TERAHERTZ BAND FEL WITH ADVANCED BRAGG REFLECTORS*

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Abstract

Periodical Bragg structures may be considered as an effective way of controlling the electromagnetic energy fluxes and provision of spatially coherent radiation in the free electron lasers with oversized interaction space. A new scheme of terahertz band FEL with advanced Bragg resonator exploiting the coupling between the two counter-propagating modes and the quasi-cutoff is considered. Advanced Bragg resonator provides effective mode selection over the transverse index and may be used for realizing a powerful long pulse FEL at terahertz frequency band.

INTRODUCTION

Reflectors based on Bragg coupling of counter propagating waves on the periodic structures are widely used both in quantum [1, 2] and classical [3,4] electronics. In the millimeter wavelength range, Bragg structures based on hollow metallic waveguides with periodic corrugation of inner surface allow one to combine the effective electron beam transportation with selective resonance system. However, the advance in shorter wave bands is limited because at large values of the oversize factor the coupling between numerous pairs of propagating modes occurs. As a result, the radiation generated by the electron beam would represent an uncontrolled mixture of the waveguide modes.

As a solution, we have suggested in [5] to use an advanced Bragg structure (ABS). Unlike the traditional variants of Bragg structures in ABS coupling between propagating and cut-off waves takes place. As a result such a resonator provides effective mode control over both longitudinal and transverse coordinates. In [6] a new scheme of terahertz band FEL with hybrid planar resonator is considered consisting of advanced input Bragg mirror and traditional output Bragg mirror. Advanced Bragg mirror exploiting the coupling between the two counter-propagating modes and the quasi cutoff one provides mode selection over the transverse index. Main amplification of the wave by the electron beam takes place in the regular section of the resonator. Small reflections from the output traditional Bragg mirror are sufficient for oscillator self-excitation.

Unlike the case considered in [6], in the present paper we study a single section model (see Fig. 1). But important factor that for propagating waves transverse profiles (profiles over axis y directed between plates) are not fixed is taken into account.

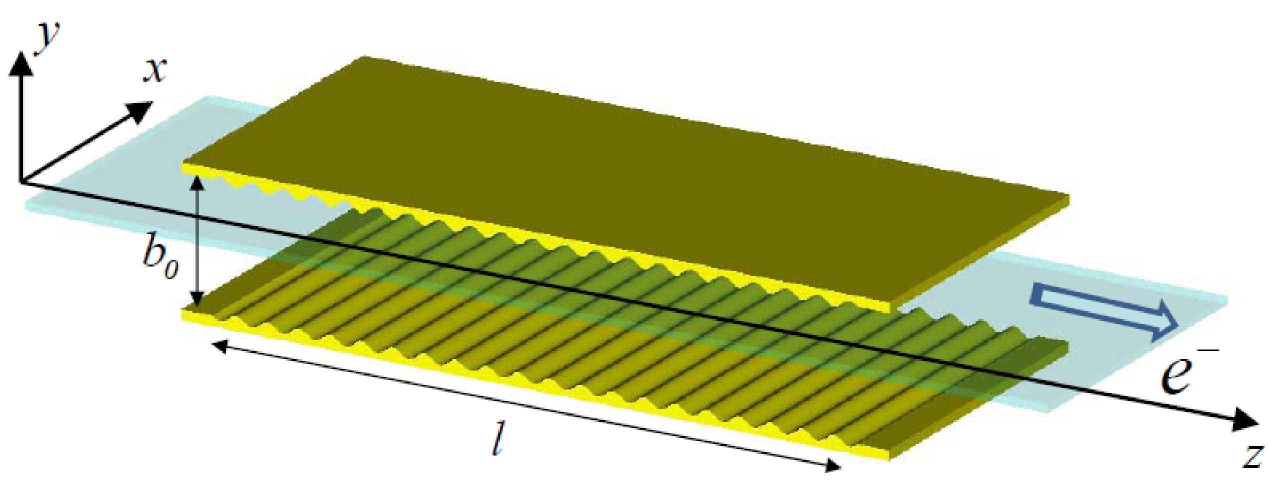

Figure 1: Scheme of a planar FEL with advanced Bragg resonator.

MODEL AND BASIC EQUATIONS

An advanced Bragg structure is formed by two parallel plates with shallow periodic corrugation of the inner walls:

$$a(z) = \frac{a_1}{2}\cos(\bar{h}_1 z) \tag{1}$$

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where $\bar{h} = 2\pi/d$, a_1 and d are the depth and the period of the structure correspondingly (this period is two times larger than in traditional Bragg reflectors). Under the Bragg resonance condition

$$\bar{h} \approx h, \qquad (2)$$

where h is the wavenumber of the propagating wave, which is satisfied when the mean distance between plates is given by $b_0 = nd_l/2$ (where n is integer), the field can be presented as a sum of the two quasi-optical beams propagating in opposite directions

$$\vec{E}_A = \vec{y}_0 \operatorname{Re}\left(A_+(t,z,y)e^{i(\omega t - hz)} + A_-(t,z,y)e^{i(\omega t + hz)} \right) \qquad (3)$$

and a cutoff ᅟᅟ$_n$ mode:

$$\vec{E}_B = \vec{z}_0 \operatorname{Re}\left(B(t,z)\sin\left(\frac{\pi n y}{a_0}\right)e^{i\omega t}\right). \qquad (4)$$

Here $\vec{y}_0$ is the unit vector directed normal to the plates, $\vec{z}_0$ is the unit vector along the resonator axis (see Fig. 1).

Further we consider the nonlinear dynamics of the planar FEL with advanced Bragg structure. We assume that the sheet electron beam interacts with the synchronous wave A_+ under the resonance condition

$$\bar{\omega} - hv_\parallel \approx h_w v_\parallel ,$$

where $h_w = 2\pi/d_w$, and d_w is the undulator period.

Non-stationary equations for the amplitudes of coupled waves can be presented in the form

$$\begin{aligned} &-2i\frac{\partial A_+}{\partial z} + 2i\frac{\partial A_+}{c\partial t} + \frac{\partial^2 A_+}{h\partial y^2} = -h\ a_1 B\delta(y) + i\frac{4\pi}{c}J_\omega \\ &2i\frac{\partial A_-}{\partial z} + 2i\frac{\partial A_-}{c\partial t} + \frac{\partial^2 A_-}{h\partial y^2} = -h\ a_1 B\delta(y) \qquad . \qquad (5) \\ &\frac{\partial B}{c\partial t} + \frac{i}{2h}\frac{\partial^2 B}{\partial z^2} + i\Delta B + \sigma B = ih^2 a_1 \left(A_+ + A_-\right)\Big|_{y=0} \end{aligned}$$

Here $\Delta = (\bar{\omega} - \omega_c)/c$ is the mismatch between cutoff frequency $\omega_c = \pi nc / b_0$ and the Bragg frequency, $\sigma = \frac{h\nu}{b_0}$ is the Ohmic losses parameter for the cutoff mode, ν is the skin depth (Ohmic losses for propagating waves $A_\pm$ are negligibly small), $\delta(y)$ is the delta function, J_ω is the HF electron current. To derive Eqs. (5) which describe coupling between the two propagating and a cutoff mode we use the concept of the surface magnetic current developed in [7, 8].

Taking into account boundary conditions for amplitude of propagating waves on the metallic plates

$$\left.\frac{\partial A_\pm}{\partial y}\right|_{y=0,b_0} = 0 ,$$

these waves can be expanded in a Fourier series

$$A_\pm = \sum_{n=0}^{\infty} A_n^\pm(t,z)cos\left(\frac{n\pi}{b_0}y\right) \qquad (6)$$

Each Fourier term in (6) with its own index n may be considered as a normal wave of regular planar waveguide.

As a result Egs. (5) can be transformed to the form

$$\begin{aligned} &\frac{\partial \hat{A}_n^+}{\partial Z} + \frac{\partial \hat{A}_n^+}{\partial \tau} + iFn^2\hat{A}_n^+ = \frac{2i\alpha\hat{B}}{1+\delta_{0n}} + J_n \\ &-\frac{\partial \hat{A}_n^-}{\partial Z} + \frac{\partial \hat{A}_n^-}{\partial \tau} + iFn^2\hat{A}_n^- = \frac{2}{1+\delta_{0n}}i\alpha\hat{B} \qquad (7) \\ &\frac{\partial \hat{B}}{\partial \tau} + \frac{iC}{2}\frac{\partial^2 \hat{B}}{\partial Z^2} + i\hat{\Delta}\hat{B} = i\alpha\sum_{n=0}^{\infty}\left(\hat{A}_n^+ + \hat{A}_n^-\right) \end{aligned}$$

Here we used the following normalized variables and parameters

$$\hat{A}_n^\pm = \frac{A_\pm eK\mu}{\gamma mc\omega C^2},\ \hat{B} = \frac{BeK\mu}{\gamma mc\omega C^2}\sqrt{\frac{N_A}{N_B}},$$

$$\hat{b} = \sqrt{2C}hb_0,\ F = \frac{\pi^2}{\hat{b}^2},\ Z = hCz,\ \tau = \omega Ct,\ \ \hat{\Delta} = \Delta/hC,$$

$N_{A,B}$ are the norms of coupled modes, $\hat{\alpha}, \hat{\sigma} = \alpha, \sigma/hC, \alpha = \frac{ha_1}{\sqrt{2}b_0}$ is the coupling coefficient, $C = \left(\frac{eI_0}{mc^3}\frac{c\lambda^2K^2\mu}{4\pi^2\gamma_0 N_A}\right)^{1/3}$ is the gain parameter, $K = \frac{eH_u}{h_w mc^2}$, H_u is the wiggler field amplitude, $\mu \approx \gamma^{-2}$ is the bunching parameter, $\beta_\parallel = v_\parallel / c$ is the electron translational velocity, γ is the relativistic mass factor. δ_{0n} is the Kronecker delta. The HF-current harmonics in Eq. (7) $J_n = \frac{2}{1+\delta_{0n}}\frac{1}{\pi}\int_0^{2\pi} e^{-i\theta(y)}cos\left(\frac{n\pi y}{b_0}\right)d\theta_0$ can be found from the averaged electron motion equations

$$\left(\frac{\partial}{\partial Z} + \frac{1}{\beta_\parallel}\frac{\partial}{\partial \tau}\right)^2 \theta = \operatorname{Re}\left(\sum_{n=0}^{\infty}\hat{A}_n^+(t,z)cos\left(\frac{n\pi}{b_0}y\right)e^{i\theta}\right) \qquad (8)$$

For propagating waves boundary conditions at resonator edges take a form

$$\hat{A}_+\big|_{Z=0} = \hat{A}_0,\ \ \hat{A}_-\big|_{Z=L} = 0, \qquad (9)$$

For the cutoff mode we apply the radiation boundary conditions at the edges of corrugation [9,10]

$$\left[\hat{B} \mp \sqrt{\frac{C}{2\pi i}}\int_0^\tau \frac{e^{-\hat{\sigma}(\tau-\tau') - iS(\tau-\tau')}}{\sqrt{\tau-\tau'}}\frac{\partial \hat{B}(\tau')}{\partial Z}d\tau'\right]\Bigg|_{Z=0,L} = 0, \qquad (10)$$

where L is the normalized length of the resonator L=Chl.

Electron efficiency is determined by the following relations

$$\begin{aligned} &\eta = \frac{C}{\mu\left(1-\gamma_0^{-1}\right)}\hat{\eta} \\ &\hat{\eta} = \frac{1}{2\pi b_0}\int_0^{b_0}\int_0^{2\pi}\left(\frac{\partial\theta}{\partial Z} - \Delta\right)\Bigg|_{Z=L} d\theta_0 dY \end{aligned} \qquad . \qquad (11)$$

SIMULATION RESULTS

Results of the simulation are presented in Fig. 2 and 3. for normalized parameters $\hat{b} = 2.5$, $L = 5$, $\hat{\alpha} = 0.3$. In Fig. 2a one can see the process of establishment of steady-state single frequency oscillation regime. Amplitude of modes in steady state regime is shown in Fig. 2b. Alongside with the fundamental TEM mode several TE_n

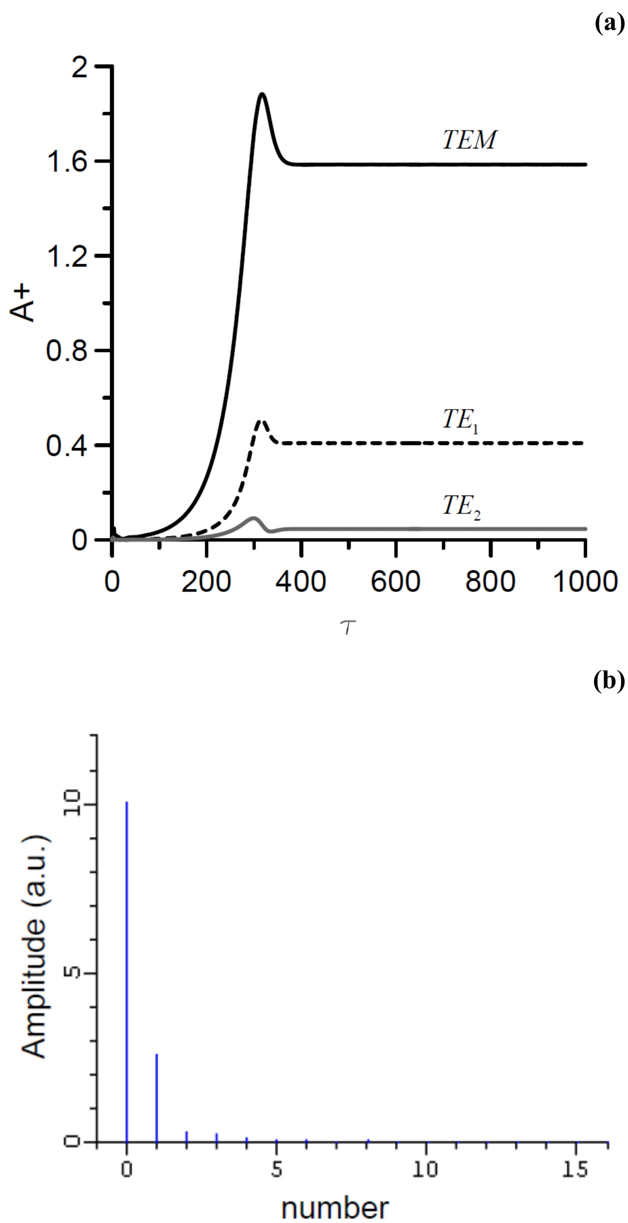

Figure 2: **(a)** Temporal dependence of normalized amplitude harmonics (TEM - solid black line, TE_1 - dashed black line and TE_2 - solid grey line) and **(b)** amplitude of harmonics in steady-state regime at $L = 5$, $\hat{b} = 2.5$, $\hat{\alpha} = 0.3$, s, $\hat{\sigma} = 0.01$.

modes are also excited by electron beams. Phases of these mode are correlated and profiles of partial wave beams $\hat{A}_{\pm}(Z,Y)$ profiles shown in Fig. 3a and 3b don't vary in time. It should be noted that due to electron beam the distribution of propagating waves over transverse Y coordinate becomes more homogeneous than the structure of the cold mode. In the absence of electron beam the amplitude of these waves fall down from corrugated plate (in our simulation this plate is located at $Y = 0$).

Let us now consider an example of a physical system corresponding normalized parameters above. We consider a FEL generating a radiation frequency 1 THz using a 5 MeV electron beam and an undulator period of 6 cm.

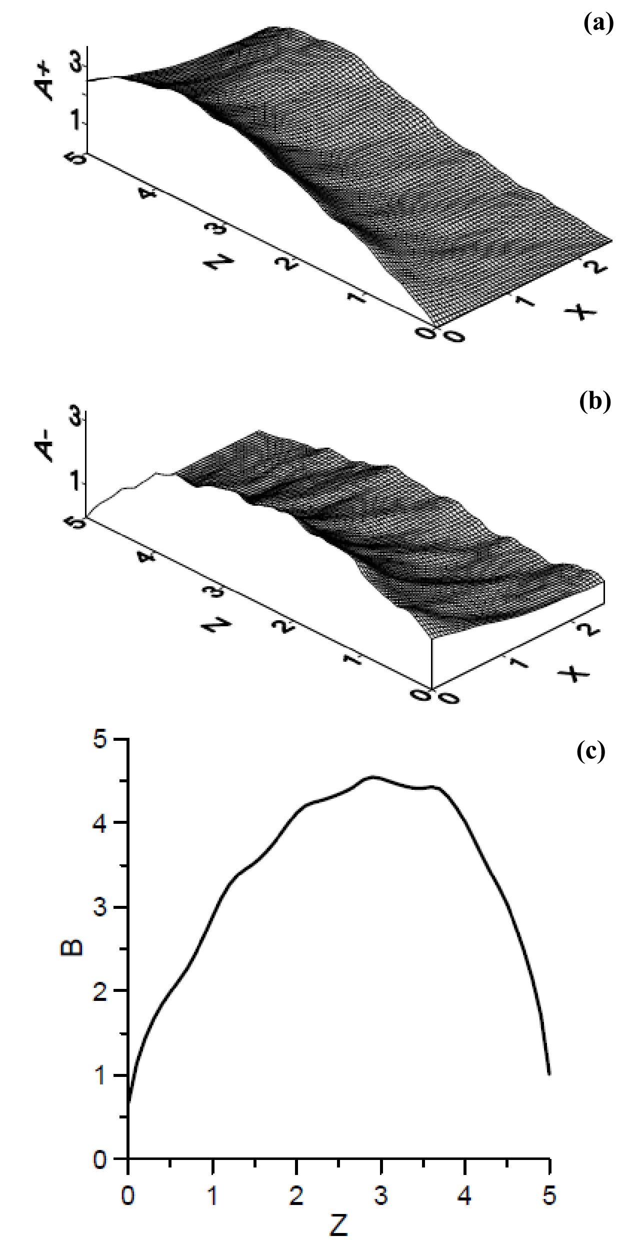

Figure 3: Spatial distribution of partial waves **(a)** A_+, **(b)** A_- and **(c)** longitudinal profile of partial wave B in the steady-state regime.

Taking a sheet beam current density 10 A/cm, a gap $b_0 = 10$ mm between the plates and an undulator field amplitude H_u=5 kOe one can obtain for the gain parameter $C \approx 1.4 \cdot 10^{-4}$. Normalized parameters $\widehat{b} = 2.5$, $L = 5$, $\widehat{\alpha} = 0.3$ would then correspond to an interaction length $l = 170$ cm and a corrugation depth $a_1 = 5\mu$m. For the simulation presented in Fig. 2 we find a normalized efficiency $\widehat{\eta} = 1.5$, which corresponds to an electron efficiency of 1%. Taking into account the Ohmic losses the output power is 0.5 MW/cm.

Note that for the effective single frequency operation it is sufficient to provide a condition that the frequency distance between cut off modes with different transverse indices q exceeds the FEL amplification band

$$c\pi / b_0 < \omega / N , \quad (12)$$

which is defined by the number of wiggler periods $N=l/d_w$ d_w inside the interaction length l. Taking into account that the FEL operation wavelength $\lambda \approx \gamma^{-2} d_w / 2$ we get a restriction for the width of the gap between resonator plates

$$b_0 < l\, \gamma^{-2} / 2 , \quad (13)$$

Obviously, this condition can be satisfied in terahertz wave band.

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