool, because it can provide valuable and detailed information on the electron beam. In principle, by detecting coherent ER, 3D distributions, divergence and microbunching may be measured. Usually, electrons in accelerators are highly collimated and monochromatic. In this case, coherent ER can be used for longitudinal and transverse beam-size monitoring. In contrast to this, electrons generated in laser-plasma interactions have different properties compared with those in conventional accelerators. Namely, in this case electrons have both divergence angle and energy distribution. We address applications of coherent ER by studying, first, the relatively simple case when only the influence of longitudinal and transverse structure factor of the electron bunch is accounted for. In particular, we give particular attention to the case when microbunching at optical wavelengths is imprinted onto an electron bunch, and we analyze the more complicated case of a bunch produced in a laser-plasma accelerator, i.e. accounting for divergence angle distribution of the beam. Energy distribution can be easily accounted for, based on this analysis. The problem of extraction of ER by a mirror, strictly related to diagnostics applications, is included too.

EDGE RADIATION IN A WAVEGUIDE

In [2] we considered edge radiation propagating in unbounded space. In [1] we extended our considerations to account for the presence of metallic surroundings (like e.g.

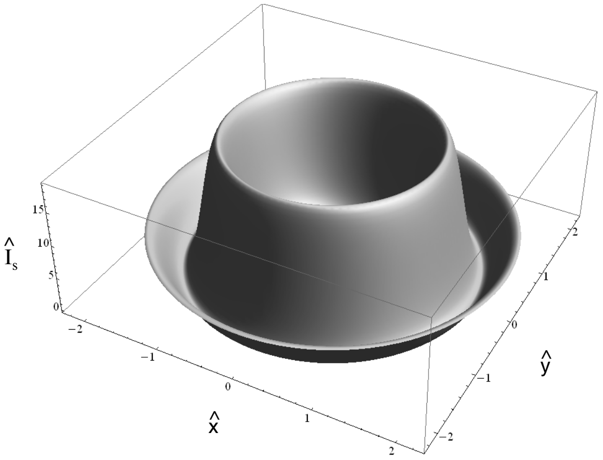

Figure 1: Three-dimensional plot of the intensity profile of the virtual source ($z/L = 0$) at $\Omega = 2\pi R^2/(\lambda L_w) = 5$ for $\Delta = 0.1$ in the limit for $\hat{\phi} = 2\pi L/(\gamma^2 \lambda) \ll 1$.

the electron vacuum chamber), which effectively act like a waveguide.

The need to account for guiding structures in edge radiation setups arises when one wants to operate in the sub-millimeter wavelength range. Emission of edge radiation in the presence of metallic boundaries has been a much less-treated subject in literature, compared to the unbounded space case. To the best of our knowledge, there is only one article reporting on edge radiation from electrons in a planar overmoded metallic waveguide (ref. [6] in [1]): the analysis is based on Lienard-Wiechert fields, modified by the presence of finite metallic boundary. To account for perfectly conducting plates, a generalization of the method of images, well-known from electrostatics is used. Instead of this approach, here we prefer to use a mode-expansion technique. We characterize radiation from particles in guiding structures with the help of the proper tensor Green's function, automatically accounting for boundary conditions. The Green's function itself and the field are then conveniently expressed in terms of the natural modes of the guiding structure. This approach is shown to provide a convenient and methodical way to deal with the vectorial character of our problem. We apply our method to the case of a homogeneous waveguide with circular cross-section. This configuration approximates the vacuum chamber to be used at XFEL and Energy Recovery Linac (ERL) facilities. Our approach, however, is very general and may be applied to other geometries, e.g. to rectangular waveguides, which approximate vacuum chambers used at SR facilities.

Once the field is found, in order to present results one should cut high spatial frequency contributions due to abrupt switching of the bending magnet fields on a scale shorter than $\sqrt[3]{R^2 \bar{\lambda}}$, because these are outside of the accuracy of the sharp-edge approximation. We may then introduce a spatial frequency filter in our expression for the field by smoothing the rectangular sharp-edge profile on a scale $\sqrt[3]{R^2 \bar{\lambda}}$ on a typical distance Δ (in units of the straight section length L). Detailed results are shown in [1]. Fig. 1 gives an example of the three-dimensional plot of the intensity profile of the virtual source at $\Omega = R^2/(\bar{\lambda} L_w) = 5$ for $\Delta = 0.1$ in the limit for $\hat{\phi} = L/(\gamma^2 \bar{\lambda}) \ll 1$.

COHERENT EDGE RADIATION

The physics of radiation processes from electron beams critically depends on the ratio of the bunch length to the wavelength of the emitted radiation. We can thus define two opposite asymptotes for the electron bunch length, corresponding to very different characteristics of radiation.

In the first limit, when the electron bunch is long compared to the radiation wavelength, one deals with the conventional case of spontaneous radiation. In this regime the electron phases are not correlated, i.e. distances between electrons are randomly distributed on a characteristic scale much longer than the radiation wavelength. Incoherent superposition of each electron contribution to the radiation field yields low-power emission, proportional to the charge in the bunch, with the usual incoherent phase-noise statistics.

In the second limit, when the electron bunch is short compared to the radiation wavelength, one deals with the case of coherent radiation. Electron phases are correlated, in the sense that random variations of distances between electrons are distributed on a characteristic scale shorter than the radiation wavelength. The bunch essentially behaves as a single point-charge, which coherently emits high-power radiation. In this case, the radiation power scales quadratically with the total electric charge in the bunch, rather than linearly as in the other limit. It is because of this quadratic effect that the radiation power can be greatly enhanced, because the bunch population is usually of order 10^{10} particles.

Coherent radiation is usually not emitted in electron storage rings, because the bunch length is of order of a centimeter. However, in linear accelerators, much shorter electron bunches of the order of 100μm can be produced in magnetic bunch compressors. In [1] we extensively discussed the physics of coherent ER distinguishing two cases, when coherent emission is due to an overall temporal bunch profile shorter than the wavelength or to short structures (microbunches of order of the radiation wavelength) superimposed to the temporal profile of the bunch.

When it comes to the use of ER as a diagnostic tool, the availability of a much larger number of photons constitutes an important advantage of coherent ER compared to the incoherent case. By detecting coherent ER, characteristics of the electron bunch such as their three-dimensional distribution and divergence can be measured. Moreover, coherent radiation presents the unique feature that electron-beam microbunching can be investigated too.

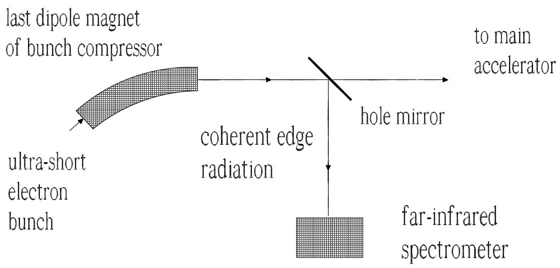

Figure 2: Electron bunch length monitor for XFEL using far-infrared coherent edge radiation.

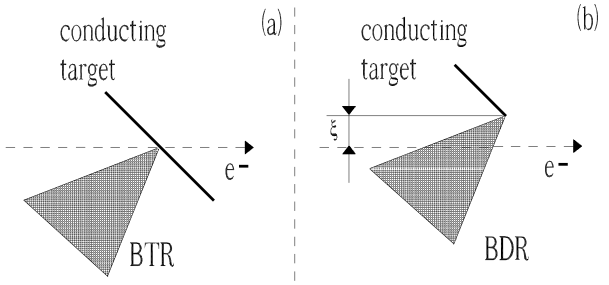

Figure 3: Geometry of backward transition radiation (BTR) (a), backward diffraction radiation (BDR) (b).

EXTRACTION OF EDGE RADIATION BY A MIRROR

Backward Transition and Diffraction Radiation

A setup where edge radiation from an upstream bending magnet is extracted by a mirror is shown in Fig. 2. In the particular case depicted in that figure, radiation is reflected by a hole mirror and sent to a far-infrared spectrometer. This example refers to a method (see reference [24] of [1]) where coherent radiation is used to monitor the bunch length at XFEL setup, but similar setups can also be used to collect incoherent radiation in the optical range. The size and shape of the mirror may vary. A hole may be present in the mirror or not, and the geometry of the experimental arrangement depends on the particular setup considered. In any case, in its basic lines, a standard edge-radiation diagnostic setup consists, similarly as in Fig. 2, of a mirror positioned at some distance after a bend and rotated by an angle $\pi/4$ in the direction of the electron bunch to allow extraction of radiation through a vacuum window. Such setup can be used for longitudinal bunch-length measurements in the mm and sub-mm range as well as for transverse electron bunch diagnostics in the optical wavelength range (see references [24-26] of [1]).

Consider the straight section after the bend. If this is long enough so that $\phi \gg 1$ (and $\delta = \sqrt[3]{R^2 \lambdabar}/L$ such that $\delta \cdot \phi \ll 1$, i.e. $\lambdabar \gg \lambdabar_c$), the radiation collected by the mirror is the usual Transition Radiation (TR) first noted by Ginzburg and Frank and considered in a number of publications written during the last half century. In recent years there has been a good deal of interest in TR, because this form of radiation has found useful applications, e.g. in diagnostic for ultrarelativistic beams (see [28-32] of [1]). These papers describe an electron crossing an interface between two media with different dielectric constants and they refer, in particular, to the case of the boundary between vacuum and an ideal conductor. As a consequence of the crossing, time-varying currents are induced at the boundary. These currents are responsible for TR (see Fig. 3 left). The metallic mirror, that is treated as the source of TR, is usually modelled with the help of a Physical Optics approach. This is a well-known high-frequency approximation technique, often used in the analysis of electromagnetic waves scattered from large metallic objects. Surface currents entering as the source term in the propagation equations of the scattered field are calculated by assuming that the magnetic field induced on the surface of the object can be characterized using Geometrical Optics, i.e. assuming that the surface is locally replaced, at each point, by its tangent plane.

As mentioned before, one typically assumes that the Ginzburg-Frank formula can be used, i.e. the field distribution is proportional to $K_1[\omega r/(\gamma c)]$. In terms of parameters introduced in this paper, this approximation has sense only when $\phi \gg 1$. However, even for this case, one should always specify the transverse region of applicability of the usual TR formulas, which is typically neglected (this region is for $r \ll \phi \lambdabar \gamma = L/\gamma$, with $\phi \gg 1$). Moreover, mainly due to an increased electron energy, today setups often meet, in practice, condition $\phi \lesssim 1$ even in the optical wavelength range. In this case one should account for the whole setup when discussing the reflected radiation, and not only for the metallic mirror. In other words, the problem of extraction of edge radiation reduces to the well-studied TR problem only asymptotically, i.e. for $\phi \gg 1$. We will still keep on talking about TR or, with some abuse of language, even when usual TR formulas do not apply anymore ($\phi \lesssim 1$, or $r \gtrsim L/\gamma$).

When dealing with the above-discussed setup, one usually talks about Backward Transition Radiation (BTR). BTR is widely used for different purposes, because it allows for low background radiation levels. In [2] we considered a whole range of setups for BTR, without treating their geometry in detail, and gave an algorithm to deal with these systems in all generality. In fact, in all cases, the main problem is in the specification of the electric field distribution at some position where a mirror is present (the hole mirror in the example of Fig. 2). The only element to be accounted for, aside for the mirror itself, is an upstream dipole magnet. Note that the mirror is tilted as in Fig. 2, i.e. it is not perpendicular with respect to the optical axis. In principle, one may account for this tilting, but in practice the projection of the mirror on the optical axis is negligible compared with the distance to the dipole, and one may consider the screen as not tilted.

Note that Diffraction Radiation (DR) appears when charged particles move in the vicinity of a medium (e.g. a conductive mirror) at some given impact parameter ξ, and has been recently suggested as a possible tool for non-invasive bunch diagnostics. DR is obviously related to TR and, in particular, it can be treated starting from the knowledge of the electric field distribution at the mirror, and subsequently applying physical optics techniques. BDR in the optical wavelength range has been measured and applied for transverse electron beam diagnostics. A schematic comparison between a BDR and a BTR emission is shown in Fig. 3.

It should be stressed that specification of the field at the target position must be considered as the first step to the solution of a more complicated problem, i.e. the characterization of the field at a detector position. Such first step is considered separately, because the field at the target plane is independent of the type of target and detector. Once the field at the target position is known, the full problem can be solved with the help of Physical Optics techniques, where one should account for diffraction effects due to the particular shape of the mirror.

A virtual source method was applied in reference [1] to deal with the problem of field characterization at the target position. Our algorithm consists of the following steps: (i) propagate the field from the upstream virtual source to the mirror, (ii) add the field from the downstream virtual source.

Note that, in contrast to the study case of a straight section between two bends considered before, numerical methods involving direct integration of Maxwell's equations (e.g. with the help of SRW) down the optical axis up to the mirror will fail, because electromagnetic sources must be propagated up to the observation plane, and integration of the Green's function up to that position yields a singularity (independently of the value of δ). In our virtual source approach we also deal with a singularity, but this is isolated in a single term, and expressed by means of an analytical function, allowing straightforward presentation of results. The same approach has been proposed in reference [33] of [1] to discuss the setup in Fig. 2.

Impact on Setups at XFEL Facilities

An important application of the previously discussed method is in relation to electron beam diagnostics in XFEL setups. We presented a study of electron beam diagnostics for XFEL setups elsewhere in this conference.

During the last decade, the electron beam energy in linac-based facilities increased up to around 10 GeV and novel effects, which were considered negligibly small at lower energies, became important. In fact, at high energies, condition $L \gg \gamma^2 \lambdabar$ becomes unpractical even for optical wavelengths, and the effect of the presence of an upstream bending magnet on TR emissions must be considered. Moreover, when measuring the long-wavelength coherent BTR, i.e. when the radiation wavelength is comparable to, or longer than the electron bunch length, the influence of the upstream bending magnet could be significant even at lower energies. These facts led to several misconceptions that can be found till recently.

In general, monitoring methods based on coherent radiation take advantage of the sensitivity of the coherent spectrum on the bunch form factor, which is related to the bunch length. Coherent radiation from a given setup is thus collected with the help of mirrors and its energy measured, for example with the help of pyroelectric detectors. In order to analyze results, whatever the setup considered, one always has to characterize the field distribution at the collecting mirror position. Hence the relevance of the previous study concerning diagnostics techniques. Novel concepts in electron beam diagnostics relying on coherent Optical Transition Radiation from density-modulated electron bunches have been presented in these conference in WEPC46 and WEPC47.

REFERENCES

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