DEEP SATURATION DYNAMICS IN A FREE ELECTRON LASER

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Abstract

The Free Electron Laser (FEL) regime is investigated long after saturation using the Colson-Bonifacio model. This regime is actually an out-of-equilibrium metastable state, which slowly relaxes toward thermodynamical equilibrium. This specific dynamics is characterized by a strong regularity, unexpected for an interaction between waves and large number of particles, as well as by low-dimensional phase-space structures in the electron-beam phase space. In this context, the switch from regimes associated to high gain (for small electron-beam energy spread) or very low gain (for large energy spread) is interpreted as out-of-equilibrium phase transitions, a phenomenon which was recently described by a mechanism of entropy maximization.

INTRODUCTION

The wave-particle interaction in a FEL has the particularity to be *long-ranged*, which means that the collective field generated by the particles dominate over the local effects (e.g. space-charge effects). Such interactions are characterized by the existence of out-of-equilibrium nearly stable states in which the system often gets trapped. Typically, starting from an unstable state, systems with long-range interactions [1, 2] violently relax toward a long-lived regime, called Quasi-stationary State (QSS), whose features can significantly differ from those of thermodynamic equilibrium. Although the system will eventually reach equilibrium, the QSSs lifetime has been shown to diverge with the number of particles [3, 4], which potentially makes them the sole regimes accessible experimentally.

Since it concentrates the main features of long-range interactions with amenable mathematical treatment, the Hamiltonian Mean-Field (HMF) model [5] was widely used to explore the properties of these systems. Apart from the properties of very slow relaxation toward equilibrium, out-of-equilibrium phase transitions were detected and adequately predicted by a statistical theory [6]. Furthermore, a strong regularity was shown to characterize these states, where the particles are nearly trapped on low-dimensional structures of phase space [7], each having a nearly periodic motion.

In the case of FEL, starting from an unstable state with negligible intensity, the wave first grows exponentially before it reaches saturation. Then the wave starts oscillating for very long times - the so-called QSS. Note that the fluctuations will eventually reduce to thermodynamic fluctuations), but only over very long times.

In this paper, we describe the QSS features in the FEL, with a focus on the low-dimensional structures which emerge from the particles dynamics. The out-of-equilibrium statistical theory is briefly reviewed, which gives a different insight into the quasi-stationary dynamics. Finally, the out-of-equilibrium phase transition in the FEL is presented, where dynamical and statistical approaches provide crossed information.

THE COLSON-BONIFACIO MODEL AND ITS VLASOV FORMULATION

In the one-dimensional limit, the FEL dynamics is well described by the Colson-Bonifacio model [8], whose N-particle description reads:

$$\begin{aligned}
\frac{d\theta_j}{dz} &= p_j, \\
\frac{dp_j}{dz} &= -2\sqrt{I}\cos(\theta_j-\phi), \\
\frac{d\phi}{dz} &= \frac{1}{\sqrt{I}}\sum_{j=1}^{N}\sin(\theta_j-\phi), \\
\frac{dI}{dz} &= 2\sqrt{I}\sum_{j=1}^{N}\cos(\theta_j-\phi),
\end{aligned} \tag{1}$$

where z is the rescaled undulator length, (θ_j, p_j) are the ponderomotive phase and relative momentum (with respect to the resonant one) of particle j, while (ϕ, I) correspond to the phase and intensity of the radiated wave. For large number of particles, the system is equally described by the Vlasov equation, where the particles are represented by a density function in phase-space $f(\theta, p)$. The associated equations of motion are:

$$\begin{aligned}
\frac{\partial f}{\partial z} &= -p\frac{\partial f}{\partial \theta} + 2\sqrt{I}\cos(\theta-\phi)\frac{\partial f}{\partial p}, \\
\dot{\phi} &= \frac{1}{\sqrt{I}}\int d\theta dp f\sin(\theta-\phi), \\
\dot{I} &= 2\sqrt{I}\int d\theta dp f\cos(\theta-\phi),
\end{aligned} \tag{2}$$

We here consider water-bag initial conditions, which means that the particles are uniformly spread in the

$[-\alpha;+\alpha] \times [-\Delta p;+\Delta p]$ part of phase-space. Then, the (rescaled) energy of the waterbag is $\epsilon = \Delta p^2/6$, while the initial bunching is $b_0 = \sin(\alpha)/\alpha$. As can be observed in

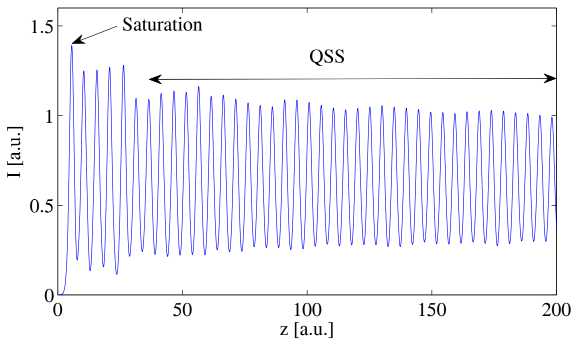

Figure 1: Evolution of the intensity inside the undulator, for the N-body model with $N = 100000$ for an initially unbunched water-bag ($b_0 = 0$) with no energy spread ($\epsilon = 0$).

Fig. 1, the intensity initially quickly grows (linear regime characterized by an exponential growth of the wave), after which the intensity saturates and starts oscillating with a large amplitude of fluctuations.

The N-particle model and the Vlasov equation naturally yield the same dynamics for large number of particles. However, for a finite number of bodies, the so-called granularity progressively drives the system toward thermodynamic equilibrium, while the Vlasov system will remain trapped into a stationary state. The time required for the N-body system to reach equilibrium has been shown to diverge with the number of particles, although the rate of divergence may differ depending on the initial conditions [3, 4].

PHASE-SPACE STRUCTURES AND REGULARITY OF THE DEEP SATURATION DYNAMICS

Despite the complexity of phase space, which is $2N+2$-dimensional for the N-particle system, and infinite-dimensional in the Vlasov limit, the particle dynamics features can be investigated by a technique close to stroboscopic plot, which can be assimilated to a Poincaré section: Each time the intensity crosses its average saturated value, going from below to above, the position and momentum of each particle is recorded. In Fig. 2 are plotted the positions of a few particles in the (θ, p) space.

Thus, it appears that most particles are nearly trapped on one-dimensional structures of the (θ, p) space. These structures are similar to KAM-tori [9], which are structures on which particles are trapped for infinite times in the case of two-degree-of-freedom models. Howver, in the present case, because of finite-N effects (the so-called "granularity"), the particles slowly diffuse through the (θ, p)-space, an effect which gives the tori a finite "thickness".

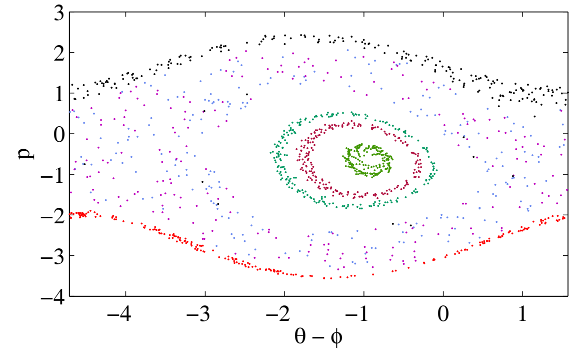

Figure 2: Poincaré section of the N-body dynamics: A few particles positions in phase-space are represented for each time the intensity crosses its saturated intensity (each color codes for a different particle).

This phenomenon has been demonstrated to depend on the number of particles involved in the interaction for a similar model, the so-called Hamiltonian Mean-Field [5]: In particular, this thickness was shown to tend toward zero when the number of particles grows to infinity (see Fig. 3). This indicates that both the HMF model and the Colson-Bonifacio model saturated dynamics are characterized by low-dimensional chaos, and that the more particles there are, the closer the system dynamics is to a low-dimensional one.

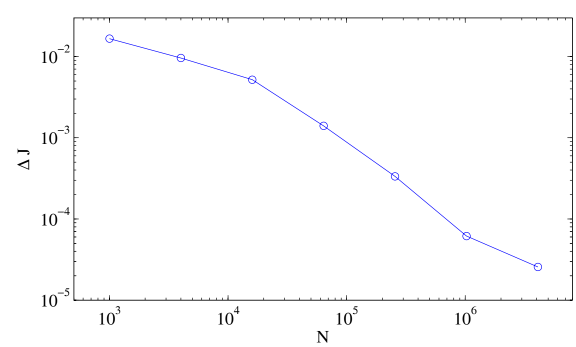

Figure 3: Thickness of the tori as a function of the number of particles for the HMF model [7]. It is equivalent to measuring the variation of energy of an individual particle over a given period of time.

LYNDEN-BELL PREDICTION FOR THE OUT-OF-EQUILIBRIUM DYNAMICS

In order to predict some features of the saturated regime, such as the intensity of the laser field, one can resort to a theory originally devised by Lynden-Bell [10]. This theory, suited for collisionless dynamics (i.e. obeying a Vlasov equation), is based on the optimization of an entropy functional, under macroscopic constraints of mass, energy and momentum. Starting from an initial waterbag, one obtains as a result $\bar{f}(\theta, p)$, a distribution function for the electrons in phase-space valid on a coarse grained level, and the associated laser field I.

This framework has been used to predict the saturated intensity of the FEL [11], as a function of the detuning parameter in particular. More recently, it allowed to evidence the out-of-equilibrium phase transition present in the FEL [12]. Indeed, QSS are then interpreted as being dynamically close to one of the two solutions of Lynden-Bell's theory. The first one, called LB0, has zero field intensity, and the electrons are not organized coherently (no micro-bunching). The other one has a finite field intensity and a finite bunching factor. However, the first solution may be unstable, and the wave will then eventually grows. Depending on the initial conditions, the system may be able to evolve toward one solution or the other. Figure 4 shows the evolution of the saturated intensity for a bunching factor and a wave intensity initially zero. In this case, the system naturally follows the evolution of the LB0 solution predicted by Lynden Bell's theory.

A complete predictive theory is still lacking at the present time, which will determine which solution the system will choose. Still, the QSS predictions given by Lynden-Bell's theory provides a phase-space density distribution to which the phase-space portrait of the FEL will tend, and that can be compared to experimental or numerical results.

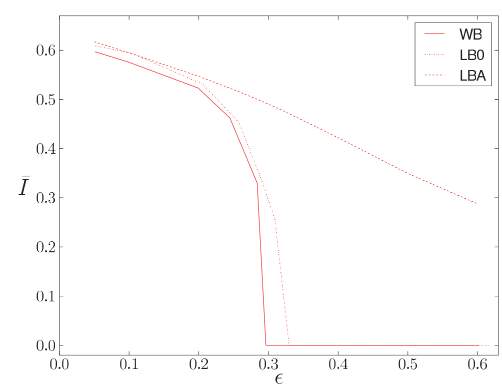

Figure 4: The prediction of the saturated intensity I from Lynden-Bell's theory (LBA and LB0), for an the initial bunching $b_0 = 0$. The WB curve corresponds to Vlasov simulations of the dynamics.

OUT-OF-EQUILIBRIUM PHASE TRANSITION

In the HMF model, an out-of-equilibrium phase transition was evidenced when the energy of the system is increased [6], which was interpreted as a change in stability of Lynden-Bell solutions. This approach provides a framework where both intensity and particles distribution can be described. It however lacks of insight into the dynamics, since it describes a *stationary* state of the Vlasov equation. Thanks to the Poincaré section technique, one can observe that the changes in the macroscopic observables (macroscopic field such as the field intensity, bunching factor) are associated to changes in the phase-space structures of the particles dynamics (see [7] for details on the HMF model).

Recent works on the Colson-Bonifacio model allowed to evidence a similar phase transition in the FEL [12]. In the case of a waterbag with low energy spread (see Fig. 2), the resonance created by the wave traps most particles. Whereas at high energies (larger energy spread Δp), as can be observed in Fig. 5, the saturated intensity collapses, which results in a tiny resonance. Less particles get trapped in the wave potential, whereas more particles have ballistic trajectories, nearly without interaction with the field. Unfortunately, this transition could not yet be interpreted in term of change in stability for Lynden-Bell solutions, but rather as a dynamical switch between different existing solutions.

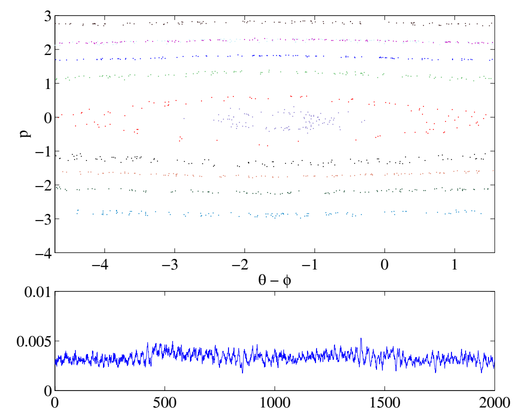

Figure 5: Upper panel: Poincaré section at large energy spread ($\Delta p = 3$ and $\alpha = 0$); each color codes for a different particle. Lower panel: Evolution of the intensity.

CONCLUSION AND PERSPECTIVES

In this Proceedings, we described the saturated regime of the FEL, using both the Poincaré section technique and the statistical theory of Lynden-Bell, which provide crossed information. A natural extension of this work is to make the bridge between the two approaches, linking the specific phase-space structures observed with the predictions of the statistical theory. Finally, the out-of-equilibrum phase transition in the FEL still lacks a comprehensive framework, and its understanding will be the object of future works.

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