FEL AND OPTICAL KLYSTRON GAIN FOR AN ELECTRON BEAM WITH OSCILLATORY ENERGY DISTRIBUTION*

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Abstract

If the energy spread of a beam is larger then the Pierce parameter, the FEL gain length increases dramatically and the FEL output gets suppressed. We show that if the energy distribution of such a beam is made oscillatory on a small scale, the gain length can be considerably decreased. Such an oscillatory energy distribution is generated by first modulating the beam energy with a laser via the mechanism of inverse FEL, and then sending it through a strong chicane. We show that this approach also works for the optical klystron enhancement scheme. Our analytical results are corroborated by numerical simulations.

INTRODUCTION

If the energy spread of a beam is larger then the Pierce parameter, the FEL gain length rapidly increases with the rms energy spread. This can be easily illustrated with a 1D model [1]

$$\mu - \nu - \int d\eta \frac{dV(\eta)/d\eta}{\eta - \mu} = 0, \tag{1}$$

where $V(\eta)$ is the distribution function of the beam over the energy normalized by unity, $\int V(\eta)d\eta = 1$, η is the dimensionless energy deviation relative to the nominal one, $\eta = (\gamma - \gamma_0)/\rho\gamma_0$, γ is the Lorentz factor, γ_0 is the nominal beam energy in units of mc^2, ρ is the Pierce parameter, $k_u = 2\pi/\lambda_u$ with λ_u the undulator period, and ν is the relative frequency detuning. The parameter μ is the complex growth rate of the radiation field in the undulator measured in units $2\rho k_u$.

For a Gaussian distribution function

$$f = (\sqrt{2\pi}\sigma_\eta)^{-1} e^{-\eta^2/2\sigma_\eta^2}, \tag{2}$$

where σ_η is the rms energy spread of the beam in dimensionless energy units. It is easy to find that in the limit $\sigma_\eta \to 0$ (that is for $f = \delta(\eta)$) the optimal detuning is $\nu = 0$ and $\mu \approx \mu_0 = (-1 + i\sqrt{3})/2 = -0.5 + 0.87i$. For $\sigma_\eta = 1$ and an optimized detuning, $\mathrm{Im}\,\mu = 0.44$. In the limit of large σ_η, the growth rate $\mathrm{Im}\,\mu$ becomes small, and the imaginary part of the integral in (1) is approximately given by the residue of the integral taken at $\eta = \mathrm{Re}\,\mu$, $\mathrm{Im}\,\mu = \pi \left.\frac{df}{d\eta}\right|_{\eta=\mathrm{Re}\,\mu}$. Noting that the real part of μ can be varied by changing detuning ν in (1) we conclude that the maximum value of $\mathrm{Im}\,\mu$ is given by [2]

$$\max \mathrm{Im}\,\mu = \pi \max \left|\frac{df}{d\eta}\right| = 0.61\sqrt{\frac{\pi}{2}}\frac{1}{\sigma_\eta^2}. \tag{3}$$

As numerical calculations show, this asymptotic dependence of $\max \mathrm{Im}\,\mu$ gives a good approximation to the exact value when $\sigma_\eta \gtrsim 2$.

In this paper we show that even when the energy spread of the beam is large, the gain length for the FEL instability can be considerably decreased if the energy distribution function is made oscillatory over energy. As we show in the next section, this kind of energy distribution function can be created using a laser beam with a tuned undulator, and a strong chicane.

GENERATING OSCILLATORY DISTRIBUTION FUNCTION

A system that creates an oscillatory energy distribution function is shown in Fig. 1: it consists of an undulator and a laser beam, which are synchronized with the electron beam

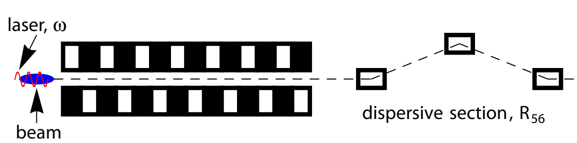

Figure 1: An undulator followed by a chicane. The beam energy is modulated in the undulator due to interaction with a laser beam.

in such a way that the electron beam energy becomes modulated over energy after the passage through the modulator. Typically, the bunch length is much larger than the laser wavelength λ_L, and one can locally consider a longitudinally uniform beam, neglecting variation of the beam current over the distance of several laser wavelength. The undulator is followed by a chicane whose strength is characterized by the parameter R_{56}.

Assuming a Gaussian distribution function (2) before the undulator, the distribution function after the chicane is (see, e.g., [3])

$$f(\zeta, \eta) = \frac{1}{\sqrt{2\pi}\sigma_\eta} e^{-\frac{1}{2\sigma_\eta^2}(\eta - A\sin(\zeta - B\eta))^2}, \tag{4}$$

where $A = \Delta\gamma/\rho\gamma_0$, $B = R_{56}q\rho$, $\zeta = qz$, $\Delta\gamma$ is the amplitude of the energy modulation in units mc^2, and $q = 2\pi/\lambda_L$ is the wave number of the laser. Note that

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normalization of energy and the strength of the chicane involves the Pierce parameter ρ.

We consider an example of a large initial energy spread $\sigma_\eta = 4$ and modulate the beam with $A = 4$. After passing through the chicane, the energy distribution function of the beam becomes oscillatory as shown in Fig. 2 for the case $B = 2$. Note that the distribution function depends on the longitudinal coordinate ζ in the beam being a periodic function of z with the period λ_L. In Fig. 2 we show two plots corresponding to locations $\zeta = 0$ and $\zeta = 0.5\lambda_L$.

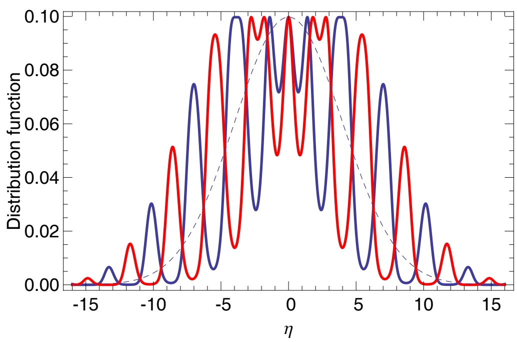

Figure 2: Distribution functions of the beam after the chicane at $z = 0.5\lambda_L$ (blue solid line) and $z = 0$ (red line) for $B = 2$. For comparison, the dashed line shows the original Gaussian with $\sigma_\eta = 4$.

The number of oscillations in energy is proportional to the dimensionless strength of the chicane B, and the width of the fine structure on the energy distribution is inversely proportional to B. Note also that modulation of the beam energy increases the rms energy spread in the beam from σ_η to $\sqrt{\sigma_\eta^2 + A^2/2}$. In the above example this means that the energy spread of the beam is increased from the initial $\sigma_\eta = 4$ to the rms value 4.9.

GAIN LENGTH FOR OSCILLATORY BEAM DISTRIBUTION

Using Eq. (1) we numerically calculated the parameter $\mathrm{Im}\,\mu$ for the distribution functions corresponding to the chicane strengths $B = 1$, 2, and 4.5. The results of such calculations for the case $B = 2$ are shown in Fig. 3.

For a smooth gaussian distribution function, as it follows from Eq. (3) for $\sigma_\eta = 4$, the inverse growth length is $\mathrm{Im}\,\mu = 0.047$. One can see from Fig. 3 that the maximum value of $\mathrm{Im}\,\mu$ increases (from ≈ 0.05) to 0.22, more than 4 times. Calculations carried out for $B = 1$ and 4.5 give the maximum values of $\mathrm{Im}\,\mu$ equal to 0.17, and 0.25, respectively.

It is important to emphasize that, as seen from Fig. 3, the position of the maximum growth rate varies with the coordinate z in the beam (it is a periodic function of z with the period λ_L). Due to the slippage of radiation relative to the beam, if the slippage length is not small compared

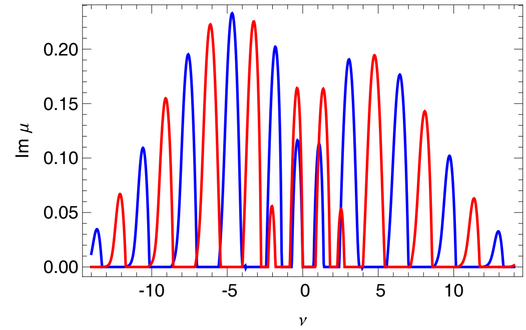

Figure 3: Plot of $\mathrm{Im}\,\mu$ versus detuning ν for $z = 0$ (blue solid line) and $z = 0.5\lambda_L$, for $B = 2$.

with λ_L, this can lead to a detuning of the radiation field in the process of its exponential growth. The effect of the slippage can be estimated in the following way. One can see from Fig. 3 that, for a given detuning ν, a shift in z by half a laser wavelength changes the growth rate from its maximum to almost zero. If this shift happens on the gain length L_g, it will strongly suppress the FEL process. Hence the condition, when the slippage can be neglected is

$$\lambda_r \frac{L_g}{\lambda_u} \ll \frac{1}{2}\lambda_L \, , \tag{5}$$

where λ_r is the wavelength of the radiation, and on the left we have an estimate of the slippage on the gain length.

To compare with our analytical theory, we perform 1D FEL simulations using the following parameters: electron energy is 1.2 GeV, peak current is 2 kA, the normalized emittance is 1 μm, undulator period is 3 cm and the beta

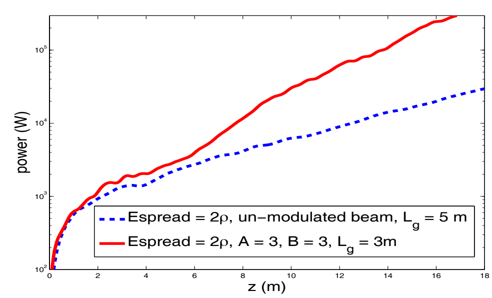

Figure 4: Results of 1D simulations. FEL power as a function of distance z from the entrance to the undulator for two cases: a Gaussian beam with an initial energy spread of $\sigma_\eta = 2$ (blue line) and the same beam after passing through the system shown in Fig. 1 with $A = 3$ and $B = 3$ (red line).

function in the undulator is 4 m. We choose the energy modulator laser wavelength to be 2.4 μm, and the final radiation wavelength to be about 5 nm. With these parameters, we have $\rho = 2.2 \times 10^{-3}$ and choose the beam energy spread $\sigma_\eta = 2\rho = 4.4 \times 10^{-3}$ for the simulation. We also use $A = 3$ and $B = 3$ for the modulated case. As shown in Fig. 4, the beam with the oscillatory energy distribution has a gain length of about 3 m, while the beam with the Gaussian distribution has a gain length of about 5 m. Note that the above 1D theory predicts about a factor of 2 enhancement in gain length using these parameters. The gain enhancement effect is slightly reduced in these simulations presumably due to slippage effect not taken into account in the theory.

3D COMPUTER SIMULATIONS

We used the three dimensional (3D) FEL simulation code Genesis 1.3 [4] to check the gain enhancement effect for a beam with an oscillatory energy distribution. The parameters of the beam and the undulators were chosen close to the LCLS soft x-ray (1.5 nm) parameters, with undulator period 3 cm, K = 3.5, electron energy 4.3 GeV, and an normalized emittance of 0.4 μm. With these parameters, we have $\rho = 1.6 \times 10^{-3}$. With an initial energy spread $\sigma_\eta = 1$ (corresponding to the rms energy spread of 6.9 MeV with real parameters), Genesis simulations give a gain length of about 3.4 m (see Fig. 5).

To generate an oscillatory energy distribution of the electrons, we choose a 4 μm wavelength laser to interact with the electron beam in the modulator. This laser wavelength satisfies the condition in Eq. (5). The amplitude of the energy modulation is equal to $A = 3$. The chicane is set at $B = 3$. After passing through the chicane, the energy distribution of the electron beam becomes oscillatory with the rms energy spread increased to $\sigma_\eta = 2.34$ (corresponding to 16.1 MeV). This oscillatory-distribution beam is then read into Genesis for FEL simulations. The results of the

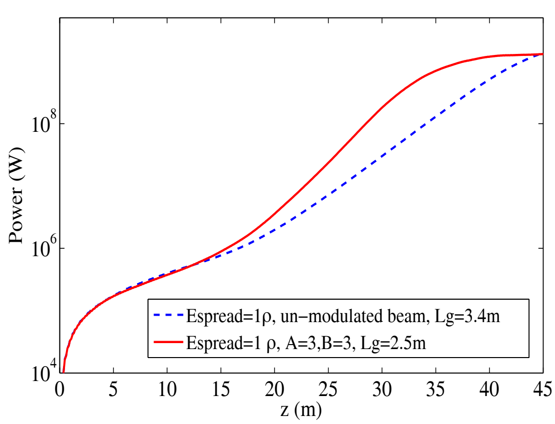

Figure 5: FEL power as a function of distance z from the entrance to the undulator for two cases: a Gaussian beam with an initial energy spread of $\sigma_\eta = 1$ (blue dashed line) and the same beam after passing through the system shown in Fig. 1 with $A = 3$ and $B = 3$ (red solid line).

simulations are shown in Fig. 5: the gain length is equal to 2.5 m in this case. The gain enhancement is about 1.36 compared with the Gaussian beam. Shown in Fig. 6 is the longitudinal phase space distribution of the particles at the

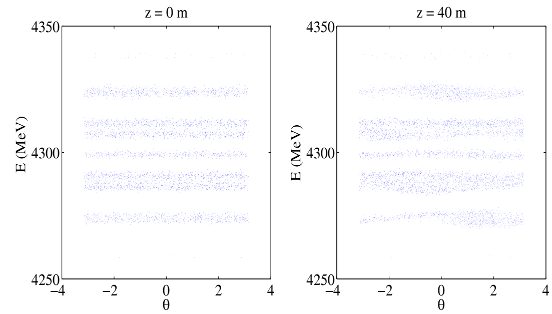

Figure 6: Particle distribution at the entrance of the undulator and at $z = 40$ m, for the case of the oscillatory distribution function of the beam. The horizontal coordinate θ is the longitudinal position normalized by $\lambda_r/2\pi$.

entrance of the undulator and right after saturation ($z = 40$ m) point for the modulated case.

We also performed Genesis simulation for another case with two times larger initial energy spread of $\sigma_\eta = 2$ and the same parameters $A = 3$ and $B = 3$. The results of that simulation which show the gain enhancement for the oscillatory case of about 1.6, are shown in Fig. 7.

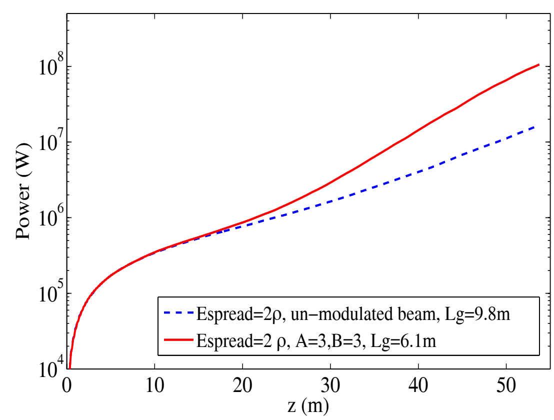

Figure 7: FEL power as a function of distance z from the entrance to the undulator for two cases: a Gaussian beam with an initial energy spread of $\sigma_\eta = 2$ (blue dashed line) and the same beam after passing through the system shown in Fig. 1 with $A = 3$ and $B = 3$ (red solid line).

OPTICAL KLYSTRON FEL

As discussed earlier, if the beam energy spread is much less than the FEL ρ parameter, the high gain FEL process is not sensitive to the detailed energy distribution. However, a high-gain optical klystron (OK) can take advantage of a very small energy spread (much smaller than ρ) to speed up the bunching process [5]. Here we investigate whether such a scheme can benefit from an oscillatory energy distribution.

The 1D theory for a high-gain optical klystron FEL including SASE effects can be found in Ref. [6]. Here we write down the OK enhancement factor to the radiation field E_ν at the resonant frequency $\omega = \omega_r$ and neglect the phase matching effect:

$$R \equiv \frac{E^{\rm OK}}{E^{\rm no\ OK}} = \frac{1 - \int d\eta \frac{dV(\eta)/(d\eta)}{(\mu-\eta)^2} e^{-iD\eta}}{1 + 2\int d\eta \frac{V(\eta)}{(\mu-\eta)^3}}\,, \tag{6}$$

where $D = R_{56}k_r\rho$, and we use the same notation as in Eq. (1).

Treating $|\eta| \ll |\mu_0|$ in Eq. (6) and integrating the numerator by part, we have

$$R \approx \frac{1}{3}\left[1 + \left(2 - \frac{iD}{\mu_0^2}\right)\int d\eta V(\eta) e^{-iD\eta}\right]. \tag{7}$$

The gain enhancement comes mainly from the last term that is proportional to the dispersion strength D:

$$R_3 = -\frac{iD}{3\mu_0^2}\int d\eta V(\eta) e^{-iD\eta}\,. \tag{8}$$

For an oscillatory energy distribution as described in Eq. (4), the distribution function varies along the longitudinal position z as shown in Fig. 2. Assuming the modulation wavelength is much longer than the relevant slippage length in the FEL undulator, we can choose a representative z-location for the energy distribution as

$$\begin{aligned} V(\eta) \approx& f\left(\zeta = \frac{\pi}{2}, \eta\right) \\ =& \frac{1}{\sqrt{2\pi}\sigma_\eta}\exp\left[-\frac{(\eta - A\cos(B\eta))^2}{2\sigma_\eta^2}\right] \\ \approx& \frac{1}{\sqrt{2\pi}\sigma_\eta}\exp\left(-\frac{\eta^2}{2\sigma_\eta^2}\right)\left[1 + \frac{\eta A}{\sigma_\eta^2}\cos(B\eta)\right]. \end{aligned} \tag{9}$$

The last approximation was obtained by considering $A < \sigma_\eta$. Putting this energy distribution into Eq. (8), we obtain

$$\begin{aligned} R_3 = -\frac{iD}{3\mu_0^2}\Big[& e^{-D^2\sigma_\eta^2/2} - \frac{iA}{2}(D+B)e^{-(D+B)^2\sigma_\eta^2/2} \\ & - \frac{iA}{2}(D-B)e^{-(D-B)^2\sigma_\eta^2/2}\Big]. \end{aligned} \tag{10}$$

The first term is the OK gain for a smooth Gaussian energy distribution (i.e., $A = 0$). Its amplitude is maximized when $D = \pm 1/\sigma_\eta$. The second and the third terms are maximized when $D = \pm(1/\sigma_\eta + B)$ for an oscillatory energy distribution. The ratio of the optimized second/third term to the optimized first term is

$$\left|\frac{R_3(A)}{R_3(A=0)}\right| = \frac{A}{2}\left(\frac{1}{\sigma_\eta} + B\right) \approx \frac{AB}{2} \tag{11}$$

for $AB \gg 1$. Thus, an oscillatory energy distribution will improve the OK gain factor compared to a smooth energy distribution. This is true even when $\sigma_\eta \to 1$.

We check our approximate analytical result with 1D FEL simulations. The simulation is carried out in the SASE mode at 5 nm radiation wavelength using parameters described for Fig. 4, except that we take $\sigma_\eta = 0.3$. We then modulate the electron beam with $A = 0.9$ at $\lambda_L = 2.4$ μm and pass the beam through a chicane with $B = 15$. After the chicane, the modulated beam is sent through the FEL undulator in the optical klystron configuration: after the beam interacts with the radiation in the first part of the undulator, a chicane is introduced to bunch the beam at 5 nm before sending into the second part of the undulator. The bunching gain factor vs the dispersion strength at the beginning of the second undulator is shown in Fig. 8. The bunching maximizes at $D \approx 16$, which is in reasonable agreement with the expected optimal $D = 1/\sigma_\eta + B = 18$.

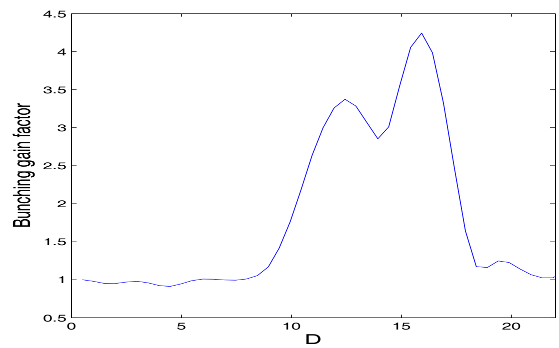

Figure 8: Bunching gain factor as a function of dispersion strength D.

CONCLUSIONS

In this paper we demonstrated that an oscillatory energy distribution function of an electron beam exhibits a shorter FEL gain length than a smooth Gaussian distribution. An oscillatory distribution function can be obtained by means of a laser beam interacting with the electron beam in an undulator-modulator followed by a chicane. The proposed method of shortening of the gain length might be useful, in particular, for FELs based on electron beams generated in a laser-plasma wakefield accelerator which are characterized by relatively large energy spread [7, 8].

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