THREE-FREQUENCY UNDULATOR RADIATION AND FREE-ELECTRON LASER GAIN

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Abstract

In this paper we study three-frequency undulator radiation and free-electron laser gain for higher harmonics. The three-frequency undulator gives higher intensities and gain at odd higher harmonics. An analytical expression for the intensity for the undulator radiation and laser gain is presented for comparison with the standard planar undulator and other biharmonic undulator schemes.

INTRODUCTION

The free-electron laser [1-4] is a popular coherent electromagnetic radiation source at wavelengths from microwave to x-rays. The spectral feature of the radiation incorporates the qualities of the relativistic electron and the undulator field as well. The undulator field is the key component in the free-electron laser. The free electron laser gain is influenced by both the undulator structure and the quality of the field. In its simplest feature the undulator field is planar but several new novel schemes have been proposed [5-19] over the years for several other applications of the laser. The harmonic undulator [9-20] and the two- frequency [21] are the promising undulator scheme that has been proposed in the past. Niculescu et.al [22] has proposed the scheme of three frequency undulator scheme as a source of effective synchrotron radiation source. In this paper we reconsider the theory of three frequency undulator and discuss the small signal free electron gain.

THREE FREQUENCY UNDULATOR RADIATION

We assume that the electron moves on axis in a three frequency undulator, whose on axis field is given by [22],

$$\vec{B}=\left[0,\hat{y}B_0 a_1\left\{\sin\omega_u t+\frac{a_2}{a_1}\sin 2\omega_u t+\frac{a_3}{a_1}\sin 3\omega_u t\right\},0\right] \tag{1}$$

Where, $k_u = 2\pi/\lambda_u$, λ_u is the undulator wavelength, B_0 is peak field strength. The undulator field is composed of three harmonic fields of amplitudes $B_0 a_1$, $B_0 a_2$ and $B_0 a_3$ respectively. The velocity and electron trajectory of electron can be written as

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\omega_u t)+\delta_2\cos(2\omega_u t)+\delta_3\cos(3\omega_u t)\right]$$

$$\beta_z = 1-\frac{1+\frac{K^2}{2}\left(1+\delta_2^2+\delta_3^2\right)}{2\gamma^2}-\frac{K^2}{4\gamma^2}\left[\begin{array}{l}\cos 2(\omega_u t)+\delta_2^2\cos 2(2\omega_u t)+\\ \delta_3^2\cos 2(3\omega_u t)+2\delta_2\left\{\begin{array}{l}\cos(2\omega_u+\omega_u)t\\ +\cos(2\omega_u-\omega_u)t\end{array}\right\}\\ +2\delta_3\left\{\begin{array}{l}\cos(3\omega_u+\omega_u)t\\ +\cos(3\omega_u-\omega_u)t\end{array}\right\}\\ +2\delta_2^2\delta_3^2\left\{\begin{array}{l}\cos(3\omega_u+2\omega_u)t\\ +\cos(3\omega_u-2\omega_u)t\end{array}\right\}\end{array}\right]$$

$$x(t)=-\frac{Kc}{\gamma}\left[\left(\frac{1}{\omega_u}\right)\sin(\omega_u t)+\left(\frac{\delta_2}{2\omega_u}\right)\sin(2\omega_u t)+\left(\frac{\delta_3}{3\omega_u}\right)\sin(3\omega_u t)\right]$$

$$\begin{aligned}z(t)={}&\beta^* ct-\frac{K^2c}{4\gamma^2}\left(\frac{1}{2(\omega_u)}\right)\sin 2(\omega_u t)-\frac{K^2c}{4\gamma^2}\left(\frac{\delta_2^2}{2(2\omega_u)}\right)\sin 2(2\omega_u t)\\ &-\frac{K^2c}{4\gamma^2}\left(\frac{\delta_3^2}{2(3\omega_u)}\right)\sin 2(3\omega_u t)-\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_2}{2\omega_u+\omega_u}\right)\sin(2\omega_u+\omega_u)t-\\ &\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_2}{2\omega_u-\omega_u}\right)\sin(2\omega_u-\omega_u)t-\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_3}{3\omega_u+\omega_u}\right)\sin(3\omega_u+\omega_u)t\\ &-\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_3}{3\omega_u-\omega_u}\right)\sin(3\omega_u-\omega_u)t-\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_2\delta_3}{3\omega_u+2\omega_u}\right)\sin(3\omega_u+2\omega_u)t-\\ &\frac{K^2c}{4\gamma^2}\left(\frac{2\delta_2\delta_3}{3\omega_u-2\omega_u}\right)\sin(3\omega_u-2\omega_u)t\end{aligned} \tag{2}$$

Where,

$K=\frac{eB_0 a_1\lambda_u}{2\pi m_0 c^2}$ is the undulator parameter,

$$\delta_2=\left(\frac{\omega_u a_2}{2\omega_u a_1}\right),\ \delta_3=\left(\frac{\omega_u a_3}{3\omega_u a_1}\right)$$

and

$$\beta^* = 1-\frac{1}{2\gamma^2}\left[1+\frac{K^2}{2}\left\{1+\delta_2^2+\delta_3^2\right\}\right]$$

Using Lienard-Wiechert integral [23], we get the spectral properties of the radiation by electrons,

$$\frac{d^2 I}{d\omega d\Omega}=\frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\left\{\hat{n}\times\left(\hat{n}\times\vec{\beta}\right)\right\}\exp\left[i\omega\left(t-\frac{z}{c}\right)\right]dt\right|^2 \tag{3}$$

The brightness expression integrated over the limit 0 to T where $T=\frac{N\lambda_u}{c\beta_z}$ and now Equation (3) is reduces to

$$\frac{d^2 I}{d\omega d\Omega}=\frac{e^2\omega^2 T^2}{4\pi^2 c}\left(\frac{\sin(\nu t/2)}{\nu t/2}\right)^2\left\{\left|T_x^2\right|\right\} \tag{4}$$

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Where the intensity coefficient T_x is given by (in GBF),

$$T_x = \frac{K}{2\gamma}\left[(b_0) + \delta_2(b_1) + \delta_3(b_2)\right]J_{q_+q_-r_+r_-s_+s_-} \tag{5}$$

and b_0, b_1, b_2 and $J_{q_+q_-r_+r_-s_+s_-}$ are defines as;

$$\begin{aligned} b_0 &= \left[J_{m+1}(0,x_1) + J_{m-1}(0,x_1)\right]J_n(0,x_2)J_p(0,x_3) \\ b_1 &= \left[J_{n+1}(0,x_2) + J_{n-1}(0,x_2)\right]J_m(0,x_1)J_p(0,x_3) \\ b_2 &= \left[J_{p-1}(0,x_3) + J_{p+1}(0,x_3)\right]J_m(0,x_1)J_n(0,x_2) \end{aligned} \tag{6.a}$$

$$J_{q_+q_-r_+r_-s_+s_-} = J_{q_+}(x_4)J_{q_-}(x_5)J_{r_+}(x_6)J_{r_-}(x_7)J_{s_+}(x_8)J_{s_-}(x_9) \tag{6.b}$$

The detuning parameter ν in Equation (4) is given by,

$$\nu = \frac{\omega}{2\gamma^2}\left[1 + \frac{K^2}{2}\left(1 + \delta_2^2 + \delta_3^2\right)\right] - \phi_h \tag{7}$$

Where

$$\phi_h = m\omega_u + n(2\omega_u) + p(3\omega_u) + q_+(2\omega_u + \omega_u) + q_-(2\omega_u - \omega_u) + r_+(3\omega_u + \omega_u) + r_-(3\omega_u - \omega_u) + s_+(3\omega_u + 2\omega_u) + s_-(3\omega_u - 2\omega_u)$$

And the arguments of the Generalized Bessel functions are given as follows:

$$x_1 = -\frac{K^2\omega}{4\gamma^2}\left(\frac{1}{2(\omega_u)}\right),\quad x_2 = -\frac{K^2\omega\delta_2^2}{4\gamma^2}\left(\frac{1}{2(2\omega_u)}\right),\quad x_3 = \frac{K^2\omega\delta_3^2}{4\gamma^2}\left(\frac{1}{2(3\omega_u)}\right),$$

$$x_4 = -\frac{K^2\omega}{4\gamma^2}(2\delta_2)\left(\frac{1}{(2\omega_u + \omega_u)}\right),\quad x_5 = -\frac{K^2\omega}{4\gamma^2}(2\delta_2)\left(\frac{1}{(2\omega_u - \omega_u)}\right),$$

$$x_6 = -\frac{K^2\omega}{4\gamma^2}(2\delta_3)\left(\frac{1}{(3\omega_u + \omega_u)}\right),\quad x_7 = -\frac{K^2\omega}{4\gamma^2}(2\delta_3)\left(\frac{1}{(3\omega_u - \omega_u)}\right),$$

$$x_8 = -\frac{K^2\omega}{4\gamma^2}(2\delta_2\delta_3)\left(\frac{1}{(3\omega_u + 2\omega_u)}\right),\quad x_9 = -\frac{K^2\omega}{4\gamma^2}(2\delta_2\delta_3)\left(\frac{1}{(3\omega_u - 2\omega_u)}\right)$$

The radiation wavelength is read from the resonance condition,

$$\omega = \frac{2\gamma^2\phi_h}{1 + \frac{K^2}{2}\left(1 + \delta_2^2 + \delta_3^2\right)} \tag{8}$$

The three-frequency undulator radiation is given by Equation (4). The line shape factor $\left(\frac{\sin(\nu/2)}{\nu/2}\right)^2$ is the same as that of the conventional planar undulator. So no broadening of the spectrum occurs by the addition of the harmonic undulator fields.

FREE ELECTRON LASER GAIN

Now we calculate the small signal gain of the three frequency undulator free electron laser. Let us consider a radiation field as,

$$\vec{E} = \hat{x}E_0\cos(sk_1z - s\omega_1t + \phi) \tag{9}$$

Where, s is the harmonic number and ϕ is the phase of the electron with the radiation field. The energy transfer equation of the electron in free electron laser is given by,

$$\frac{d\gamma}{dt} = -\frac{e}{m_0c}\left[\vec{E}.\vec{\beta}\right] \tag{10}$$

We get the solution of Equation (10) with $\psi = sk_1z - s\omega_1t + \phi$

$$\frac{d\gamma}{dt} = \frac{eE_0K}{2m_0c\gamma}\left[\left\{\cos(\psi + k_uz) + \cos(\psi - k_uz)\right\} + \delta_2\left\{\begin{matrix}\cos(\psi + 2k_uz) \\ +\cos(\psi - 2k_uz)\end{matrix}\right\} + \delta_3\left\{\begin{matrix}\cos(\psi + 3k_uz) \\ +\cos(\psi - 3k_uz)\end{matrix}\right\}\right] \tag{11}$$

The electron longitudinal motion is expressed by $z = \bar{z} + \Delta z$. Substituting the longitudinal electron motion, the phase terms appearing in Equation (11) are solved and simplified as,

$$\psi \pm (k_uz) = s\xi_0 + \varphi + sk_1\Delta z - (s \mp 1)k_uz - s2k_uz - s3k_uz$$

$$\psi \pm (2k_uz) = s\xi_0 + \varphi + sk_1\Delta z - (s \mp 1)2k_uz - sk_uz - s3k_uz$$

$$\psi \pm (3k_uz) = s\xi_0 + \varphi + sk_1\Delta z - (s \mp 1)3k_uz - sk_uz - s2k_uz$$

Where $\xi_0 = \left[(k + k_u + 2k_u + 3k_u)\bar{z} - \omega_1t\right]$,

Now, Equation (11) is solved using above relation and after averaging over the undulator,

We get,

$$\frac{d\gamma}{d\tau} = \frac{eKE_0}{4\gamma m_0c\lambda_u}\left[b_0 + \delta_2b_1 + \delta_3b_2\right]J_{q_+q_-r_+r_-s_+s_-}\cos(s\xi_0 + \phi) \tag{12}$$

Expressing

$$\frac{d^2\xi_0}{d\tau^2} = \dot{\nu} = \frac{4\pi N\dot{\gamma}}{\gamma},\quad s\xi_0 = \xi_s \text{ and}$$

dimensionless optical field as,

$$|a_s| = \frac{\pi NeKE_0}{\gamma^2m_0c\lambda_u}s\left[b_0 + \delta_2b_1 + \delta_3b_2\right]J_{q_+q_-r_+r_-s_+s_-}$$

Now, equation (12) can be written as

$$\frac{d^2\xi_s}{d\tau^2} = |a_s|\cos(\xi_s + \phi) \tag{13}$$

The wave equation for the vector potential $\vec{A}$ is written as

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\vec{A} = -\frac{4\pi}{c}\vec{J}_t \tag{14}$$

The vector potential is calculated from

$$E = -\frac{1}{c}\frac{\partial A}{\partial t}$$

and substituted in Equation (14) and simplified to its final form in terms of the dimensionless optical field

$$\frac{\partial a_s}{\partial \tau} = -\frac{\pi^2 Ne^2 K^2}{\gamma^3 m_0 \lambda_u^2} s\left[b_0 + \delta_2 b_1 + \delta_3 b_2\right]^2 \left|J_{q_+q_-r_+r_-s_+s_-}\right|^2 \langle \exp(-i\xi_s)\rangle,$$

We write the dimensionless current density as

$$j_s = \frac{\pi^2 Ne^2 K^2}{\gamma^3 m_0 \lambda_u^2} s\left[b_0 + \delta_2 b_1 + \delta_3 b_2\right]^2 \left|J_{q_+q_-r_+r_-s_+s_-}\right|^2$$

to express the gain as

$$G_s = -\frac{j_s}{\nu_0^3}\left[2 - 2\cos(\nu_0) - \nu_0 \sin(\nu_0)\right] \tag{15}$$

RESULTS AND DISCUSSION

A new three-frequency undulator scheme as proposed [22] has been studied. The electron longitudinal velocity gets modulated at frequencies of $\omega_u, 2\omega_u, 3\omega_u$ as well as at sum-difference frequencies i.e. $2\omega_u + \omega_u, 2\omega_u - \omega_u, 3\omega_u + \omega_u$, $3\omega_u - \omega_u, 3\omega_u + 2\omega_u and 3\omega_u - 2\omega_u$. As a consequence the system generates several emission harmonics at these oscillations seen in eq. (8). The intensity enhancement at the third harmonic is displayed in fig.1. The free electron laser gain is calculated for this undualtor scheme and the analysis shows that substantial gain can be extracted at the third harmonic.

To conclude we have analyzed the undulator radiation and free electron gain with a three-frequency undulator scheme. Several issues have been reflected in the analysis. First, the system generates several harmonic due to sum- difference frequency emissions. Second, the detuning function remains unchanged. So spectrum broadening is not induced by the three frequency scheme. Third, the undulator supports high intensity and gain at higher odd harmonic enabling one to extract higher gain at higher harmonic. Fourth, one can get even harmonic axis. However due to less intensity and gain, these harmonics are not be distinctly marked.

The harmonic undulator consisting of second harmonic and third harmonic fields is primarily dominant at odd harmonics [17-19]. In view of the results of the study we propose alternate scheme of the three- frequency undulator scheme with odd harmonic integers such as

$$\vec{B} = \left[0, \hat{y} B_0 a_1 \left\{\sin \omega_u t + \frac{a_3}{a_1}\sin 3\omega_u t + \frac{a_5}{a_1}\sin 5\omega_u t\right\}, 0\right]$$

Another useful extension to D.Iracane, et.al [21] may be realized as,

$$\vec{B} = \left[0, \hat{y} B_0 a_1 \left\{\sin \omega_{u_1} t + \frac{a_2}{a_1}\sin \omega_{u_2} t + \frac{a_3}{a_1}\sin \omega_{u_3} t\right\}, 0\right].$$

Where, $\omega_{u_1} \neq \omega_{u_2} \neq \omega_{u_3}$. The desired three-frequency magnetic field can be achieved by arranging high–permeability shims inside the gap of conventional planar undulator [12,24-25]. The spectral brightness and free-electron laser gain with these undulator scheme will be further investigated and reported in future works.

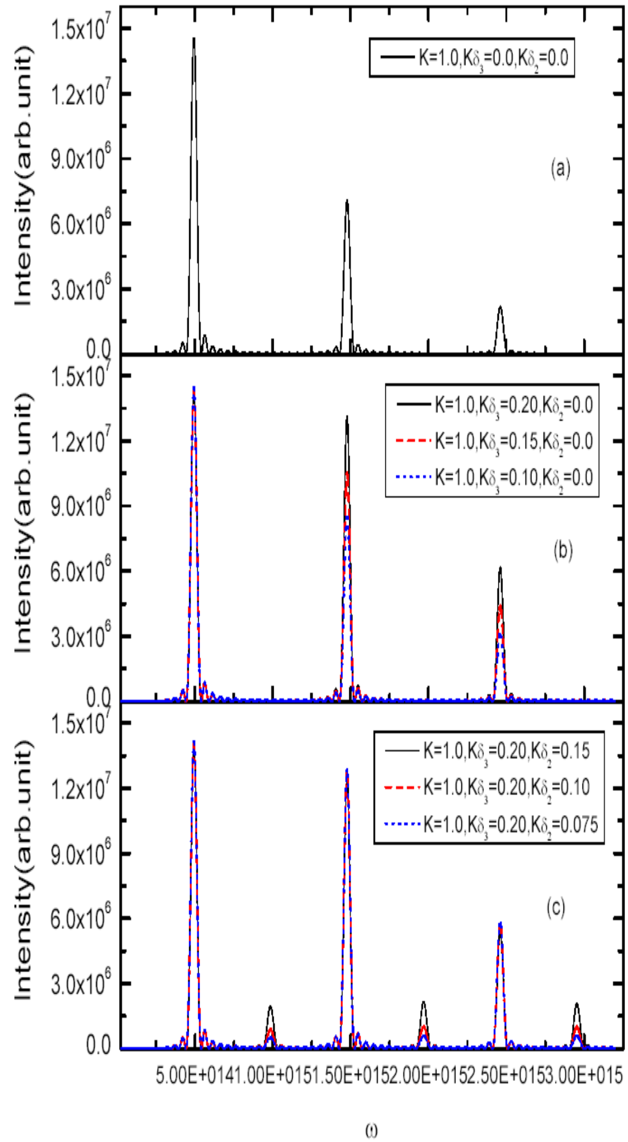

Figure 1: (a).Intensity versus frequency without $K\delta_2$ and $K\delta_3$. (b).Intensity versus frequency for various values of $K\delta_3 = 0.20, 0.15, 0.10$ respectively. (c).Intensity versus frequency for constant value of $K\delta_3 = 0.20$ and various values of $K\delta_2 = 0.15, 0.10, 0.075$ respectively.

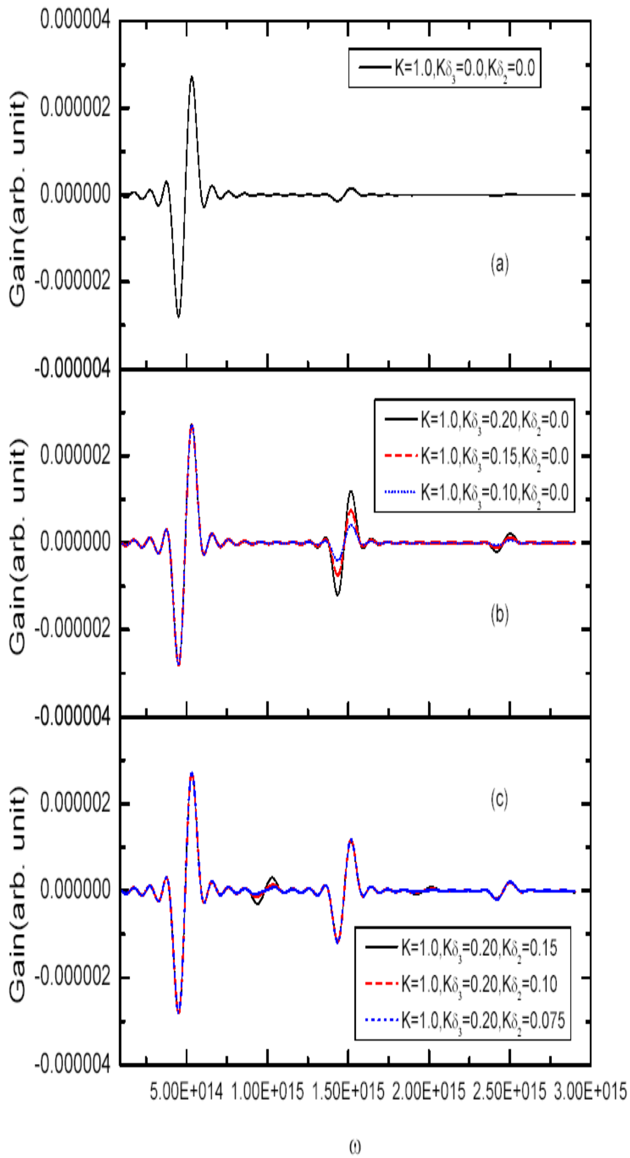

Figure 2: (a).Gain curves versus frequency without $K\delta_2$ and $K\delta_3$.(b).Gain curves versus frequency for various values of $K\delta_3 = 0.20, 0.15, 0.10$ respectively. (c).Gain curves versus frequency for constant value of $K\delta_3 = 0.20$ and various values of $K\delta_2 = 0.15, 0.10, 0.075$ respectively.

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