IMPACT ON A SEEDED HARMONIC GENERATION FEL OF AN INITIAL CHIRP AND CURVATURE IN THE ELECTRON BUNCH ENERGY DISTRIBUTION

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Abstract

In a harmonic generation free electron laser (HG FEL), the electron bunch entering the undulator can have an initial energy curvature besides an initial energy chirp. Solving the Vlasov-Maxwell equations within the 1D model, we derive an expression for the Green function for the seeded HG FEL process for the case of the electron bunch having both an energy chirp and an energy curvature. We give an asymptotic closed form which is a good approximation in the exponential growth regime, and a series expression that allows the evaluation of the field envelope along the undulator in both lethargy and exponential growth regime. The latter is useful to study the HG FEL behavior in the short modulator, like that of the FERMI@Elettra project. The FEL radiation properties such as central frequency shift and frequency chirp are studied considering Gaussian laser seeds of different temporal duration with respect to that of the Green function. The energy chirp and curvature of the electron bunch result in a time dependent bunching factor for the FEL start-up process in the radiator of the HG FEL like the FERMI@Elettra. The coherence properties of the FEL are examined.

INTRODUCTION

For a Harmonic Generation Free Electron Laser (HG FEL) a high quality electron bunch with low (slice) emittance, low (slice) energy spread, but high peak current and high centroid energy are needed. During the process of acceleration, bunch compression and transportation, the electron bunch is subject to the radio frequency curvature and wakefields effects. Thus, the energy profile of the electron bunch can undergo modifications, and in particular it can acquire a linear chirp and a quadratic energy curvature, that can have an impact on the FEL process. Figure 1 represents the schematic of an HG FEL: in the modulator a laser seed imprints an energy modulation on the electrons; in the dispersive section the energy modulation is converted to density modulation, so that the electrons current distribution functions contains a frequency content at higher harmonics. However, due to the energy chirp and curvature, the electron bunching at the radiator entrance presents a quadratic phase that can have a strong impact on the FEL radiation. In this paper, within the 1D Vlasov-Maxwell equations framework, as done in Ref. [1] for the Self-Amplified Spontaneous Emission (SASE) FEL and Ref. [2] for a seeded FEL, in case of a linear chirped electron bunch, we derive time dependent Green functions considering the laser seed and an initial denisity modulation as sources for the FEL process, without using asympthotic approximations, for the case of both linear energy chirp and quadratic curvature on the electrons. Further we give formulas to evaluate the phase of the bunching at the radiator entrance for a given electrons energy profile. Finally we discuss the results, considering the case of the FERMI@Elettra HG FEL with different possible electron distributions at the modulator entrance.

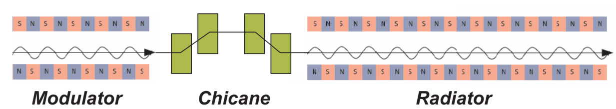

Figure 1: HG FEL layout.

DERIVATION

In order to study the evolution of HG FEL, we analyze the coupled Vlasov and Maxwell equations, which describe the interaction between the electrons and the electromagnetic field [3], both in the modulator and in the radiator. We solve this set of equations providing a series expansion solution for the FEL Green function for the initial conditions of laser seed and electron bunching, in case of a linear chirp and curvature on the electrons energy. To evaluate the bunching at the undulator entrance we will consider its amplitude using the formula provided in [4, 5] with total phase advance in both the modulator and the dispersion section. Due to the overall energy chirp and curvature in the electron bunch, the phase accumulated in the non zero length modulator and in the dispersive section are explicitly included.

Coupled Vlasov-Maxwell Equations

We adopt the notations of Refs. [1, 2]. We use the dimensionless variables $Z = k_w z$ and $\theta = (k_0 + k_w)z - \omega_0 t$, where z is the longitudinal coordinate, $k_w = 2\pi/\lambda_w$, with λ_w the undulator period, $k_0 = 2\pi/\lambda_0$, with λ_0 the radiation wavelength and $\omega_0 = k_0 c$, with c the velocity of

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light in the vacuum. As a measure of the energy deviation, we also introduce the quantity $p = 2(\gamma - \gamma_0)/\gamma_0$, where γ is the Lorentz factor of an electron of the bunch and γ_0 the Lorentz factor in resonance condition. For a planar undulator, the latter quantity satisfies the relation $\lambda_0 = \lambda_w(1 + K^2/2)/(2\gamma_0^2)$, where the undulator parameter is $K \approx 93.4 B_w \lambda_w$, with B_w the peak magnetic field in Tesla and λ_w the undulator period in meters. The electron distribution function is denoted as $\psi(\theta, p, Z)$ and the FEL electric field is written in the form $E(\theta, Z) = A(\theta, Z)e^{i(\theta - Z)}$, with $A(\theta, Z)$ being the slow varying envelope function. Following [1, 2], the one dimensional linearized Vlasov-Maxwell equations are expressed by:

$$\frac{\partial \psi}{\partial Z} + p\frac{\partial \psi}{\partial \theta} - \frac{2D_2}{\gamma_0^2}\left(Ae^{i\theta} + A^* e^{-i\theta}\right)\frac{\partial \psi_0}{\partial p} = 0, \quad (1)$$

$$\left(\frac{\partial}{\partial Z} + \frac{\partial}{\partial \theta}\right) A(\theta, Z) = \frac{D_1}{\gamma_0} e^{-i\theta} \int dp \psi(\theta, p, Z), \quad (2)$$

where the asterisk denotes the complex conjugate, the integral is defined on the whole p domain, and, in SI units:

$$D_1 = \frac{e a_w n_0 [JJ]}{2\sqrt{2} k_w \varepsilon_0}, \text{ and } D_2 = \frac{e a_w [JJ]}{\sqrt{2} k_w m c^2}, \quad (3)$$

with e and m being the charge and the mass of the electron, respectively, ε_0 the vacuum permittivity, n_0 the electron bunch density, and $[JJ] = J_0[a_w^2/2(1 + a_w^2)] - J_1[a_w^2/2(1+a_w^2)]$, where $a_w = K/\sqrt{2}$ is the dimensionless rms undulator parameter, while J_n is the Bessel functions of the first kind of n-th order. Finally, the function ψ_0 in Eq. (1) is defined as a solution of the equation $\frac{\partial \psi}{\partial Z} + p\frac{\partial \psi}{\partial \theta} = 0$. For an electron bunch with both linear energy chirp and curvature, we take $\psi_0 = \delta\left(p + \mu\theta_0 + \nu\theta_0^2/2\right)$ with $\theta_0 = \theta - pZ$ and $\mu = \frac{2}{\gamma_0 \omega_0} \frac{d\gamma}{dt}\Big|_{t=0}$ and $\nu = -\frac{2}{\gamma_0 \omega_0^2} \frac{d^2\gamma}{dt^2}\Big|_{t=0}$ characterizing the linear chirp and the curvature respectively. We solve Eq. (1) for a small perturbation ψ_1 of ψ_0 neglecting $\frac{\partial \psi_1}{\partial p}$ term obtaining:

$$\psi_1 = \frac{D_2}{\gamma_0^2} e^{i\theta} \frac{\partial \psi_0}{\partial p} \int_0^Z dZ_1 e^{ip(Z_1 - Z)} A\left[\theta - p\left(Z - Z_1\right), Z_1\right] \\ e^{i(\theta - pZ)} F(\theta - pZ) \quad (4)$$

The $e^{i(\theta - pZ)} F(\theta - pZ)\big|_{Z=0} = e^{i\theta} F(\theta)$ can be used to take into account an initial electron distribution function perturbation as FEL start-up. We will consider initial condition $F_b \propto \delta\left(p + \mu\theta_0 + \nu\theta_0^2/2\right)$ for an initial density modulation. Substituting Eq. (4) into Eq. (2), integrating over p, supposing $\mu Z \ll 1$ and $\nu\theta Z \ll 1$, and using the Laplace transform:

$$f(s, \theta) = \int_0^{+\infty} e^{-sZ} A(\theta, Z) dZ, \quad (5)$$

one obtains:

$$\frac{\partial f(s,\theta)}{\partial \theta} + \left(s - \frac{i(2\rho)^3}{\left[s - i(\mu\theta + \nu\theta^2/2)\right]^2}\right) = \\ A(\theta, 0) + \frac{iD_1/\gamma_0 F_b(\theta)}{s - i\theta(\mu - \nu\theta/2)} \quad (6)$$

where ρ denotes the Pierce parameter [8].
Equation (6) is solved by:

$$f(s, \theta) = \int_{-\infty}^{\theta} d\theta_1 e^{\int_{\theta_1}^{\theta} s - \frac{i(2\rho)^3}{\left[s - i(\mu\theta_2 + \nu\theta_2^2/2)\right]^2} d\theta_2} \times \\ \left[A(\theta_1, 0) + \frac{iD_1/\gamma_0 F_b(\theta_1)}{s - i(\mu\theta_1 + \nu\frac{\theta_1^2}{2})}\right] \quad (7)$$

Using the following convenient notation:

$$\hat{z} = 2\rho Z, \hat{s} = \rho\theta, \hat{\xi} = \rho(\theta - \theta_1), \hat{\alpha} = -\mu/(2\rho^2), \hat{p} = s/(2\rho) \quad (8)$$

$$A(\hat{s}, \hat{z}) = \frac{1}{\pi i} \int_0^{\hat{z}/2} d\hat{\xi} \int_{c_{\hat{p}}} d\hat{p}\, e^{\hat{p}(\hat{z} - 2\hat{\xi})} \\ e^{\frac{4i\hat{\beta}}{(2i\hat{p}\hat{\beta} - \hat{\alpha}^2)^{\frac{3}{2}}}\left(\frac{atan(\hat{\alpha} - \hat{s}\hat{\beta})}{\sqrt{2i\hat{p}\hat{p} - \hat{\alpha}^2}} - \frac{atan[\hat{\alpha} - \hat{\beta}(\hat{s} - \hat{\xi})]}{\sqrt{2i\hat{p}\hat{p} - \hat{\alpha}^2}}\right)} \\ e^{\frac{4i}{a^2 - 2i\hat{p}\hat{\beta}}\left(\frac{\hat{\alpha} - \hat{s}\hat{\beta}}{2i\hat{p} - 2\hat{s}\hat{\alpha} + \hat{s}^2\hat{\beta}} + \frac{\hat{\alpha} - \hat{\beta}(\hat{s} - \hat{\xi})}{-2i\hat{p} + (\hat{s} - \hat{\xi})[2\hat{\alpha} - \hat{\beta}(\hat{s} - \hat{\xi})]}\right)} \\ \left[A(\hat{s} - \hat{\xi}) + \frac{iD_1/(\gamma_0\rho)F_b(\hat{s} - \hat{\xi})}{2\hat{p} + i(\hat{s} - \hat{\xi})[2\hat{\alpha} - \hat{\beta}(\hat{s} - \hat{\xi})]}\right] \quad (9)$$

where $c_{\hat{p}}$ denotes the integration path for the inverse Laplace transform. Eq.(9) gives the electric field envelope along the undulator as sum of a seed and a density modulation term. A time dependent Green function can be determined performing the contour integral before the convolution with the source term, and the latter equation rewritten as

$$A(\hat{s}, \hat{z}) = \int_0^{\hat{z}/2} d\hat{\xi} \left[G_s A(\hat{s} - \hat{\xi}) + G_b F_b(\hat{s} - \hat{\xi})\right] \quad (10)$$

The Green function for the start up from a laser seed with electron bunch having linear energy chirp and curvature has been determined in [6] using a saddle point approximation method to perform the contour integral. A more accurate result, especially for short undulators, can be determined by performing the inverse Laplace transform exactly. This can be accomplished for each source term, by exploiting the residual theorem in conjunction with a series expansion of the integrand function. In particular for starting up from a laser seed, we introduce an auxiliary integral that fulfills the conditions of the Jordan's Lemma, which is related to the original integral. The integrand function presents two essential singularities and the integral is evaluated expanding the integrand into Laurent series of $\hat{p}$ and determining analytically the $\hat{p}^{-1}$ coefficient. Similarly, but without using an auxiliary integral, a formula for the bunching Green function has been derived.
The Green function for a seeded FEL and the one for the initial density modulation have been found as:

$$G_s = \sum_{J=1}^{\infty} \sum_{W_l=0}^{J - \sum_{k=1}^{l-1} W_k} \frac{(\hat{z} - 2\hat{\xi})^{\sum_{h=1}^{\infty} hW_h + 2J - 1}}{(\sum_{h=1}^{\infty} hW_h + 2J - 1)!} \times \\ \frac{(2i\hat{\xi})^{J - \sum_{h=1}^{\infty} W_h}}{J - \sum_{h=1}^{\infty} W_h} \frac{T(l)}{W_l!} + \delta(\hat{\xi} - \hat{z}/2) \quad (11)$$

$$G_b = \sum_{H=0}^{\infty}\sum_{J=0}^{\infty}\sum_{W_l=0}^{J-\sum_{k=1}^{l-1}W_k} \frac{(\hat{z}-2\hat{\xi})^{\sum_{h=1}^{\infty}hW_h+2J+H+1}}{(1+H+2J+\sum_{h=1}^{\infty}hW_h)!} \times \frac{T(l)}{W_l!}R(H)\delta_{(J,\sum_{h=0}^{\infty}W_h)} \quad (12)$$

where l are non negative integers and

$$T(m) = \sum_{n=0}^{m} \frac{(\hat{s}^{2m-n+1}-(\hat{s}-\hat{\xi})^{2m-n+1})}{(2m-n+1)2^{m-n-1}n!(m-n)!} \times i^{m+1}(m+1)!(-\hat{\alpha})^n\hat{\beta}^{m-n} \quad (13)$$

$$R(H) = \frac{D_1}{\rho\gamma_0}\frac{i^{3H+1}}{2^H}((\hat{s}-\hat{\xi})(2\hat{\alpha}-\hat{\beta}(\hat{s}-\hat{\xi})))^H \quad (14)$$

Bunching at the Undulator Entrance

To study the evolution of the FEL pulse along the radiator, we need to determine an expression for the bunching at the radiator entrance. To this aim we consider an electron bunch with both energy linear chirp and curvature having a Gaussian energy distribution on a fixed phase coordinate, characterized by the rms Lorentz factor σ_γ. Through the first modulator the laser seed induces an energy modulation $\Delta\gamma$ on the Lorentz factor. The amplitude of the bunching factor for the n-th harmonic has been evaluated analytically in [4], for a given strength of the dispersive section $\frac{d\theta}{d\gamma}$ as

$$|B_n| = e^{-\frac{1}{2}n^2\sigma_\gamma^2(\frac{d\theta}{d\gamma})^2} J_n\left(n\Delta_\gamma\frac{d\theta}{d\gamma}\right) \quad (15)$$

with

$$\frac{d\theta}{d\gamma} = \frac{2\pi N_w}{\gamma_0} + \frac{k_0+k_w}{\gamma_0}R_{56}, \quad (16)$$

which includes the phase advance both in the modulator and the dispersion section [5] with N_w being the number of period in the modulator and R_{56} characterizing the dispersion strength in the chicane. Beside the amplitude of the density modulation, we determine a phase accumulated through the modulator and the dispersive section due to the overall energy chirp and curvature on the electron bunch. For a short undulator we can evaluate with good approximation the phase from the linearized pendulum equation $\frac{d\theta}{dz} = k_w 2\frac{\gamma-\gamma_0}{\gamma_0}$. For a N_w periods modulator, this yields a contribution $\Delta\theta_M$ to the bunching phase:

$$\Delta\theta_M(\theta) = 4\pi\frac{\gamma(\theta)-\gamma_0}{\gamma_0}N_w. \quad (17)$$

Similarly the dispersive section gives its own contribution to the phase $\Delta\theta_C$. For a give R_{56} value it can be calculated as

$$\Delta\theta_C(\theta) = (k_0+k_w)\frac{\gamma(\theta)-\gamma_0}{\gamma_0}R_{56}. \quad (18)$$

At the n-th harmonic, the bunching factor will be finally evaluated as

$$B_n(\theta) = |B_n|e^{in[\Delta\theta_M(\theta)+\Delta\theta_C(\theta)]}. \quad (19)$$

DISCUSSION

The bandwidth of the seeded FEL Green function with both energy linear chirp and energy second-order curvature has been calculated in [6], while the effects on the radiation like central frequency shift and frequency chirp for seed of different lengths are discussed in [7]. The non asymptotic approximated formula in Eq. (11) presents a Dirac delta pulse at $\hat{\xi} = \hat{z}/2$ which takes into account the seed traveling at the velocity of light through the undulator. Furthermore Eq. (11) allows us to use a wider range of values for $\hat{\alpha}$ and $\hat{\beta}$ since for the approximated one an order analysis was adopted to determine the saddle point.
When convoluted with a constant bunching at the undulator entrance, Eq. (12) gives an electric field envelope that grows linearly during the start-up and then evolves in the exponential growth regime as shown in Figures 2 and 3.

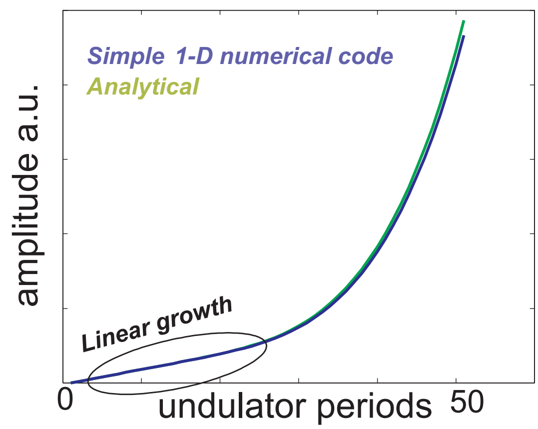

Figure 2: Electric field envelope peak growth as function of the undulator periods.

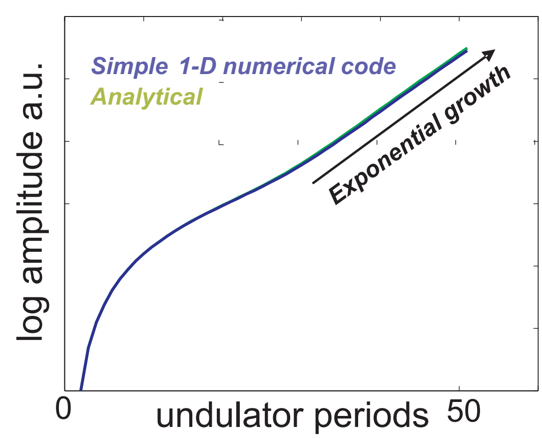

Figure 3: Electric field envelope peak growth as function of the undulator periods with log scale.

We consider now an HG FEL with the set of parameters employed in the FERMI@Elettra project. The modulator is 19 periods long, and it is tuned at a wavelength of 240 nm, the R_{56} parameter characterizing the dispersive section is 30 μm and the radiator is 400 periods long and is tuned on the 10-th harmonic of the modulator wavelength. We consider two possible initial distribution functions for the electrons at the modulator entrance, that have been generated with LiTrack [10] and an ideal flat-energy electron bunch. Fitting the particle distribution with a second order polynomial, we calculate the linear chirp and quadratic curvature parameters needed for our calculations, which are reported in Table 1. The r.m.s. uncorrelated energy spread for electron distributions is set to 150 keV. The laser seed at the undulator entrance is supposed to be constant in amplitude, and longer than the electron bunch, and induces a modulation of 1.5 MeV. To show the effects on the linear

Table 1: Electron Bunches Parameters

Bunch	**Energy**	**Lin. Chirp**	**Curvature**
FB	1140 MeV	0 MeV/fs	0 MeV/fs^2
B1	1140 MeV	1.4 10^{-5} MeV/fs	3.2 10^{-6} MeV/fs^2
B2	1140 MeV	2.2 10^{-3} MeV/fs	6.5 10^{-6} MeV/fs^2

chirp and curvature after the radiator, we use the Wigner function [9] defined as:

$$W(t,\omega,z) = \int E(t-\tau/2,z)E^{*}(t+\tau/2,z)e^{-i\omega\tau}d\tau \tag{20}$$

Figures 4, 5 and 6 shows the Wigner function plot for the three different bunches considered. The ideal flat bunch yields an unchirped FEL pulse with the shortest bandwitdh. In the other cases instead, the FEL pulse presents a larger bandwidth and a strong frequency chirp, in fact, the curvature on the electron energies yields a parabolic behavior of the phase of the bunching at the radiator entrance, which gives a frequency chirped radiation. The pulse generated by the bunch B2 as shown in Fig. 6 presents a larger frequency chirp and a shorter temporal length compared to the pulse of B1 in Fig. 5 due to the larger curvature on the electrons energies.

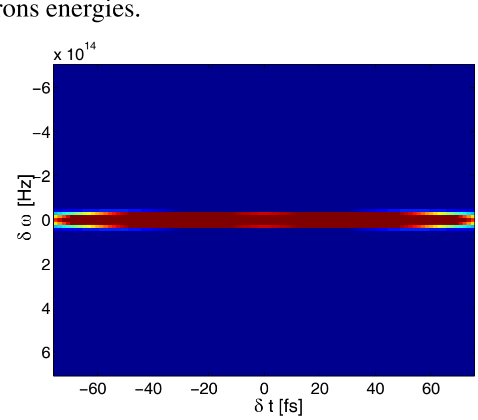

Figure 4: Wigner function plot for the FB case.

CONCLUSION

In this paper we derived, without using a saddle point approximation, the Green functions for an FEL considering

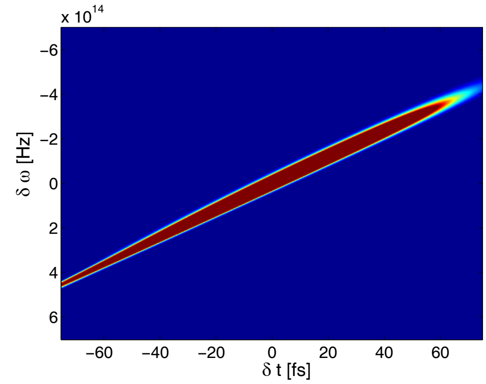

Figure 5: Wigner function plot for the B1 case.

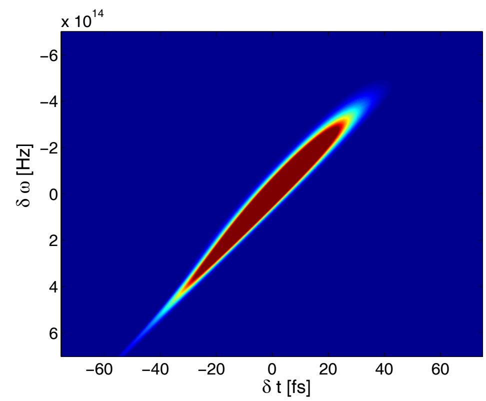

Figure 6: Wigner function plot for the B2 case.

as possible initial start-up conditions a laser seed and an initial density modulation. For the HG FEL configuration we evaluated the bunching at the radiator entrance in both amplitude and phase. A curvature on the electron energies, yields a parabolic phase behavior which is translated into a frequency chirp on the FEL pulse in the radiator.

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