ANALYSIS ON THE GAIN OF A COMPACT CHERENKOV FREE-ELECTRON LASER*

D. Li[#], K. Imasaki, ILT, Osaka, 565-0871, Japan
M. Asakawa, Kansai University, Osaka, 564-8680, Japan

Abstract

A compact Cherenkov free-electron laser is studied for a double-slab structure with no incident field or electromagnetic feedback mechanism. The simplified model is composed of a rectangular wave-guide partially filled with two lined parallel dielectric slabs and a sheet electron beam. The interaction between the electron beam and the electromagnetic mode is described with the macro-particle approach. The coupled equations are derived and solved numerically with the parameters of an ongoing experiment, demonstrating the amplification of emitted power from spontaneous emission.

INTRODUCTION

There is increasing interest in the development of the terahertz radiation sources in recent years because the requirements of applications in medical, industrial and material science [1-9]. As is known, the Cherenkov free-electron lasers have an advantage over the usual undulator free-electron lasers, and they can generate terahertz radiation with low energy electron beam [10], which means that the large electron-beam-serve system is not necessary. We planned to develop a compact terahertz Cherenkov free-electron laser with moderate (~10 mW) average power. To achieve this goal, a compact electron beam source is being developed, and a double-slab Cherenkov free-electron laser device is being studied. For a preliminary lasing experiment, this device is designed to generate the millimeter wave. The device is composed of a rectangular wave-guide loaded with double dielectric slabs, and between them is the vacuum space for electron beam to go through. The slab is with a thickness of 650 µm and the vacuum width is 1000 µm. The dielectric medium is chosen as silicon since it has a relatively high dielectric constant, ε_r=11.6, with which the device can produce radiation from millimeter to terahertz wave. The electron beam source generates a round beam with an average radius of 300 µm. The beam current is about 1 mA and the energy is 50 keV.

Such a device can be designed to operate at a resonator mode or an amplifier mode. In this paper, we focus on the amplifier mode. The incident field is not involved, and the emitted field is amplified from the spontaneous emission. The analysis of the dispersion relation and the small-signal gain for the double-slab structure has been carried out based on the hydrodynamic model [11]. This paper aims at the nonlinear analysis of the interaction between the electromagnetic wave and the electron beam. Macro-particle approach is used to describe the interaction, and the system of equations is numerically solved, demonstrating the evolution of emitted wave from spontaneous emission to saturation.

*Work supported by KAKENHI (20656014)
[#]dazhi_li@hotmail.com

FUNDAMENTAL THEORY

The system to be analyzed consists of a rectangular wave-guide partially filled with two lined parallel dielectric slabs and a sheet electron beam travelling in the vacuum area between the two slabs, as shown in Fig. 1. It is assumed that dielectric slabs are much wider than the

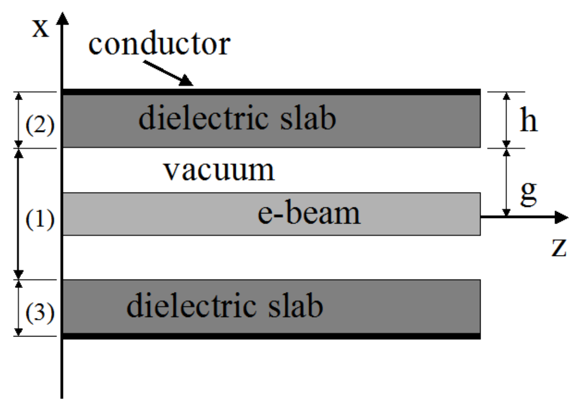

Figure 1: Double-slab Cherenkov free-electron laser

beam width in the y direction, and outside the slabs is the pad of perfect conductor. The beam electrons interact with the electromagnetic mode of such a structure, causing energy to be transferred between the beam and the electromagnetic mode. The mode is restricted to the TM polarization because a nonzero z component of electric field is required for the energy exchange interaction.

We derived the electromagnetic fields in the vacuum region (1) and they are

$$E_{z1} = \frac{c^2\Gamma_1^2}{j\omega}(C_{1,1}\sin\Gamma_1 x + C_{1,2}\cos\Gamma_1 x) \quad (1)$$

$$H_{y1} = \frac{\Gamma_1}{\mu_0}(-C_{1,1}\cos\Gamma_1 x + C_{1,2}\sin\Gamma_1 x) \quad (2)$$

where $\Gamma_1^2 = \omega^2/c^2 - k^2$.

In the dielectric regions (2) and (3), the electromagnetic fields read

$$E_{z2} = \frac{c^2\Gamma_2^2}{j\omega\varepsilon_r}(C_{2,1}\sin\Gamma_2 x + C_{2,2}\cos\Gamma_2 x) \quad (3)$$

$$H_{y2} = \frac{\Gamma_2}{\mu_0}(-C_{2,1}\cos\Gamma_2 x + C_{2,2}\sin\Gamma_2 x) \quad (4)$$

and

$$E_{z3} = \frac{c^2\Gamma_3^2}{j\omega\varepsilon_r}(C_{3,1}\sin\Gamma_3 x + C_{3,2}\cos\Gamma_3 x) \quad (5)$$

$$H_{y3} = \frac{\Gamma_3}{\mu_0}(-C_{3,1}\cos\Gamma_3 x + C_{3,2}\sin\Gamma_3 x) \quad (6)$$

where $\Gamma_2^2 = \Gamma_3^2 = \varepsilon_r\, \omega^2/c^2 - k^2$.

Considering the boundary conditions, we get the passive (no beam) dispersion relation

$$\begin{aligned}&(\sin\Gamma_1 g + M\cos\Gamma_1 g)(\cos\Gamma_1 g + N\cdot\sin\Gamma_1 g)\\ &=(\cos\Gamma_1 g - M\sin\Gamma_1 g)(-\sin\Gamma_1 g + N\cdot\cos\Gamma_1 g)\end{aligned} \quad (7)$$

where

$$M = \frac{\Gamma_2}{\Gamma_1\varepsilon_r}\frac{-\sin\Gamma_2 g + \tan\Gamma_2(g+h)\cos\Gamma_2 g}{\cos\Gamma_2 g + \tan\Gamma_2(g+h)\sin\Gamma_2 g}$$

$$N = \frac{\Gamma_3}{\Gamma_1\varepsilon_r}\frac{\sin\Gamma_3 g + \tan\Gamma_3(g+h)\cos\Gamma_3 g}{\cos\Gamma_3 g + \tan\Gamma_3(g+h)\sin\Gamma_3 g}.$$

The linear analysis tells that the electron beam interacts with the mode with phase velocity equal to the average velocity of the electron beam, i.e., $v_{av} = \omega/k$. With the parameters mentioned above, a numerical search provides the synchronous point (ω_0, k_0). For a 50 keV electron beam, we found the synchronous frequency is f_0 =45.96 GHz, and the wave number k_0 = 2332.34 m^{-1}. Using (ω_0, k_0) and the above expressions, the ratio of the other coefficients to $C_{1,2}$ can be achieved, and we note them as $C_{1,1}/C_{1,2} = 0$, $C_{2,1}/C_{1,2} = Q_{2,1}$, $C_{2,2}/C_{1,2} = Q_{2,2}$, $C_{3,1}/C_{1,2} = Q_{3,1}$, $C_{3,2}/C_{1,2} = Q_{3,2}$, where $Q_{m,n}$ is real. The Γ_1 is found to be imaginary, and we rewrite it as $\Gamma_1 = j\beta_x$, where β_x is real.

INTERACTION DYNAMICS

To evaluate the interaction between the electron beam and the electromagnetic mode, and the coefficient $C_{1,2}$ varies with the interaction distance written as $C(z)$. This amplitude function $C(z)$ is determined by the power exchange interaction between the electron beam and the electromagnetic mode. An equation describing this interaction can be derived by starting with Poynting's theorem [12],

$$\nabla\cdot\vec{S} + \frac{\partial W}{\partial t} = -\vec{J}\cdot\vec{E} \quad (8)$$

where $\vec{S}$ is the power density, W is the local energy density, and the term $-\vec{J}\cdot\vec{E}$ describes the interaction mechanism. For this calculation, the desired solution is the steady state, for which the explicit time-dependent term in Eq. (8) vanishes. Additionally, all power flow is in the z direction, and it has been assumed that the electron motion is in the z direction as well. This allows Eq. (8) to be simplified to

$$\frac{d}{dz}S_z = -J_z E_z \quad . \quad (9)$$

We can integrate over the cross-section of the system, and furthermore, the system is assumed to operate at a single frequency ω. For the present purpose we average out over one period of the wave $T = 2\pi/\omega$, thus, we have

$$\begin{aligned}&\frac{d}{dz}\left[\frac{1}{T}\int_0^T dt\int_{-(g+h)}^{g+h}dx\int_0^{2d}dyS_z\right],\\ &= -\frac{1}{T}\int_0^T dt\int_{-(g+h)}^{(g+h)}dx\int_0^{2d}dyJ_zE_z\end{aligned} \quad (10)$$

where $2d$ is the electron beam dimension in y direction. The first term is the total average power, which propagates along the system. According to the definition $S_z = E_x H_y$, the integration of left-hand side gives

$$\frac{1}{2}P\frac{d}{dz}C^2(z) = -\frac{1}{T}\int_0^T dt\int_{-(g+h)}^{g+h}dx\int_0^{2d}dyJ_zE_z \quad (11)$$

where

$$\begin{aligned}P = 2d\cdot[&\frac{kc^2\beta_x C^2(z)}{2\omega\mu_0}\left(\sinh(2\beta_x g) + 2\beta_x g\right)\\ &-\frac{kc^2\Gamma_2 C^2(z)}{4\omega\varepsilon_r\mu_0}((Q_{22}^2 - Q_{21}^2)\sin(2\Gamma_2(g+h))\\ &-2\Gamma_2 h(Q_{21}^2 + Q_{22}^2) - 2Q_{21}Q_{22}\cos(2\Gamma_2(g+h))\\ &+(Q_{21}^2 - Q_{22}^2)\sin(2\Gamma_2 g) + 2Q_{21}Q_{22}\cos(2\Gamma_2 g))\\ &-\frac{kc^2\Gamma_3 C^2(z)}{4\omega\varepsilon_r\mu_0}((Q_{32}^2 - Q_{31}^2)\sin(2\Gamma_3(g+h))\\ &-2\Gamma_3 h(Q_{31}^2 + Q_{32}^2) + 2Q_{31}Q_{32}\cos(2\Gamma_3(g+h))\\ &+(Q_{31}^2 - Q_{32}^2)\sin(2\Gamma_3 g) - 2Q_{31}Q_{32}\cos(2\Gamma_3 g))].\end{aligned}$$

The electron beam interacts with the longitudinal electric field in the vacuum region, i.e., E_{z1}, then the Eq. (11) can be written as

$$\begin{aligned}&\frac{1}{2}P\frac{d}{dz}C^2(z) =\\ &-\frac{1}{T}\int_0^T dt\int_{-(g+h)}^{g+h}dx\int_0^{2d}dyJ_zE_{z1}(x,z,t)\end{aligned} \quad (12)$$

The electron beam current density is given by

$$\begin{aligned}&J_z(x,y,z,t) =\\ &-e\sum_i v_i(t)\delta[z - z_i(t)]\delta(x - x_i)\delta(y - y_i)\end{aligned} \quad (13)$$

where i is summed over the electron in the beam, and (v_i, x_i, y_i, z_i) are the z component of the velocity and spatial coordinates, respectively, of the i^{th} electron. Motion in the directions orthogonal to the propagating z axis is ignored in this treatment. With the definition of the current density and the expression for E_{z1}, the integration of the right-hand side of Eq. (12) is straightforward and it reads

$$\frac{1}{2}\frac{d}{dz}C^2(z) = -\Omega C(z)\langle \sin(\omega\tau_i(z) - kz)\cdot\cosh(\beta_x x_i)\rangle \quad (14)$$

where

$$\tau_i(z) = \tau_i(0) + \int_0^z d\zeta \frac{1}{v_i(\zeta)},\ \Omega = \frac{c^2 \beta_x{}^2 I}{\omega P},$$

$\langle\cdots\rangle = N^{-1}\sum_{i=1}^{N}\cdots$, N is the total number of electrons in one period T, and $I = eN/T$ is the instantaneous current. Defining $\chi_i(z) = \omega\tau_i(z) - kz$, gives

$$\frac{1}{2}\frac{d}{dz}C^2(z) = \mathrm{Im}[-\Omega C(z)\langle e^{j\chi_i(z)}\cosh(\beta_x x_i)\rangle] \quad (15)$$

This equation can be written as

$$\frac{1}{2}[C(z)\frac{d}{dz}C^*(z) + C^*(z)\frac{d}{dz}C(z)] = \frac{1}{2}j\Omega[C(z)\langle e^{j\chi_i(z)}\cosh(\beta_x x_i)\rangle - C^*(z)\langle e^{-j\chi_i(z)}\cosh(\beta_x x_i)\rangle], \quad (16)$$

and it can be rewritten as

$$C(Z)[\frac{d}{dz}C^*(z) - j\Omega\langle e^{j\chi_i(z)}\cosh(\beta_x x_i)\rangle] + C^*(z)[\frac{d}{dz}C(z) + j\Omega\langle e^{-j\chi_i(z)}\cosh(\beta_x x_i)\rangle] = 0 \quad (17)$$

Since it has to be satisfied for any $C(z)$, we conclude that

$$\frac{d}{dz}C(z) = -\Omega\left\langle e^{j(\frac{\pi}{2}-\chi_i(z))}\cosh(\beta_x x_i)\right\rangle \quad (18)$$

which describes the dynamics of the amplitude and phase of the electromagnetic field and its dependence on the distribution of particles.

The equation of motion of electrons is more convenient to use the single particle energy conservation since one dimension motion is assumed, and we have

$$\frac{d}{dz}\gamma_i(z) = \frac{e\beta_x^2\cosh(\beta_x x_i)}{m\omega}C(z)\,\mathrm{Im}[e^{j\chi_i(z)}] \quad (19)$$

To complete the description of the particles' dynamics we have to determine the dynamics of the phase term χ_i. According to its definition, we find

$$\frac{d}{dz}\chi_i(z) = \frac{\omega}{v_i} - k \quad . \quad (20)$$

The last three equations form a closed set of equations, which describes the interaction.

NUMERICAL COMPUTATION

To numerically solve the equations, it is necessary to determine the initial conditions. At the beginning of the interaction region, fluctuations in the charge density of the electron beam induce spontaneous emission, or shot noise. The shot noise was widely studied [13~15], and it is more convenient to analytically estimate the electron beam shot noise at the interaction region input. We apply those ideas to the system desired here.

The shot noise fluctuations are governed by Poisson statistics, and it has the form

$$\langle\delta n^2\rangle = \frac{I}{e}t \quad , \quad (21)$$

where $\langle\delta n^2\rangle$ is the mean squared fluctuation in the number of electrons in the observation time t, and I is the average current. These number fluctuations will give fluctuations in the time-averaged quantities computed in the dynamical equation. For Eq. (18), the time-averaged quantity at $z = 0$ can be roughly written as [15]

$$\left\langle e^{j(\frac{\pi}{2}-\chi_i(z))}\cosh(\beta_x x_i)\right\rangle \approx \frac{1}{\langle\delta n^2\rangle^{\frac{1}{2}}}\sum_i^{\langle\delta n^2\rangle^{1/2}} e^{j(\frac{\pi}{2}-\chi_i(z))}\cosh(\beta_x \bar{x}) \quad , \quad (22)$$

where $\bar{x}$ is an average value of electrons' position. Substituting Eq. (21) into Eq. (22), we have

$$\left\langle e^{j(\frac{\pi}{2}-\chi_i(z))}\cosh(\beta_x x_i)\right\rangle \approx \frac{1}{(It/e)^{\frac{1}{4}}}\cos(\beta_x \bar{x}\ ) \quad (23)$$

The term $\cosh(\beta_x \bar{x})$ is found to be of order 1 and usually ignored. Taking the observation time as a single cycle of the electromagnetic mode, Eq. (18) can be integrated over a single cycle to yield

$$C_{shot-noise}(z=0) \approx -\Omega\frac{2\pi}{k}\cdot\frac{1}{\left(2\pi I/\omega e\right)^{\frac{1}{4}}} \quad (24)$$

It is straightforward to use the finite difference methods to solve the coupled equations. The beam was specified to have a uniform square cross section of 600×600 μm^2. The number of particles used in simulation was 10^3. At the entrance $z = 0$, we assume that monoenergetic electrons are randomly distributed in phase space and positions in x direction. Equations for electron dynamics for a given field along the interaction region are solved at first. Knowing the modified electron distribution in phase space by solving Eq. 20, the field in the next space step is obtained by solving Eq. 18. The results of this calculation are given in Fig.2 and 3.

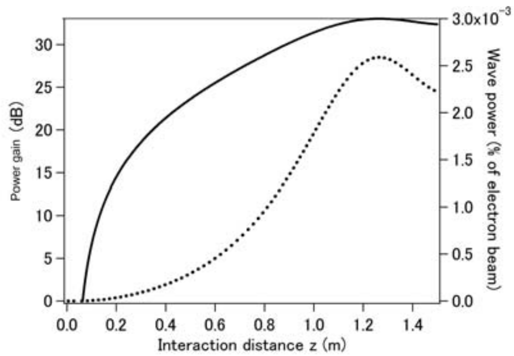

Figure 2: Evolution of power gain (solid line) an radiation power (dotted line) along the interaction region.

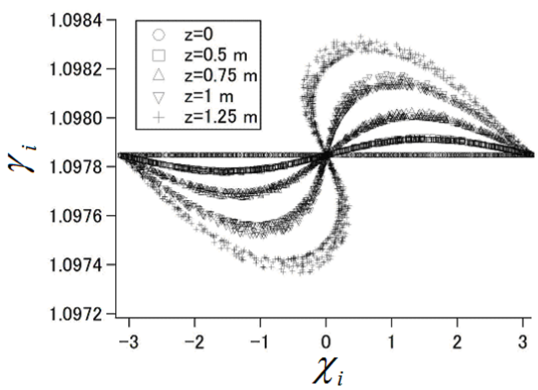

Figure 3: Plots of the energy γ and phase$_i$ χ of the beam electrons for a series of distances along the interaction region.

The power gain is shown in Fig.2. The saturation takes on around 1.2 m of the interaction distance, where the power gain comes to its maximum ~37 dB. Also plotted in Fig. 2 is the wave power, demonstrating the amplification of emitted power from spontaneous emission to saturation. The variation of electrons' phase in phase space is shown in Fig. 3. The electrons in-phase with the wave are decelerated while those anti-phase are accelerated. With the increase of the interaction distance, the bunching process continues and the electrons' energy spread grows. At last, the electrons are strongly bunched and the energy spread (peak-to-peak) comes to $\triangle\gamma = 9.7\mathrm{x}10^{-4}$ (γ_{max}=1.09833, γ_{min}=1.09736).

CONCLUSION

We studied the wave-beam interaction in a double-slab Cherenkov free-electron laser with using the macro-particle approach. A set of coupling equations are derived and numerically solved to demonstrate the amplification of the wave from spontaneous radiation to saturation, with the parameters of our ongoing experiment. The power gain was worked out and the radiation power was estimated, as well as the interaction process was well understood. The results are helpful for designing and understanding the ongoing experiment.

REFERENCES

[1] S.P. Mickan and X.-C. Zhang, Int. J. High Speed Electron. Syst. **13** (2003) 13
[2] M. Abo-Bakr, J. Feikes, K. Holldack, et.al., Phys. Lett. **88** (2002) 254801
[3] G.P. Williams, Rev. Sci. Instrum. **73** (2002) 1461
[4] G. Ramian, Nucl. Instr. Meth. Phys. Res. Sect. **A318** (1992) 225
[5] E. Brundermann, D. R. Chamberin, and E. E. Haller., Infrared Phys. Technol. **40** (1999) 141
[6] Van der meer, A. F. G, Nucl. Instr. Meth. Phys. Res. Sect. **A528** (2004) 8
[7] S. J. Allen, K. Craig, et.al., Nucl. Instr. Meth. Phys. Res. Sect. **A393** (1997) 220
[8] Y. U. Jeong, G. M. Kazakevitch, et.al., Nucl. Instr. Meth. Phys. Res. Sect. **A507** (2003) 125
[9] Y. U. Jeong, G. M. Kazakevitch, et.al., Nucl. Instr. Meth. Phys. Res. Sect. **A528** (2004) 88
[10] Walsh J. and Johnson B., Phys. Rev. Lett. **53** (1984) 779
[11] D.Li, G. Huo, K. Imasaki and M. Asakawa, Nucl. Instr. Meth. Phys. Res. Sect. A (in press)
[12] *Beam-wave interaction in periodic and quasi-periodic structures*, edited by L. Schachter (Springer-Verlag, Berlin, 1997)
[13] L. H. Yu and S. Krinsky, Nucl. Instr. Meth. Phys. Res. Sect. A285, (1989) 119
[14] L. H. Yu and S. Krinsky, Nucl. Instr. Meth. Phys. Res. Sect. A407, (1998) 261
[15] Curt A. Flory, Appl. Phys. Lett. 99, (2006) 054903