EVOLUTION OF ELECTRON BEAM PHASE SPACE DISTRIBUTION IN A HIGH-GAIN FEL

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Abstract

FEL-based coherent electron cooling (CEC) offers a new avenue to achieve high luminosities in high energy colliders such as RHIC, LHC, and eRHIC. Traditional treatments consider the FEL as an amplifier of optical waves with specific initial conditions, focusing on the resulting field. CEC requires knowledge of the phase space distribution of the electron beam in the FEL. We present 1D analytical results for the phase space distribution of an electron beam with an arbitrary initial current profile.

INTRODUCTION

Coherent electron cooling (CEC) ([1] and references therein) is a new proposed method of cooling hadron beams in high energy storage rings. In contrast with electron cooling which becomes weaker with increasing energy, and stochastic cooling which is restricted by the bandwidth of the system, the cooling decrement for coherent electron cooling does not directly dependent on the hadron beam energy. This makes CEC ideal for implementation in next generation high energy hadron colliders, offering boosts in luminosity in colliders such as the LHC and Tevatron, and future electron-hadron colliders such as eRHIC, ELIC, and LHeC.

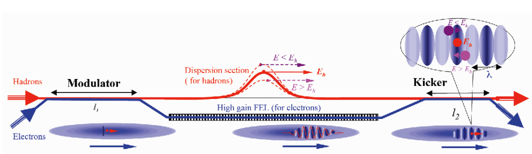

Figure 1: Schematic of CEC implementation.

CEC is structurally similar to stochastic cooling, and is schematically depicted in Figure 1. The hadron beam is combined with an electron beam with identical center of mass velocity, and leaves an imprint of the individual hadrons as charge perturbations in the electron beam. That beam is then passed through an FEL which amplifies the initial signal and modulates it into a wave packet. In the kicker section, the hadrons are recombined with the electron beam after passing through a chicane to create energy-dependent positioning. In this section, hadrons with too much or too little energy receive an energy-dependent kick, reducing the overall energy spread.

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Table 1: Proposed Parameters of a Proof-of-principle ERL and for eRHIC

	PoP	eRHIC
Energy $\mathcal{E}_0$	22 MeV	136.2 MeV
Gain Length Γ	0.486 m	2 m
Electron density n_0	1×10^{18} m^{-3}	2.08×10^{18} m^{-3}
Pierce parameter ρ	0.0097	.004
Space Charge $\hat{\Lambda}_p^2$	0.155	.0097

To implement this scheme, a detailed understanding of the phase space evolution of the initial phase space perturbation created by the hadron beam must be developed. In the case of the proof-of-principle system, with $\gamma \sim 45$, space charge effects will be important. We present initial results for the perturbation's evolution through a high-gain FEL using a 1D theory that accounts for space charge, as well as a correction to the self-consistent FEL equations of motion to first order in the Pierce parameter.

THEORETICAL RESULTS

A focus on the evolution of the phase space distribution of an electron bunch in a high-gain FEL to obtain the equations of motion for the output laser field [2]. This treatment develops the couples Maxwell-Vlasov equations, and combines them into a single current equation. Beginning with an approximate one-dimensional hamiltonian,

$$\mathcal{H} = c\sqrt{(p_z - \frac{e}{c}A_z)^2 + \left(\frac{e}{c}\right)^2 \left(\vec{A}_w + \vec{A}_\perp\right)^2 + m^2c^2} \tag{1}$$

where $\vec{A}_w$ is the vector potential on axis for a helical wiggler, $\vec{A}_\perp$ is the laser field, A_z is the longitudinal space charge field, and an appropriate gauge is chosen to make the scalar potential zero. Linearizing in the laser field and defining the phase space distribution $f = f_0 + f_1$ where f_0 is the thermal background and f_1 is the growing perturbation and $|f_1| \ll |f_0|$, the resulting linearized Vlasov equation is

$$\frac{\partial f_1}{\partial z} + \frac{1}{c}\left\{1 + \frac{1}{\gamma_z^2}\left(1 + 2\frac{\mathcal{E}}{\mathcal{E}_0}\right)\right\}\frac{\partial f_1}{\partial t} + \left\{-\frac{c}{\mathcal{E}_0}\left(1 + \frac{\mathcal{E}}{\mathcal{E}_0}\right)\left[\left(\frac{e}{c}\right)^2 \vec{A}_w \cdot \frac{\partial \vec{A}_\perp}{\partial t}\right] - eE_z\right\}\frac{\partial f_0}{\partial \mathcal{E}} = 0 \tag{2}$$

where $\mathcal{H} = \mathcal{E}_0 - \mathcal{E}$, $\gamma_z^{-2} = (1+K^2)\gamma_0^{-2}$ and $K^2 = (eA_w/mc^2)^2$. Introducing the general Fourier transform

$$G(z,t) = \frac{1}{\sqrt{2\pi}} \int d\nu\; e^{\imath k_w z + \imath\nu\omega_r(z/c-t)} \times \tilde{G}(z,\nu) \quad (3)$$

the slow-varying Maxwell equation and space charge is solved as

$$\begin{aligned} \vec{A}_w \cdot \vec{A}_\perp(z,\nu) &= \vec{A}_w \cdot \vec{A}_\perp(0,\nu) \\ &+ \frac{\pi K \imath}{\nu\omega_r\gamma_0} e^{\imath k_w z} \int_0^z \tilde{j}_1(z',\nu) dz' \end{aligned} \quad (4)$$

and

$$\tilde{E}_z = \frac{4\pi\imath}{\nu\omega_r}\tilde{j}_1 \quad (5)$$

By direct substitution into the Vlasov equation, this gives the Fourier transformed linearized Maxwell-Vlasov equation. Integrating over energy to obtain an equation for the current, we obtain

$$\begin{aligned} \mathcal{J}' = -ec\frac{\rho}{1+\hat{C}\rho} \int d\hat{P} \tilde{f}_1|_{\hat{z}=0} e^{-\imath(\hat{C}-\hat{P})\hat{z}} + \\ \int d\hat{P} \frac{d\hat{F}}{d\hat{P}} \int_0^{\hat{z}} d\hat{z}' e^{\imath(\hat{C}-\hat{P})(\hat{z}'-\hat{z})} \times \\ \left\{ \left(1 + \frac{\rho}{1+\hat{C}\rho}\right) \left(\mathcal{U}_\perp^{(0)} - \mathcal{J}\right) - \imath\hat{\Lambda}_p^2 \mathcal{J}' \right\} \end{aligned} \quad (6)$$

where $\mathcal{J} = \int_0^{\hat{z}} \tilde{j}_1 d\hat{z}'$.

The solution of this equation is carried out by use of Laplace transform techniques, keeping terms to first order in ρ. The Laplace transform yields an equation with the numerator having as source terms an initial energy modulation and an initial laser field. For applications to CEC, the initial laser field term $\mathcal{U}_\perp^{(0)}$ is set to zero, and we study the amplification of the initial charge density perturbations. Dropping the laser field term, the Fourier transformed solution is given by Equation 7

Assuming that any energy distribution in the initial modulation leads to an exponential damping term, the evolution is determined by the roots in terms of s of the equation

$$\begin{aligned} s + \int d\hat{P} \frac{d\hat{F}}{d\hat{P}} \frac{1 + \frac{\rho}{1-\hat{C}\rho}\hat{P}}{s + \imath(\hat{C}-\hat{P})} + \\ s\imath\hat{\Lambda}_p^2 \int d\hat{P} \frac{d\hat{F}}{d\hat{P}} \frac{1}{s+\imath(\hat{C}-\hat{P})} = 0 \end{aligned} \quad (8)$$

The small correction due to ρ is included, and results in a small correction to the growth exponents and phase evolution. For CEC, the phase information is an important quantity, and inclusion of some small correction may prove useful in future work.

As a particular case, we look at the situation of a cold background beam, i.e. $\hat{F}(\hat{P}) = \delta(\hat{P})$. In this case, the denominator becomes a cubic equation given by

$$s(s+\imath\hat{C})^2 - \left(\imath + \frac{1}{2}\frac{\rho}{1-\rho\hat{C}}(s+\imath\hat{C})\right) + s\hat{\Lambda}_p^2 = 0 \quad (9)$$

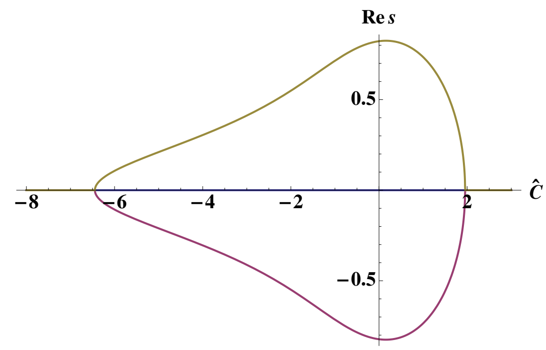

Figure 2: The real component of the roots for a cold beam. For the strong space charge effects present in the proof of principle, there is no growth for short wavelengths.

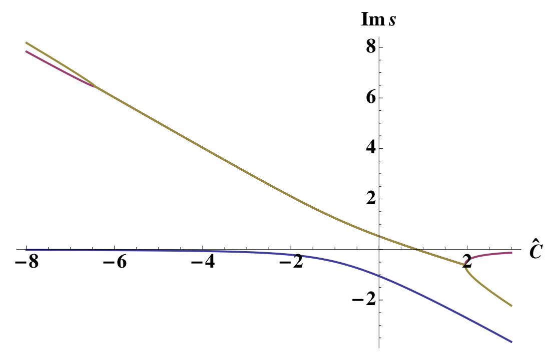

Figure 3: The imaginary component of the roots for a cold beam. The space charge effects have little effect on the phase compared to the zero space charge case.

The inclusion of the ρ parameter to first order offers small corrections to existing results in [2]. For particular application, we consider the eRHIC parameters given in Table 1.

The resulting roots are depicted in Figures 2 and 3. Compared to standard results, the space charge effects strongly reduces amplification at shorter wavelengths. The space charge effects have a minimal effect upon the phase evolution, but suppresses the peak growth, and narrows the bandwidth of growth substantially. This narrow bandwidth poses some potential troubles for the proof-of-principle, but will not be present at the operating energies for eRHIC.

APPLICATIONS TO ERHIC

eRHIC is a proposed upgrade that would allow collisions of spin-polarized electrons with hadrons, with, for example, 325 GeV protons and 20 GeV spin-polarized electrons [4]. Due to technical restrictions in spin-polarized electron current, the application of CEC to reduce hadron beam emittance is critical to reach the desired luminosities for the eRHIC upgrade. The appeals of CEC are the weak dependence of cooling time on energy, compared to traditional electron cooling, and its large bandwidth compared to stochastic cooling. To achieve this cooling us-

$$\int \tilde{j}_1 dz' = \int_{\gamma-\imath\infty}^{\gamma+\imath\infty} ds\, e^{s\hat{z}} \times \frac{-\frac{ec}{2}\frac{\rho}{1-\hat{C}\rho}\int d\hat{P}\tilde{f}_1|_{\hat{z}=0}\frac{1}{s+\imath(\hat{C}-\hat{P})}}{s+\int d\hat{P}\frac{d\hat{F}}{d\hat{P}}\frac{1+\frac{1}{2}\frac{\rho}{1-\hat{C}\rho}\hat{P}}{s+\imath(\hat{C}-\hat{P})}+s\imath\hat{\Lambda}_p^2\int d\hat{P}\frac{d\hat{F}}{d\hat{P}}\frac{1}{s+\imath(\hat{C}-\hat{P})}} \tag{7}$$

ing traditional electron cooling methods would take over 30 hours at eRHIC, whereas using CEC will take approximately six minutes for longitudinal cooling, and approximately twenty minutes for both longitudinal and transverse cooling [3].

This work begins the steps outlined in [5], providing a Green's function for the current growth given an initial signal. Accounting for energy spread, which for the full eRHIC upgrade is estimated at around 3×10^{-3}, and around 5×10^{-3} for the proof of principle ERL design [5], as well as the space charge effects in great detail, are necessary for working implementation of coherent electron cooling.

FUTURE WORK

Further investigations these results to CEC will require calculation for a thermal beam, the explicit solution for a particular initial condition $\tilde{f}_1(\hat{z} = 0, \nu, \hat{P})$, and the numerical generalization of results to include the full three-dimensional case. The final objective of this work is to calculate the cooling decrement of the FEL output signal on the hadron beam in the kicker of the CEC.

The first two requirements are coupled, as the input signal from the hadron beam is analytically related to the thermal distribution of the electron beam. Analytical results for the density perturbation generated by a charged hadron in an electron beam with Lorentzian and $\kappa = 2$ distributions, i.e.

$$f_0(\hat{P}) \propto \frac{1}{\left(1+(\hat{P}/\hat{P}_0)^2\right)^{\kappa}}$$

have been obtained [6], and these results are currently being generalized to obtain the phase space distribution. The current distribution is already known (see figure 4) for a variety of distributions, and generalization to obtain the phase space distribution is under way [7]. These results are three-dimensional, and will require some suitable reduction to the one-dimensional picture of the results obtained for the FEL model presented here. Having a Green's function for a thermal distribution and resulting imprinted signal is the next step in the 1D theory, and would allow direct analytical solution in frequency space for the wave packet output of the current. Three-dimensional numerical simulations for a given input is the last phase for simulations of the FEL output, and can be benchmarked against the 1D results.

Direct calculation of the cooling decrement and effects of the kicker on the energy spread of the hadron beam input is the ultimate goal of these calculations. Current calculations using simple models for the output signal indicate a cooling decrement weakly dependent on energy, but more precise calculations are desirable.

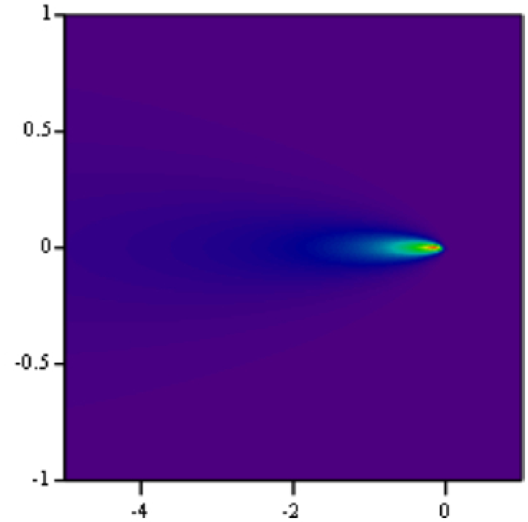

Figure 4: Taken from [6], the real space charge perturbation for a single hadron in a Lorentzian electron background. Lengths are scaled to the Debye length.

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