# COHERENT SMITH-PURCELL RADIATION: COMPARISON BETWEEN SIMULATIONS AND EXPERIMENT

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*Abstract*

The results of the CESTA experiment that used a flat, wide and intense beam to produce coherent Smith-Purcell radiation are compared with 2-D and 3-D simulations performed with the PIC code "MAGIC". The comparison provides considerable support for the paradigm proposed a few years ago by Andrews and Brau.

## INTRODUCTION

In a companion paper (TUPC65), the results of an experiment performed at the CESTA facility of the CEA that used a wide, flat and intense beam to produce coherent Smith-Purcell are presented. Here we compare some of the experimental results with simulations made with the PIC code "MAGIC". The aim of this experiment was to demonstrate the validity of the theory presented a few years ago by Andrews and Brau [1] (henceforth AB ). That theory proposed a 2-D model for producing coherent Smith-Purcell radiation from an initially continuous beam. For technical reasons, the theory concerned lamellar gratings, for which a dispersion relation (DR) relating the frequency and axial wave number could be obtained analytically. Then using perturbation theory in the vicinity of the operating point (intersection of the beam line and the DR), they could calculate the exponential growth of the resulting evanescent wave in a standard way. Subsequent to their work, we carried out a simulation of a Smith-Purcell experiment at microwave frequencies using the commercially available 2-D PIC code "MAGIC" [2, 3]. In our simulation we used a strong axial magnetic field to prevent the electrons from drifting into the grating under the influence of the image charge. Although the AB theory does not include such effects, we found quite general agreement between the theory and our simulations. A major result of the theory was that coherent S-P radiation can occur only on second and higher harmonics of the evanescent wave, and our simulations confirmed this. However, it was noted that emission of the evanescent wave from both ends of the grating was quite strong and occurred at all angles.

Other groups have performed 2-D simulations [4, 5] and striking visual effects may be found in the latter. Kumar and Kim [6] have developed a theory that agrees, in practice if not in approach, with that of AB. However, until quite recently, with the exception of Skrynnik [7] and collaborators, no evidence for the evanescent wave had been reported. At the FEL 2008 conference the Vanderbilt University-Vermont Photonics collaboration [8] reported seeing the evanescent wave, and in a grating with sidewalls, its second harmonic. D. Li and collaborators [9] performed 3-D simulations of such a grating.

The CESTA experiment [10] was designed to demonstrate the validity of the AB theory and of our simulations. Although the interest in coherent S-P radiation is concentrated in the Terahertz domain, a demonstration of the validity of the AB theory at any frequency is sufficient. The technical difficulties are another matter, but our aim was simply to confirm the theory. We should also mention that in the design and preparation of the experiment, heavy reliance was placed on both 2-D and 3-D simulations.

## THREE–D SIMULATIONS

In the AB 2-D model, only three components of the electromagnetic field are non-zero, $B_x, E_z, E_y$. Here the beam defines the *z*-axis, the grooves are parallel to the *x*-axis, and the *y*-axis is perpendicular to the grating plane. However, in 3-D, the fact that the magnetic field $B_x$ depends on *x* means that all six components of the electromagnetic field are needed. In addition, a grating of finite width acts as a finite source of radiation, and has fields which decrease asymptotically as 1/*r*, where *r* denotes the distance from the grating, in contrast to 2-D, where the fields decrease only as $r^{-1/2}$. It is therefore impossible that a 2-D simulation can yield results identical to a 3-D simulation, but there still may remain similarities that make the 2-D simulations a useful, if imperfect guide, to the 3-D reality. Of course, the main difficulty with 3-D simulations using the code "MAGIC" concerns memory requirements. While it is easy to perform a 2-D "MAGIC" simulation with a small mesh size in a comfortably large area and in a reasonable time, it is impossible to do so in 3-D. Some compromises regarding mesh size and simulation volume are needed to obtain results. Despite this, we have obtained some results using the 3-D version of "MAGIC" which are in general agreement with 2-D simulations, and others which cannot be explained in the 2-D framework. We present in the following a selection of these results. We also remind the reader that our most important 2-D predictions may found in Ref. 2.

## GRATING MODES

In the 2-D AB theory and in simulations there is a maximum frequency that can propagate on a lamellar grating. To test this, we measured the $S_{21}$ coefficient of our grating using a network analyzer. For our grating the maximum frequency is approximately 4.65 GHz according to the AB theory. We also investigated the propagation of the evanescent surface wave by making a "ping" in a groove, and observing the time signal several periods downstream. In Figure 1 we show the FFT of the time signal in red, and the transmission coefficient $S_{21}$ in red.

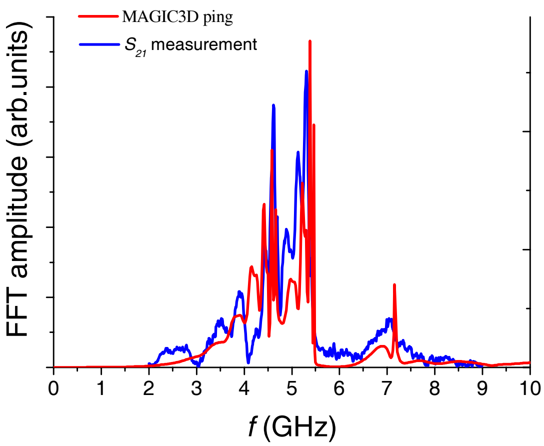


Figure 1: The measured transmission coefficient $S_{21}$ for our grating, along with the 3-D simulation results for a "ping".

The 2-D cutoff is clearly visible near 4.6 GHz, but there exists also higher frequency waves, which have sharp cutoffs at 5.5 and 7.1 GHz. Further investigation using current drivers operating at fixed frequency allowed us to show that these higher frequency modes have oscillations in the along the $x$ direction. As such, they are unlikely to be driven by a broad beam such as we use. A narrow beam might be able to excite them, however.

## BEAM PROPAGATION IN THE SOLENOID

In contrast to our simulations, where it is sufficient to tell the electrons they are in a uniform magnetic field, in our experiment it is necessary to use a solenoid to produce the magnetic field. In our experiment the knife-edged cathode was immersed in the magnetic field. We were concerned about the beam propagation, but, in accord with the 3-D simulations, the beam propagated nicely, provided a field above 0.3 T was used. On early shots, the beam dump was covered by a strip of paper that captured an image of the beam after it had traveled 45 cm. One such image is shown in Figure 2, along with the results of a 3-D simulation. In the presence of the axial field and of the strong electric field caused by the image charge on the grating, there is a modest $\vec{E}\times\vec{B}$ effect that pushes the beam to one side. In addition at the ends of the beam, the electric field has a notable x-component which makes the ends of the beam curl up and down, anti-symmetrically. The simulation reproduces this effect rather well. In a 2-D simulation there is also a drift, but no curling up, since there are no ends.

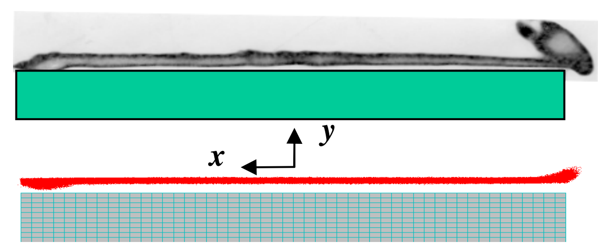


Figure 2: The upper part shows an imprint of the beam on a strip of paper placed on the beam stop just above the top of the grating. The lower part shows the result of a 3-D Magic simulation.

The most important factor in obtaining coherent S-P radiation is the bunching of the beam under the influence of the evanescent wave. In 2-D there is no possible dependence on the variable $x$, but in 3-D simulations this is not so. We show in Figure 3 the results concerning bunching according to the 3-D simulation, with a beam whose width, 10 cm, is equal to that of the grating.

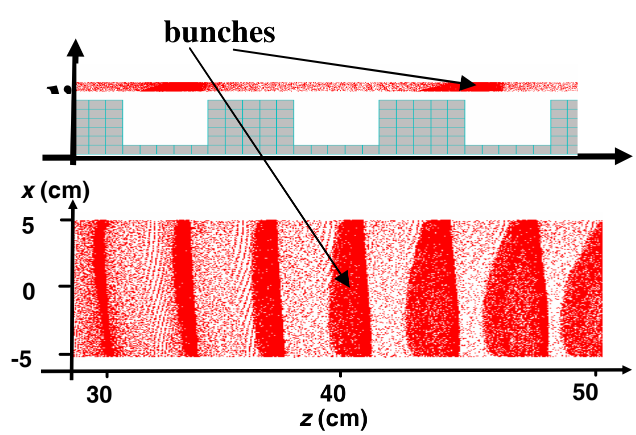


Figure 3: Images of the bunching in the $y$-$z$ plane (upper) and in the $x$-$z$ plane (lower), according to 3-D simulations.

It is apparent that the bunches have a more complicated shape than in the 2-D simulation given in Ref. 2. However, the wavelength of the bunching, about 3.7 cm, is quite consistent with our earlier results.

## COMPARING 2-D AND 3-D PREDICTIONS

The beam is bunched by the electric field component $E_z$ in the vicinity of the top of the grating. The evanescent wave is a Floquet wave with a given axial wave number $k$, together with all its partners at $k+pK$, where $K=2\pi/L$ is the grating wave number. We show in Figure 4(a) a contour map of the magnetic field along the grooves, $B_x$. according to a 3-D simulation. In Figure 4(b) the field is displayed as a function of $z$, and the corresponding spatial FFT is shown in 4(c), for both 2-d and 3-D simulations.

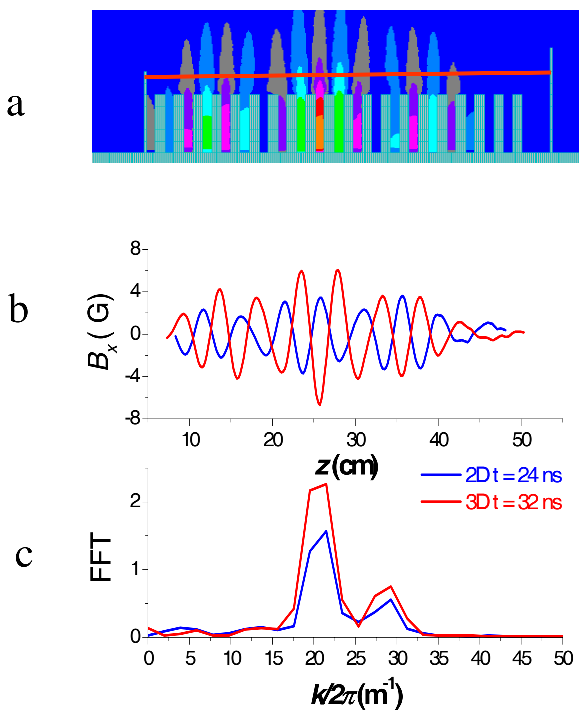


Figure 4: (a) Contour map of magnetic field component $B_x$ (parallel to grooves) in the vicinity of the grating and at $x$= 0 for 3-D simulation. (b) Graph of $B_x$ vs. $z$ for 2-D (blue) and 3-D simulations (red), at $x$ = 0, $y$ = 2 cm. (c) FFTs of preceeding, showing support near 21 and 29 m$^{-1}$, in agreement with AB theory.

We note that the beam bunching of wavelength 3.7 cm visible in Figure 3 corresponds to the smaller peak near 29 m$^{-1}$. The larger peak is actually the $p$=-1 component of the Floquet wave, since the FFT is not sensitive to signs. It is the reason the evanescent wave is backward, since that is the dominant component. There is overall agreement between 2-D and 3-D simulations on this important aspect.

## EMISSION OF SP RADIATION

Having assured that the bunching occurs as expected we now study the emission of radiation by the grating. This is a delicate task, since the fields decrease as one moves away from the surface grating; and it becomes hard to see the variation in intensity needed to see the radiation. To overcome this composite contour maps, chosen so as to maintain visual contrast throughout the simulation area are displayed in Figure 5.

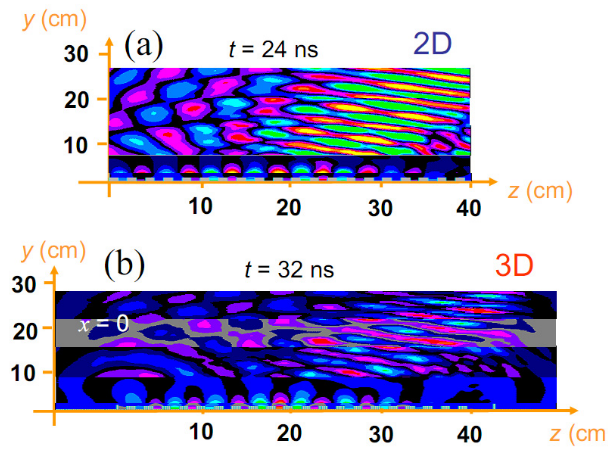


Figure 5: Composite contour plots of $B_x$ in the $y$-$z$ plane. (a) 2-D simulation, $t$ = 24 ns. (b) 3-D simulation, $t$ = 32 ns, plane $x$ = 0. Strong emission of second harmonic (3.4-cm wavelength) is apparent in both plots.

In both cases one sees coherent SP radiation with an approximate wavelength of 3.4 cm, and which corresponds to the second harmonic of the evanescent wave. However, the intense background of radiation at 6.8 cm from the evanescent wave emitted at the grating ends makes it more difficult to see it in the 3-D simulation. The radiation emerges at an angle of approximately 78° with the beam, as expected from the standard S-P relation,

$$\lambda = \frac{L}{|n|}\left(1/\beta - \cos\theta\right).$$

This equation relates the wavelength $\lambda$, the relative electron velocity $\beta$, the grating period $L$ , the angle between the beam and radiation $\theta$ , and the order of diffraction (1 in this case).

## OUTCOUPLING THE RADIATION

As stated earlier, the experiment is enclosed in a vacuum inside a large bore solenoid that provides the axial magnetic field needed to propagate the beam. This means that the image shown in Figure 5 is not subject to direct confirmation, since the radiation would strike the solenoid wall. To avoid this we introduced a movable mirror, which may be placed anywhere along the grating and oriented at any angle with respect to the grating plane. We show in Figure 6 how this mirror can deflect radiation out from the solenoid

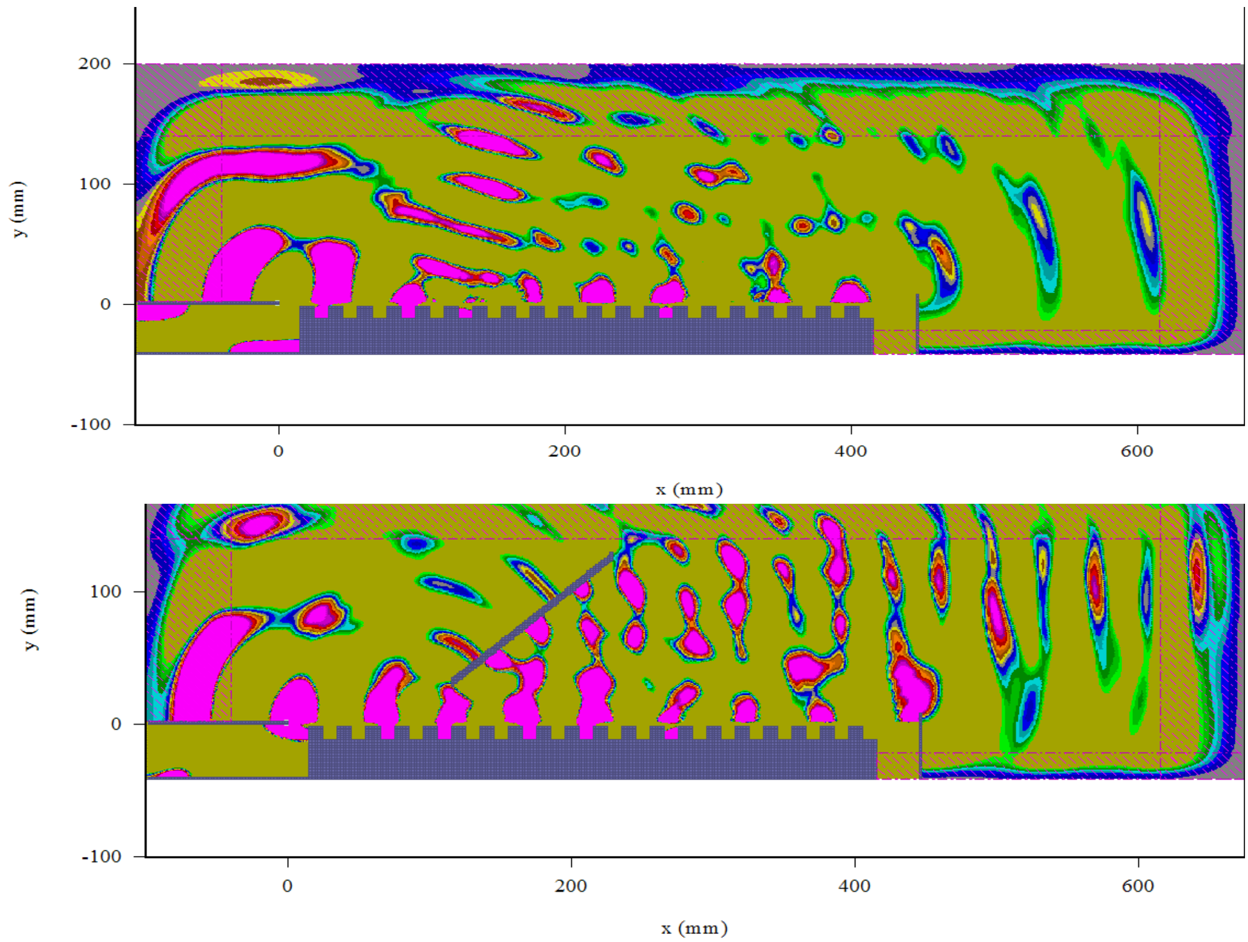


Figure 6: 2-D simulation of the radiation emitted without (upper) and with a mirror (below). In the latter, the second harmonic S-P radiation is deflected in the forward direction.

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