HARMONIC UNDULATOR RADIATION AND FREE ELECTRON LASER GAIN WITH TWO PEAK ELECTRON BEAM ENERGY DISTRIBUTION

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Abstract
In this paper we include the important influence of beam energy spread on harmonic undulator radiation. A two peak electron beam energy distribution is considered to analyze the spectrum broadening and gain reduction in the harmonic undulator free electron laser.

INTRODUCTION

An relativistic electron beam with non negligible energy spread and emittance traversing the undulator emits radiation whose spectrum is in homogeneously broadened. The reduction is more pronounced for the higher harmonics since the effect is proportional to the order of the harmonics. The spectrum distortion affects the laser gain [1].

Works on harmonic undulators [2-15] have been reported for intensity and gain enhancement at higher harmonics. Both single and the dual harmonic wiggler have been proved effective in improving the harmonic radiation. In this paper we analyse the intensity and the dependence of the spectrum broadening on the energy distribution in the case of harmonic undulators. The beam energy spread induced intensity degradation and spectrum broadening has been analyzed. The beam energy spread drives the electron away from the resonance condition thus causing a reduction in free electron laser gain. An addition of the harmonic field at $k_h = hk_u$ brings back the free electron laser to the resonance condition causing gain improvement. The scheme works effectively at higher harmonics as $h > 1$.

HARMONIC UNDULATOR RADIATION & ENERGY SPREAD

We assume that the electron moves on axis in a dual planar harmonic undulator whose on axis field is given by[13,15-16],

$$\vec{B}_u = [\hat{x}B_0 a_2 \cos(k_l z), \hat{y}B_0\{a_0 \sin(k_u z) + a_1 \sin(k_h z)\}, 0] \quad (1)$$

where $k_u = 2\pi/\lambda_u$, λ_u is the undulator wavelength, B_0 is peak field strength. The undulator field is composed of two harmonic fields of amplitudes B_0a_1 and B_0a_2 respectively, k_h, k_l are the undulator wave numbers of these additional harmonic fields with $k_h = hk_u, k_l = lk_u$ where h, l are integer numbers greater than one. The velocity and electron trajectory of electron is determined through Lorentz equation, and are given by

$$\beta_x = -\left[\frac{K}{\gamma}\cos(\Omega_u t) + \frac{K(a_1/ha_0)}{\gamma}\cos(h\Omega_u t)\right], \quad (2)$$

$$\beta_y = -\left[\frac{K(a_2/la_0)}{\gamma}\sin(l\Omega_u t)\right]$$

where, $K = \dfrac{eB_0 a_0}{m_0 c\Omega_u}$ is the undulator parameter, $\Omega_u = k_u c$. The electron trajectory is given by,

$$x(t) = -\left[\frac{Kc}{\gamma\Omega_u}\sin(\Omega_u t) + \frac{K(a_1/ha_0)c}{\gamma h\Omega_u}\sin(h\Omega_u t)\right], y(t) = \left[\frac{K(a_2/la_0)c}{\gamma l\Omega_u}\cos(l\Omega_u t)\right],$$

$$z(t) = c\beta^* t - \frac{cK^2}{8\gamma^2\Omega_u}\sin(2\Omega_u t) - \frac{cK^2(a_1/ha_0)^2}{8\gamma^2 h\Omega_u}\sin(2h\Omega_u t) + \frac{cK^2(a_2/la_0)^2}{8\gamma^2 l\Omega_u}\sin(2l\Omega_u t)$$

where

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2}{2}\left\{1 + \left(\frac{a_1}{ha_0}\right)^2 + \left(\frac{a_2}{la_0}\right)^2\right\}\right] \quad (3)$$

In writing Eq(3) for the electron longitudinal motion we have ignored the electron oscillations at $(h\Omega_u \pm \Omega_u), (l\Omega_u \pm \Omega_u)$ and $(h\Omega_u \pm l\Omega_u)$. As a consequence the radiation at the harmonics of these oscillations is not included in our analysis. The spectral properties of the undulator radiation are obtained using Lienard - Wiechert integral,

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\vec{\beta})\}\exp\left[i\omega\left(t - \frac{z}{c}\right)\right]dt\right|^2 \quad (4)$$

The brightness expression integrated over the limit 0 to t where $t=L_u/c\beta_z$, $L_u = N\lambda_u$, L_u is the total undulator length, N is number of undulator period .Eq(4) is solved to read,

$$\frac{d^2 I}{d\omega\Omega} = \frac{e^2\omega^2 t^2}{4\pi^2 c}\left\{\left|T_x^2\right| + \left|T_y^2\right|\right\}S(\nu) \quad (5)$$

generalized Bessel functions (GBF) as [4],

$$T_x = \frac{K}{2\gamma}[J_{m+1}(0,z_1) + J_{m-1}(0,z_1)]J_n(0,z_2)J_p(0,z_3) +$$
$$\frac{K(a_1/ha_0)}{2\gamma}[J_{n+1}(0,z_2) + J_{n-1}(0,z_2)]J_m(0,z_1)J_p(0,z_3)$$

$$T_y = \frac{K(a_2/la_0)}{2\gamma}[J_{p-1}(0,z_3) - J_{p+1}(0,z_3)]J_m(0,z_1)J_n(0,z_2) \quad (6)$$

The detuning parameter ν in Eq (5) is given by,

$$\nu = \frac{\omega}{2\gamma^2}\left[1 + \frac{K^2}{2}\left\{1 + \left(\frac{a_1}{ha_0}\right)^2 + \left(\frac{a_2}{la_0}\right)^2\right\}\right] - \{m\Omega_u + n(h\Omega_u) + p(l\Omega_u)\}$$

and the arguments of the GBFs and the radiation wavelength is read from the resonance condition

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$$z_1 = -\frac{K^2}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2+\left(\frac{a_2}{la_0}\right)^2\right\}\right]}\frac{m\Omega_u + n(h\Omega_u)+p(l\Omega_u)}{4\Omega_u},$$

$$z_2 = -\frac{K^2\left(\frac{a_1}{ha_0}\right)^2}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2+\left(\frac{a_2}{la_0}\right)^2\right\}\right]}\frac{m\Omega_u + n(h\Omega_u)+p(l\Omega_u)}{4(h\Omega_u)}$$

$$z_3 = \frac{K^2\left(\frac{a_2}{la_0}\right)^2}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2+\left(\frac{a_2}{la_0}\right)^2\right\}\right]}\frac{m\Omega_u + n(h\Omega_u)+p(l\Omega_u)}{4(l\Omega_u)};$$

$$\omega = \frac{2\gamma^2\{m\Omega_u + n(h\Omega_u)+p(l\Omega_u)\}}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2+\left(\frac{a_2}{la_0}\right)^2\right\}\right]} \tag{7}$$

, Eq(6) gives the dual harmonic undulator radiation. The line shape factor $((\sin(\nu t/2))/\nu\ t/2)^2$ is the same as that of the conventional planar undulator so no broadening of the spectrum occurs by the addition of the harmonic fields. The central emission frequency of the conventional planar undulator is modified by the addition of harmonics of the electron longitudinal motion at $(h\Omega_u)$ and $(l\Omega_u)$. To include the important influence of energy spread we consider the case of single harmonic field i.e B_0a_2=0, then the central emission frequency is

$$\omega = 2\gamma^2\{m\Omega_u + n(h\Omega_u)\}/\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2\right\}\right]$$ and

$$T_x = \frac{K}{2\gamma}[J_{m+1}(0,\xi_1)+J_{m-1}(0,\xi_1)]J_n(0,\xi_2) \tag{8}$$

$$+\frac{K(a_1/ha_0)}{2\gamma}[J_{n+1}(0,\xi_2)+J_{n-1}(0,\xi_2)]J_m(0,\xi_1), T_y = 0$$

Hence $$\xi_1 = -\frac{K^2}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2\right\}\right]}\frac{m\Omega_u + n(h\Omega_u)}{4\Omega_u},$$

$$\xi_2 = -\frac{K^2\left(\frac{a_1}{ha_0}\right)^2}{\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2+\left(\frac{a_2}{la_0}\right)^2\right\}\right]}\frac{m\Omega_u + n(h\Omega_u)}{4(h\Omega_u)}$$

The planar undulator emits odd harmonics on-axis i. e m =1,2,3…..The intensity at $\omega = 2\gamma^2\left\{(m\Omega_u + n(h\Omega_u))/(1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2\right\}\right\}$ $m=3, n=0$ gets superimposed at $\omega = 2\gamma^2\{m\Omega_u + n(h\Omega_u)\}/\left[1+\frac{K^2}{2}\left\{1+\left(\frac{a_1}{ha_0}\right)^2\right\}\right]$, $n=1$ to produce enhancement of the intensity at the third harmonic for h =3. Similarly the radiation can be enhanced for the fifth ($m=5$) harmonic by keeping h = 5. In the case of dual harmonic undulators it is possible to enhance the intensity simultaneously at the two harmonics. The spectral line shape distribution induced by the beam energy having an energy distribution $f(\varepsilon)$ can be written as,

$$s(\nu) = \int_{-\infty}^{\infty} d\varepsilon\left[\frac{\sin((\nu+\partial\nu_\varepsilon)/2)}{((\nu+\partial\nu_\varepsilon)/2)}\right]^2 f(\varepsilon) \tag{9}$$

where $\partial\nu_\varepsilon = 4\pi N\varepsilon$ and defines the shift induced by the energy spread. For a Gaussian type energy distribution we write two-peak electron energy beam as,

$$f(\varepsilon) = \frac{1}{\sqrt{2\pi}}\sum_i \frac{\alpha_i}{\sigma_\varepsilon^i}\exp\left(\frac{-(\varepsilon-\varepsilon_i)^2}{2\sigma_\varepsilon^{i\,2}}\right) \qquad i=1,2 \quad \text{and}$$

$$\alpha_1 + \alpha_2 = 1.0 \tag{10}$$

In Eq(10) σ is the r.m.s relative energy spread and $\varepsilon = \delta\gamma/\gamma$, $\delta\gamma = \gamma - \gamma_0$ s γ_0 being the nominal energy of the electron beam. Therefore Eq (9) can be transformed and is solved to get the line shape function for the fundamental and higher harmonics for such two peak beam given as

$$s(\nu) = 2\alpha_1\int_0^1 dt(1-t)\cos(\nu t + 4\pi N\varepsilon_1 t)\exp\left(\frac{-m^2\mu_1^2\pi^2t^2}{2}\right) \tag{11}$$
$$+2\alpha_2\int_0^1 dt(1-t)\cos(\nu t + 4\pi N\varepsilon_2 t)\exp\left(\frac{-m^2\mu_2^2\pi^2t^2}{2}\right)$$

Where $\mu_1 = 4N\sigma_\varepsilon^1$ and $\mu_2 = 4N\sigma_\varepsilon^2$

FREE ELECTRON LASER GAIN & ENERGY SPREAD

To find small signal gain of harmonic undulator free electron laser under radiation field given by,

$E = E_0\cos(\psi)\hat{x} + E_0\sin(\psi)\hat{y}$

where $\psi = n_1k_1z - n_1\omega_1 t + \phi$, n_1 is emissions harmonic number at which resonance occur. The small signal gain of the harmonic undulator free electron laser is derived by Hussain et al[23],

$$G = -\frac{j}{\nu_0^3}[2 - 2\cos(\nu_0) - \nu_0\sin(\nu_0)] \tag{12}$$

When considered the case of single harmonic field i.e B_0a_2=0, then, we get

$$j = \frac{2\pi^2Ne^2K^2L_u}{\gamma^3m_0c^2}n_1[b_0 + (a_1/ha_0)b_1]^2,$$
$$b_0 = [J_{m+1}(0,\xi_1)+J_{m-1}(0,\xi_1)]J_n(0,\xi_2),$$
$$b_1 = [J_{n+1}(0,\xi_2)+J_{n-1}(0,\xi_2)]J_m(0,\xi_1)$$

We write the gain as the derivative of the spontaneous emission line shape to get, $G = j\frac{\partial}{\partial\nu_0}[\sin c(\nu_0/2)]^2$

If the electrons beam with two peak energy spread, then gain is

$$G = 2j\left[\alpha_1\int_0^1 dt(1-t)t\sin(\nu t + 4N\pi\varepsilon_1 t)\exp\left(\frac{-m^2\mu_1^2\pi^2t^2}{2}\right)+\right. \tag{13}$$
$$\alpha_2\int_0^1 dt(1-t)t\sin(\nu t + 4N\pi\varepsilon_2 t)\exp\left(\frac{-m^2\mu_2^2\pi^2t^2}{2}\right)$$

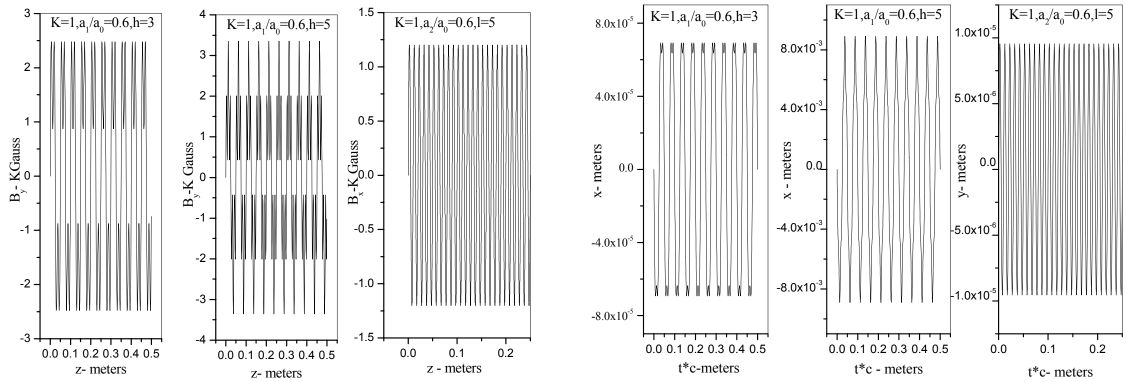

Figure 1: Magnetic field and trajectory of electron.

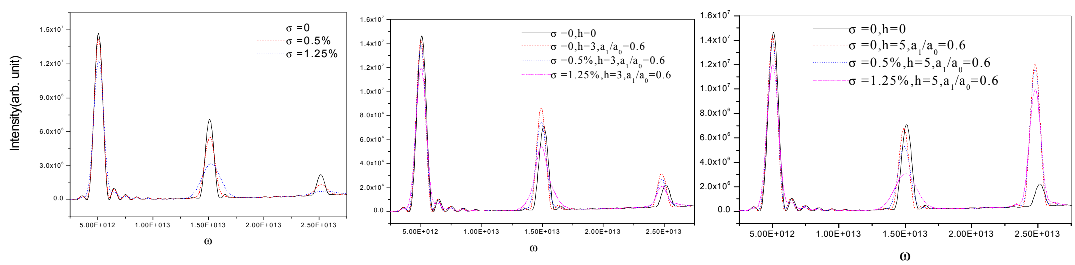

Figure 2: Intensity versus radiation frequency with one-peak beam energy distribution function.

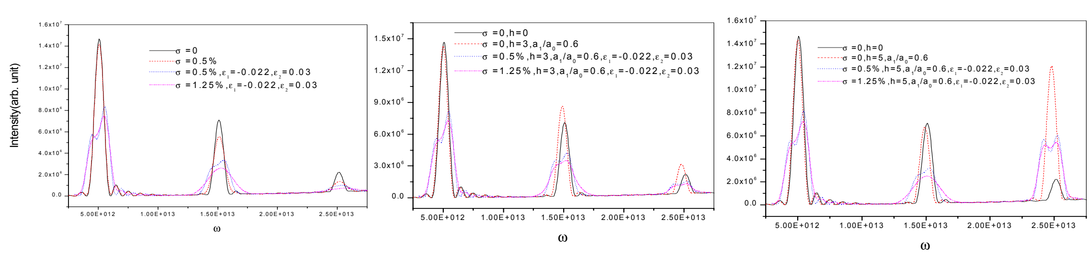

Figure 3: Intensity versus radiation frequency with two-peak beam energy distribution functions with $\alpha_1 = \alpha_2 = 0.5$.

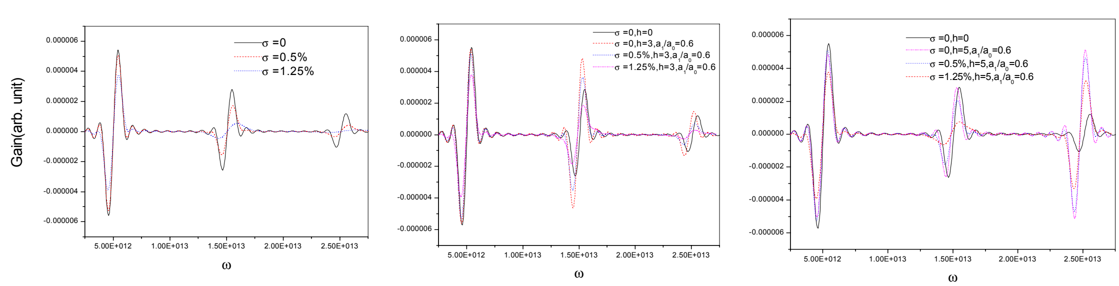

Figure 4: Gain curves versus radiation frequency with one-peak beam energy distribution function.

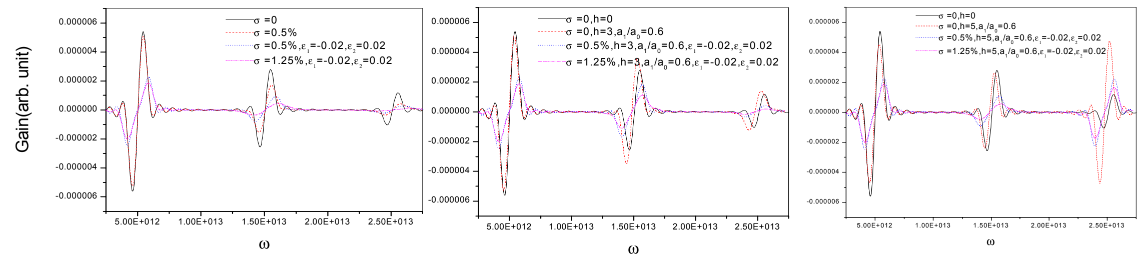

Figure 5: Gain curves versus radiation frequency with two-peak beam energy distribution function with $\alpha_1 = \alpha_2 = 0.5$

The analytical expression for gain function reduction can be written as,

$$g_{\max}(\nu_0, \mu_\varepsilon) = \frac{0.238\pi}{1 + 3.46\mu_\varepsilon^2 m^2} \quad \text{where}$$

$$g(\nu_0, \mu_\varepsilon) = \frac{\partial}{\partial \nu_0}[\sin c(\nu_0 / 2)]^2 \qquad (14)$$

RESULTS & DISCUSSION

In this paper we have considered the influence of beam energy spread on the harmonic undulator radiation. For beam and undulator parameters such as, $\gamma = 100, K = 1, \lambda_u = 5cm, h = 3$, we find the fundamental frequency as, $\omega(m=1, n=0) = 5.01 \times 10^{12}\, rad/\sec$.The device emits odd harmonics. Considering the higher harmonics we have the third harmonic is emitted at, $\omega(m=3, n=0) = 1.5 \times 10^{13}\, rad/\sec$.Due to the harmonic number $h = 3$, there is an additional contribution to the intensity at $\omega(m=0, n=1) = 1.5 \times 10^{13}\, rad/\sec$.

This explains the enhancement of the radiation for the harmonic undulator and due to the additional electron oscillation. Fig.1 gives the variation of magnetic field and trajectory of electron along the undulator length. Fig.2 illustrates the case of undulator radiation for the case of $m = 1,3,5$ harmonic with beam energy spread. For two peak beam energy distribution, the reduction of the intensity spectrum broadening is displayed in Fig.3. Fig.2(a) reflects the intensity reduction for the fundamental and higher harmonics. The intensity reduction is proportional to the square of the harmonic number. So the reduction in intensity is substantial for harmonics $m = 3, m = 5$. However with harmonic field amplitudes, the intensity at $m = 3, m = 5$ the intensity loss is compensated as shown in Fig. 2(b) and Fig.2(c) respectively. The two peak beam energy spread is analyzed in Fig. 3(a) without the harmonic field amplitudes. The intensity at the higher harmonics is improved with the harmonic field amplitudes as shown in Fig.3(b) and Fig.3(c) but the intensity improvement is less significant as compared to single peak beam energy distribution.

The influence of the beam energy spread as calculated by the derivative of the spontaneous line shape function is displayed in Fig.4-5. The gain at higher harmonics is effectively compensated by the harmonic field amplitudes with both one peak and two peak beam energy spread. An analytical expression for the gain reduction is given in Eq. (14). The beam energy spread induced gain reduction can be accounted for the electrons driven away from the resonant energy. This deviation gap is minimized by the introduction of the harmonic field undulator at frequency hk_u bringing the free electron laser back to the resonance condition. As h is a harmonic integer greater than unity the gain enhancement works for the higher harmonics only.

REFERENCES

[1] F. Ciocci, G. Dattoli, Nuclear Instruments and Methods in Physics Research B, 71, p-339,1992.
[2] F. Ciocci, G. Dattoli, K. Flottmann, J. Rossabach, Internal Report, DESY, M-93-03,1993
[3] M.G. Kong, Optics Communication, 132, p-464, 1996
[4] M. Asakawa, K. Mima, S. Naka, K. Imasaki, C. Yamanaka, Nucl.Instr.&Meth., A318, p-538, 1992
[5] M.J. Schmitt, C.J. Elliot, Phys. Rev. A, Vol. 34, No. 6, p-3483,1986
[6] G. Dattoli, L. Giannessi, P.L. Ottaviani, H.P. Freund, S.G. Biedron, S. Milton, NIMA 495, p-48, 2002
[7] M.J. Schmitt, C.J. Elliot, IEEE J. Quantum Electron, QE-23, p-1552, 1987
[8] S. Kuruma, K. Mima, K. Ohi, S. Nakai, C. Yamanaka, Nucl.Inst.& Meth , A, 304, p-638, 1991
[9] R.W. Warren, Nuclear Instruments and Methods in Physics, A, 304, p-512, 1991
[10] Vikesh Gupta, G. Mishra, NIMA, 574, p-150, 2007
[11] Vikesh Gupta, G. Mishra, NIMA, Vol.556, No.1, p-350, 2006
[11] Y. Yang, Wu Ding, NIMA, 407, p-60, 1998
[12] G. Mishra, Mona Gehlot, Jeeva Khan Hussain, Journal of Modern Optics, vol. 56, p- 667,2009
[13] G. Mishra, Mona Gehlot, Jeeva Khan Hussain, NIMA, 603(3), p-496, 2009.
[14] V.I.R. Niculescu, Minola R. Leonnovici, V. Babin, Anca Scarisoreanu, Rom. Journ. Phys., Vol.53, Nos. 5-6, P-775, 2008.
[15] Jeeva Khan Hussain, Vikesh Gupta, G.Mishra, IL Nuovo Cimento B, Vol 124 B, 2009.
[16] Jeevakhan hussain, Vikesh Gupta, G Mishra, NIMA, Article in press.