CYCLOTRON & FFAG STUDIES USING CYCLOTRON CODES

M.K. Craddock (UBC and TRIUMF)
and
Y.-N. Rao (TRIUMF)

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FFAG DESIGN TOOLS

FFAG designs have generally been developed using synchrotron lattice codes – or adaptations of them – perhaps because their designers have mostly come from a synchrotron background.

But synchrotron codes are poorly adapted for use in accelerators with fixed magnetic fields:

- The central orbit is a spiral – rather than a fixed-radius ring - with the equilibrium-orbit (E.O.) radius depending on energy;
- A wide radial region of magnetic field must be characterized.

As a result, special arrangements must be made to deal with momentum-dependent effects accurately.

ORBIT-TRACKING TOOLS

Méot et al[1]. have avoided these problems by using ZGOUBI:

- an orbit tracking code originally developed for the study and tuning of mass spectrometers and beam lines.

Here, we report studies made with the cyclotron orbit codes:

- **CYCLOPS**[2], which tracks particles through magnetic fields specified on a polar grid and determines the equilibrium orbits and their optical properties
- its sister code **GOBLIN**[3] for accelerated-orbit studies.

These have the advantages of:

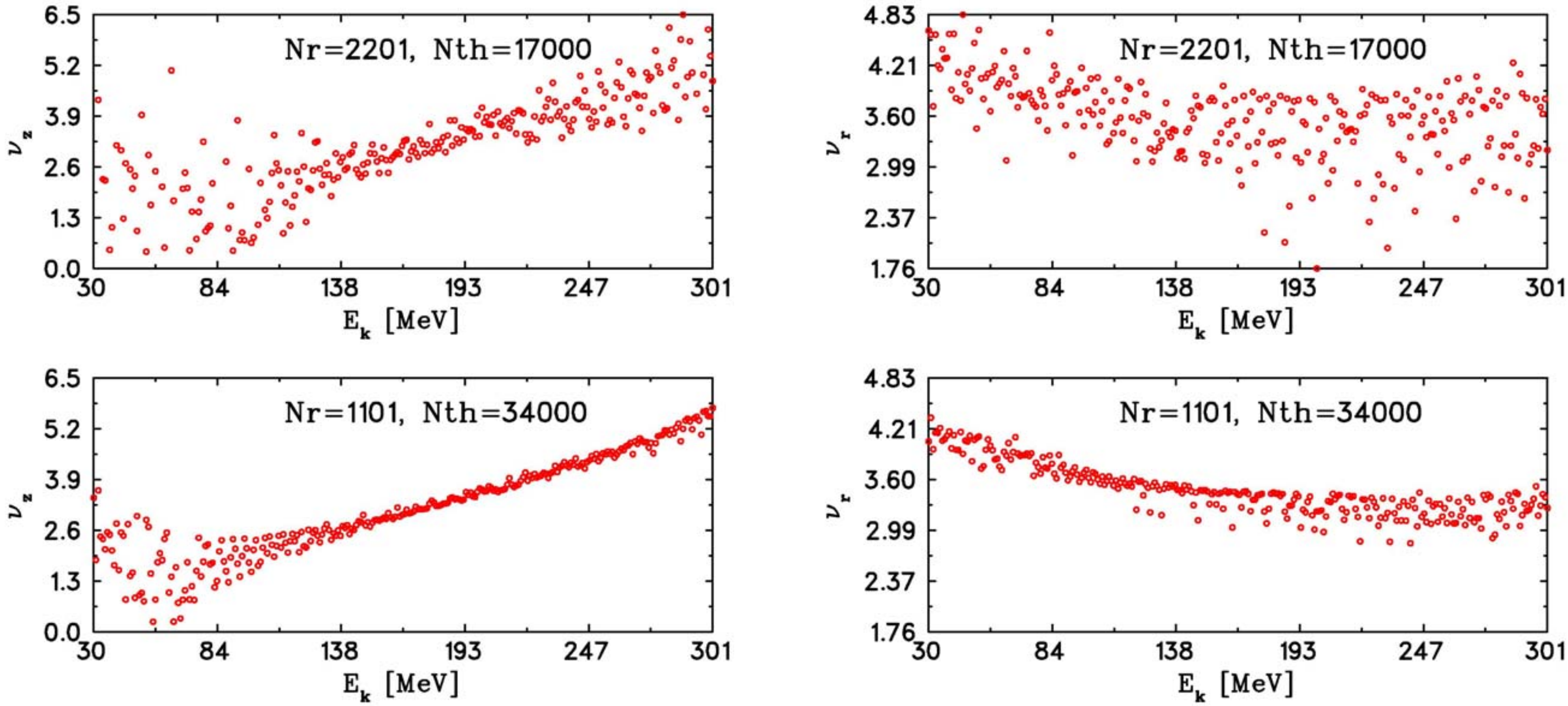
- Being designed for multi-cell machines with wide aperture magnets
- Simultaneous computation of orbit properties at all energies
- Capability of tracking through measured magnetic fields

1. F. Lemuet, F. Méot, G. Rees, Proc. PAC'05, 2693 (2005).
2. M.M. Gordon, Part. Accel. **16**, 39 (1984).
3. M.M. Gordon, T.A. Welton, ORNL-2765 (1959).

JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (1)

In 2007 Carol Johnstone & Shane Koscielniak developed an LNS FFAG, using a FODO lattice, for cancer therapy with 18-400 MeV/u carbon ions[4]. This used edge- as well as gradient-focusing to minimize the tune variation.

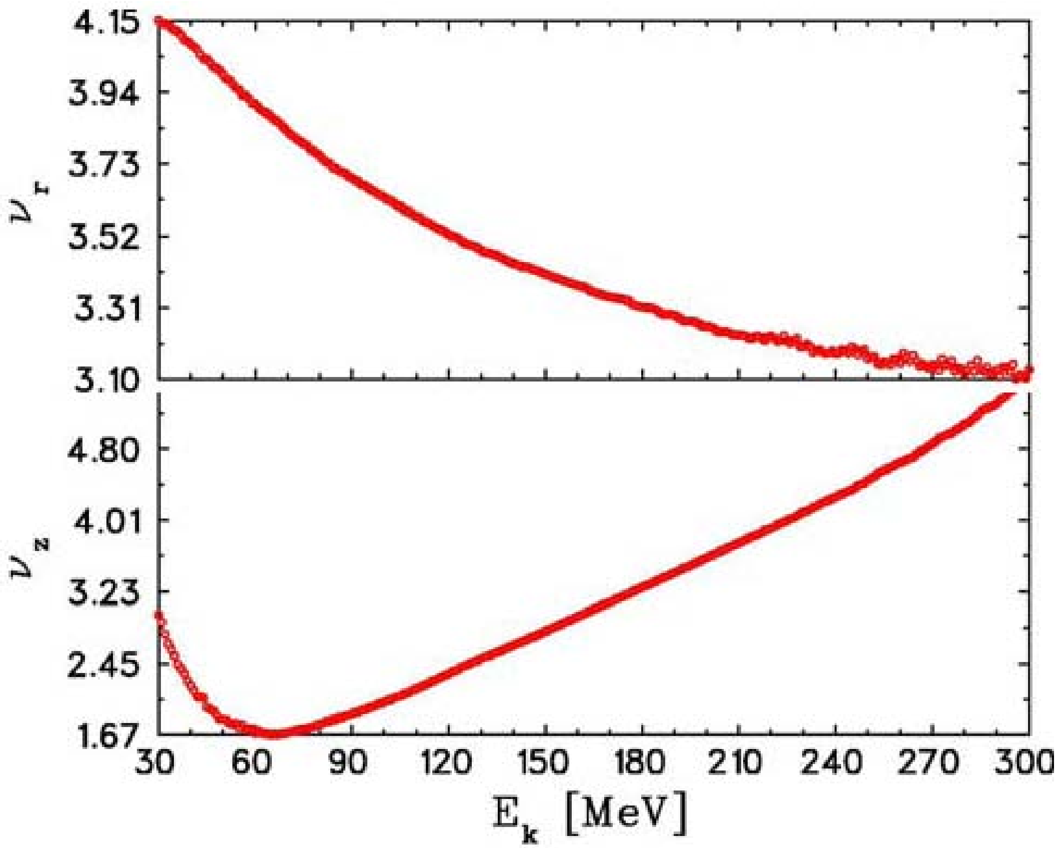
But **non-radial** hard magnet edges are tricky to model with a polar grid – and lead to noisy results from CYCLOPS – even with 37 million grid points!



4. C. Johnstone, S.R. Koscielniak, *Proc. PAC'07,* 2951-3 (2007).

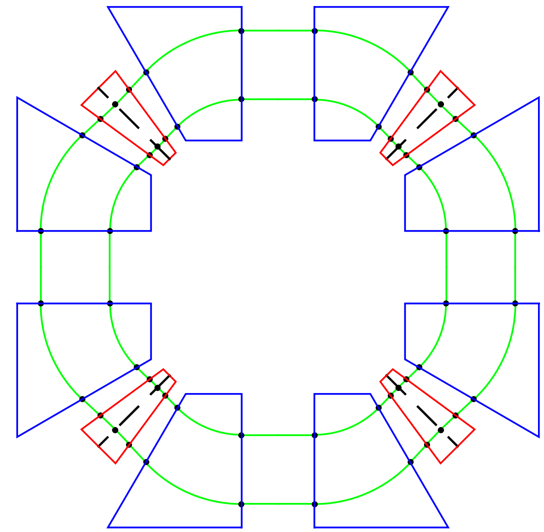
JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (2)

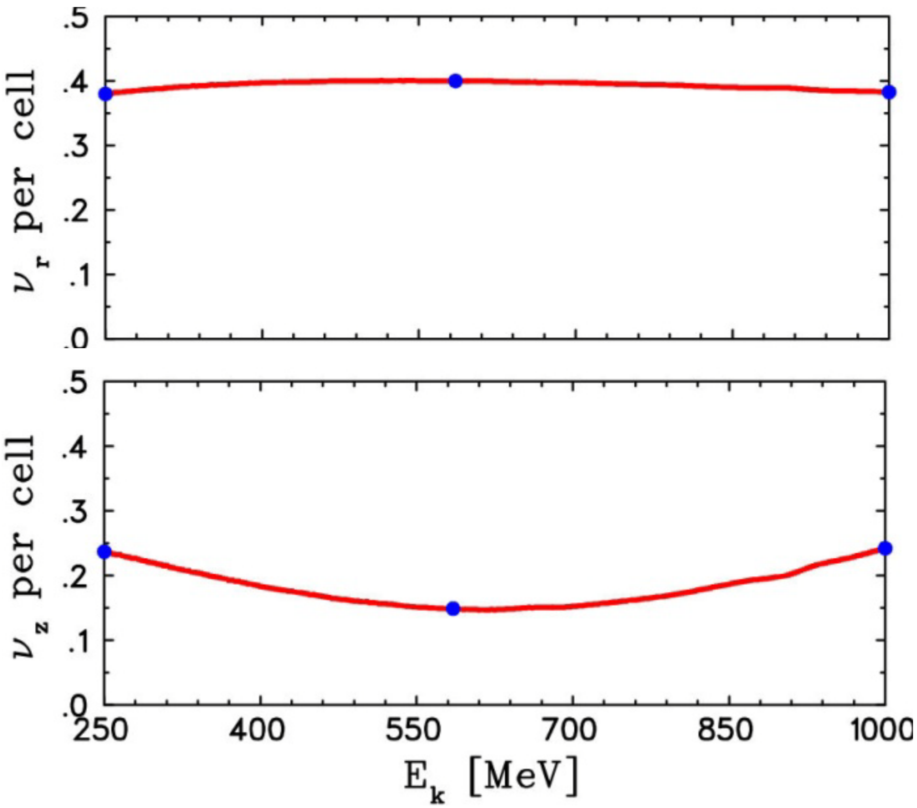
The brute-force method of reducing the mesh size was clearly inadequate.
But smoothing the hard field edges with a steep sinusoidal fall-off
– proved to be a simple but effective technique
- gave tunes that vary almost perfectly smoothly with energy.

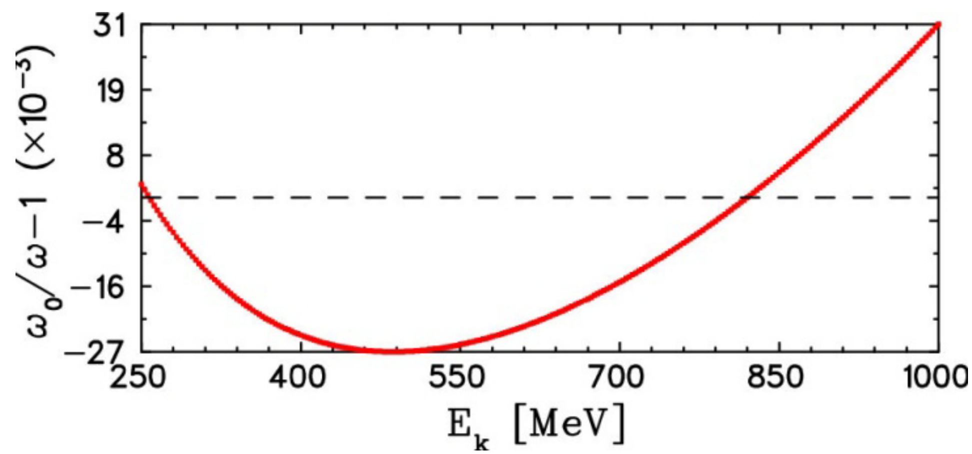


JOHNSTONE'S PROTON LNS-FFAG FOR ADSR

C. Johnstone has proposed a two-stage proton LNS-FFAG, operating at fixed frequency, to drive a sub-critical reactor. We have studied the second stage (250-1000 MeV), softening the hard-edge field minimally with an Enge function. The CYCLOPS results (—) agree well with those from COSY (●).

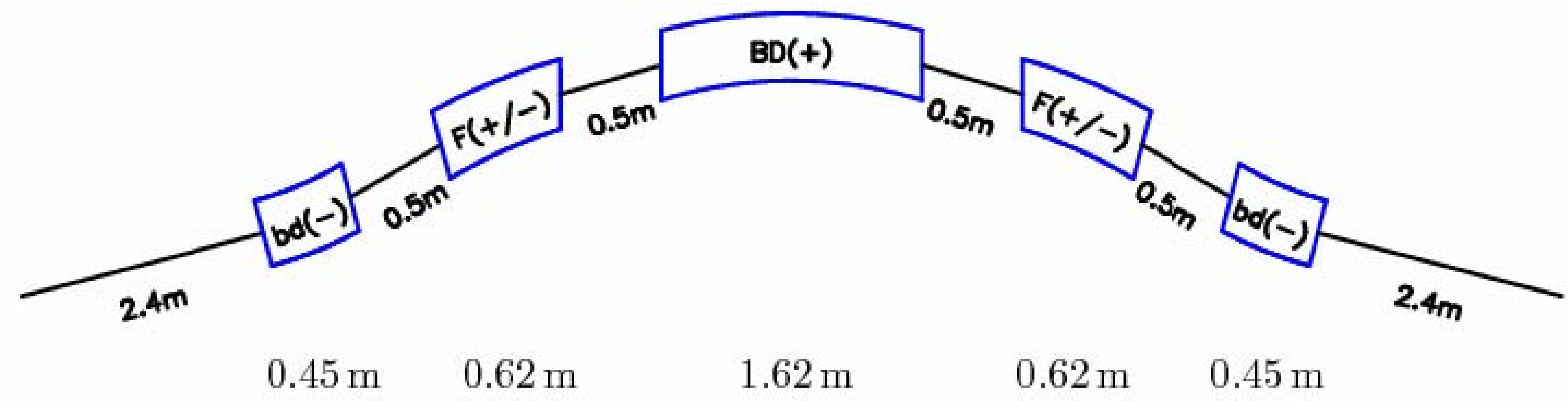






REES'S ISOCHRONOUS IFFAG

G.H. Rees[1,5] has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility.



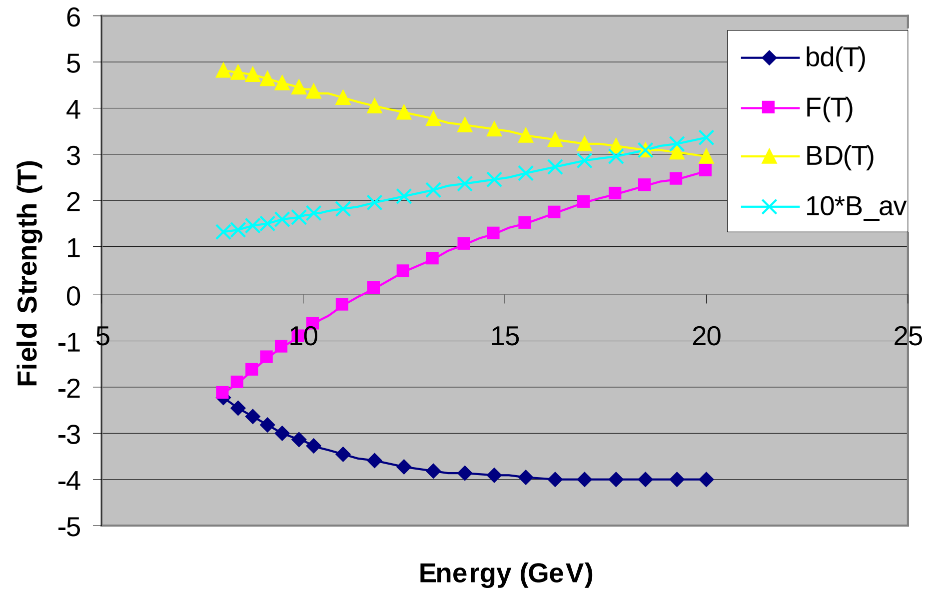
- The example shown is an isochronous design (IFFAG) for accelerating muons from 8-20 GeV in 16 turns.
- This is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies ($77 \le \gamma \le 190$) without invoking spiral magnet edge focusing [recall isochronous $\Delta\nu_z^2 = -(r/B_{av})(dB_{av}/dr) = -\beta^2\gamma^2$].
- Highest energy spiral-sector isochronous cyclotron design had $\gamma \le 15$.

5. G.H. Rees, *FFAG'04* (2004); *FFAG'05* (2005); *ICFA-Beam Dynamics Newsletter* **43**, 74 (2007)

IFFAG FIELDS & TUNES

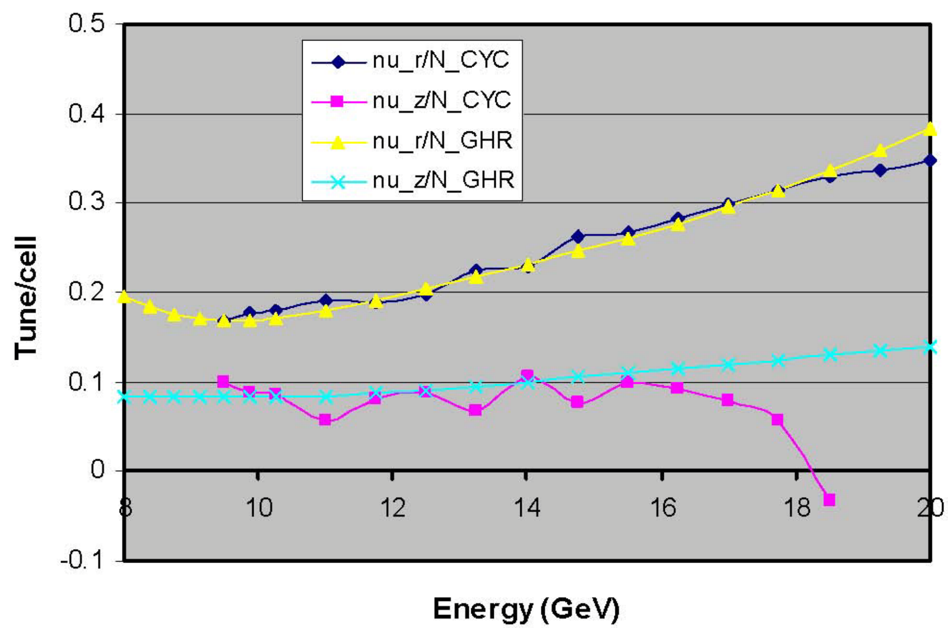
For the **fields**, note how:

- F reverses sign at ~11 GeV
- bd focusing vanishes at high E
- BD focusing vanishes at low E
- B_{av} rises linearly with E
- The vertical defocusing associated with rising B_{av} is offset by **strong AG focusing**.



For the **tunes** our initial results (♦■) were in general agreement with Rees's (▲ ×) - except above 17 GeV - but rather noisy.

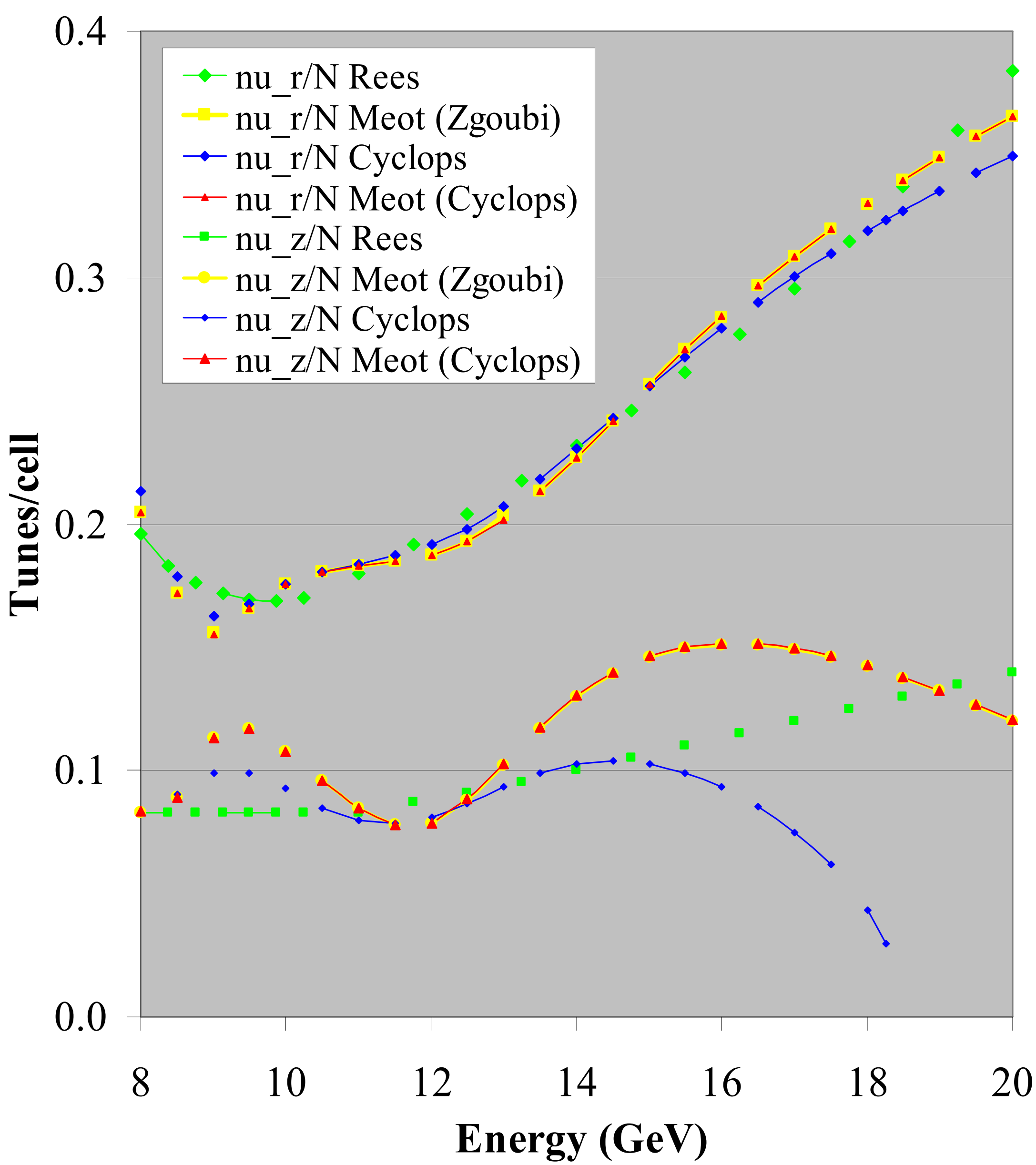
As before, field smoothing is needed to track through the non-radial hard edges accurately.



IFFAG WITH SOFTENED MAGNET EDGES

Steep sinusoidal edges remove the noise in the tune data, but not the drop-off in ν_z above 15 GeV (-◆-).

Méot's tracking results using ZGOUBI (-■-) agree with Rees's predictions (-■-) at 8, 11 & 20 GeV – but only after slight adjustments in the magnet positions and field profiles. For this adjusted configuration CYCLOPS (-▲-) gives results identical to those from ZGOUBI.



RADIAL-SECTOR CYCLOTRONS WITH REVERSE BENDS

The IFFAG is essentially an isochronous ring cyclotron with an unusually complicated magnet arrangement – 5 magnets/cell rather than 1.

An isochronous cyclotron's top energy is limited by vertical focusing:

$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2(1 + 2\tan^2\varepsilon)$$

where ε is spiral angle and the magnetic "flutter" (mean square deviation)

$$F^2 \equiv \left\langle \left(B(\theta)/B_{av} - 1 \right)^2 \right\rangle.$$

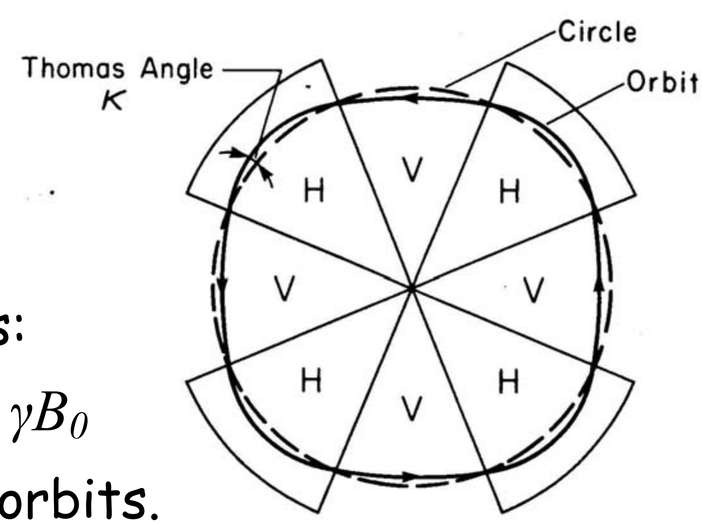
How high an energy could a radial-sector cyclotron reach by simply **converting the low-field "valley" to a reverse bend** - maximizing F^2 and introducing AG focusing?

We assume:

- N radial sectors (hill fraction h)
- Hard-edge magnets with $B \leq 5\text{T}$
- No drift spaces
- Equal and opposite hill and valley fields:

$$B_h = -B_v = B(r) = \gamma B_0$$

- Field contours following the scalloped orbits.



Denoting the angular fraction of a sector taken up by a hill as h, and ignoring scalloping effects on the orbit length and average field around it:

$$B_{av} = 2(h - ½)B .$$

The flutter is determined entirely by h, and so is the same at all energies:

$$F^2 = ¼(h - ½)^{-2} - 1 .$$

For the axial focusing to remain positive up to some maximum energy γ_m, but no further, the tune formula tells us that:

$$h - ½ = 1/2\gamma_m .$$

If the maximum magnetic field available, B_m, is applied at maximum energy γ_m , then the "central field" B_c and "cyclotron radius" R_c are given by:

$$B_c = B_m/\gamma_m^2$$

$$R_c = (m_0c/e)\,\gamma_m^2/B_m ;$$

i.e. the ring radius required increases as the square of the desired energy.

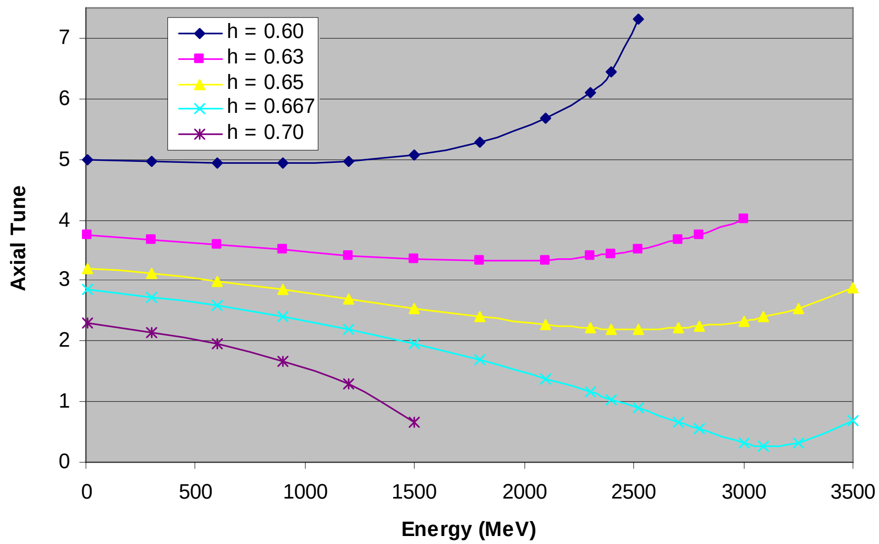
If r_{hv} is the radius at the hill/valley edge, the recipe for field strength is:

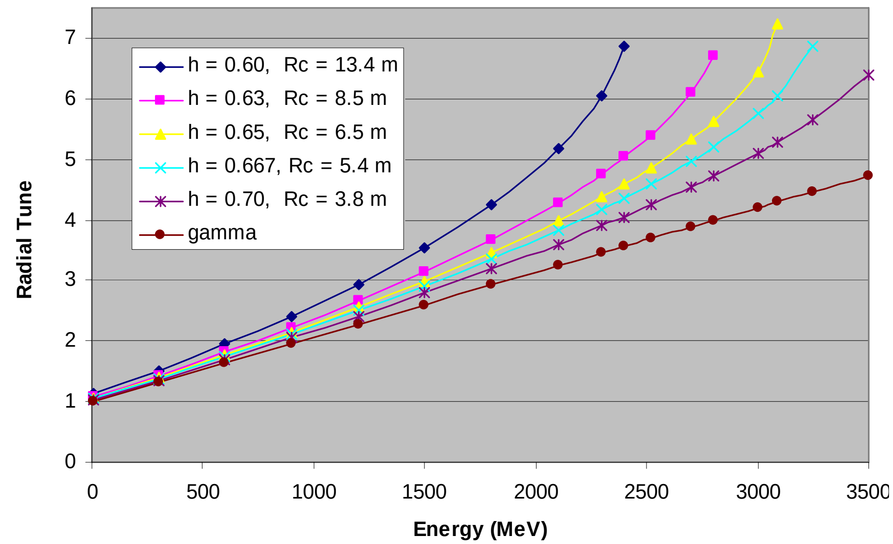
$$B(r_{hv}) = (B_m/\gamma_m)/ \sqrt{\{1 - (r_{hv}/R_c)^2\}}.$$

Symon's circumference factor, the ratio of the actual circumference to that obtainable with uniform B_m and no reverse bends: $C = \gamma_m$.

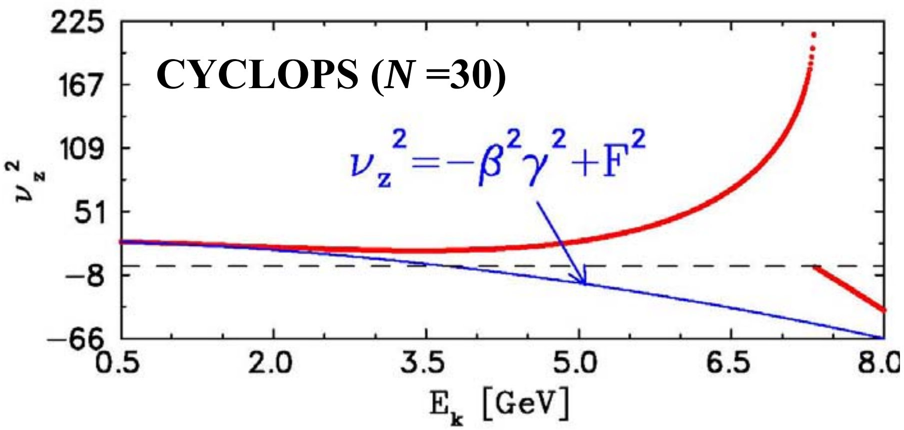
REVERSE-BEND CYCLOTRONS – SIMULATIONS

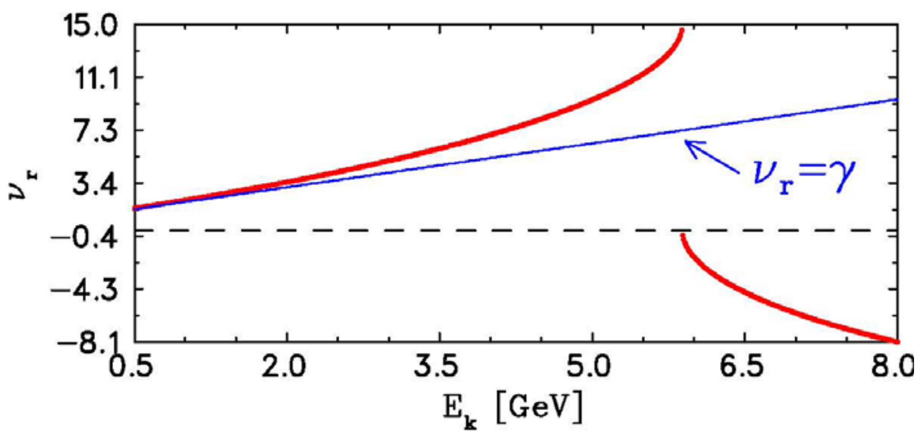
For $N = 15$ the $N/2$ stopband is a limiting factor: either we widen the hills - reducing the radius and both tunes (say $h = 0.65$, $E = 3$ GeV, $R_c = 6.5$ m),



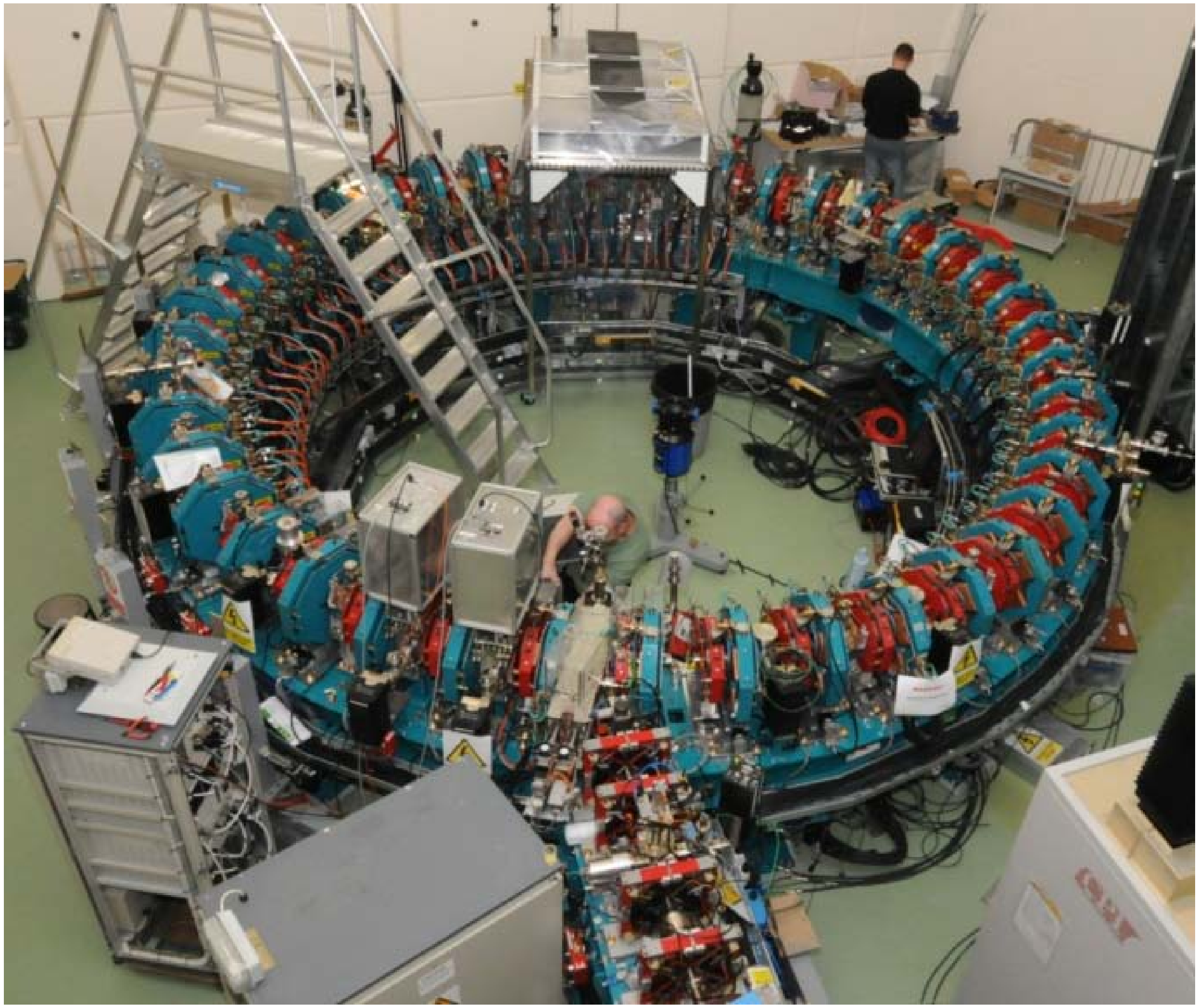


or increase the number of sectors (say to $N = 30$, with $h = 0.6$) - a more effective way of repelling $\nu_r = N/2$ ($E = 6$ GeV, but $R_c = 14.9$ m).





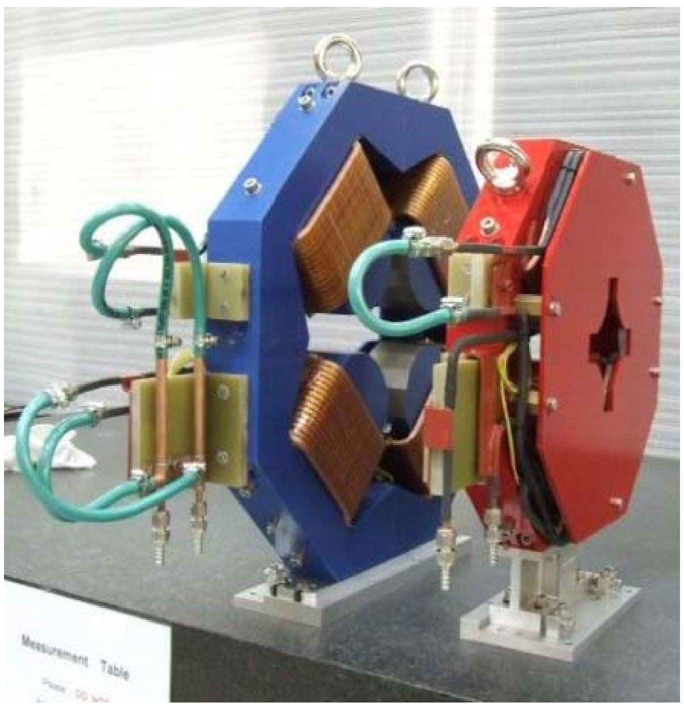
EMMA – THE FIRST NON-SCALING FFAG

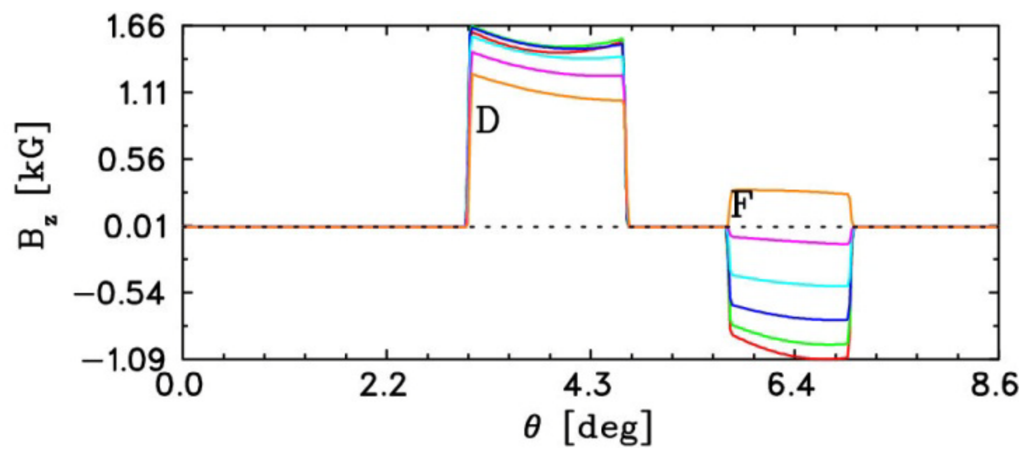


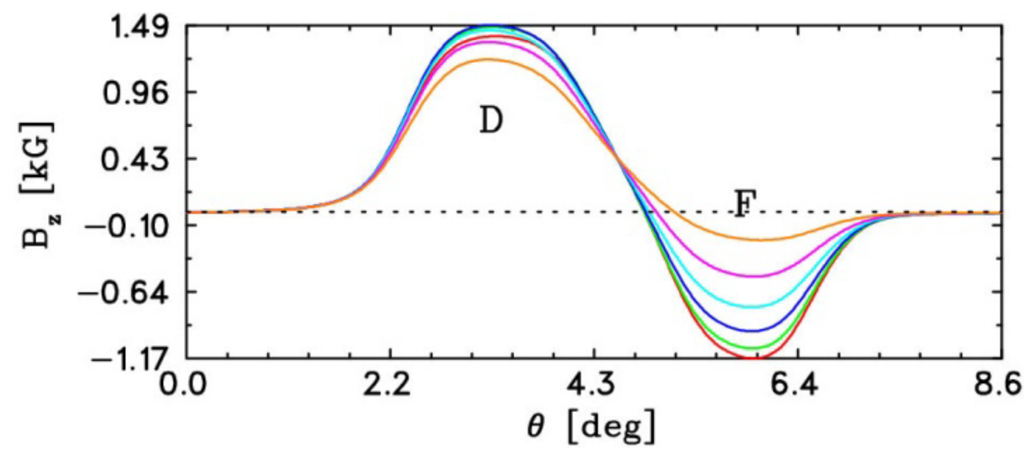
EMMA is a 10-20 MeV electron LNS-FFAG model
for a 10-20 GeV muon accelerator for a neutrino factory
- currently undergoing beam commissioning at Daresbury, UK.

EMMA MAGNETS & FIELDS

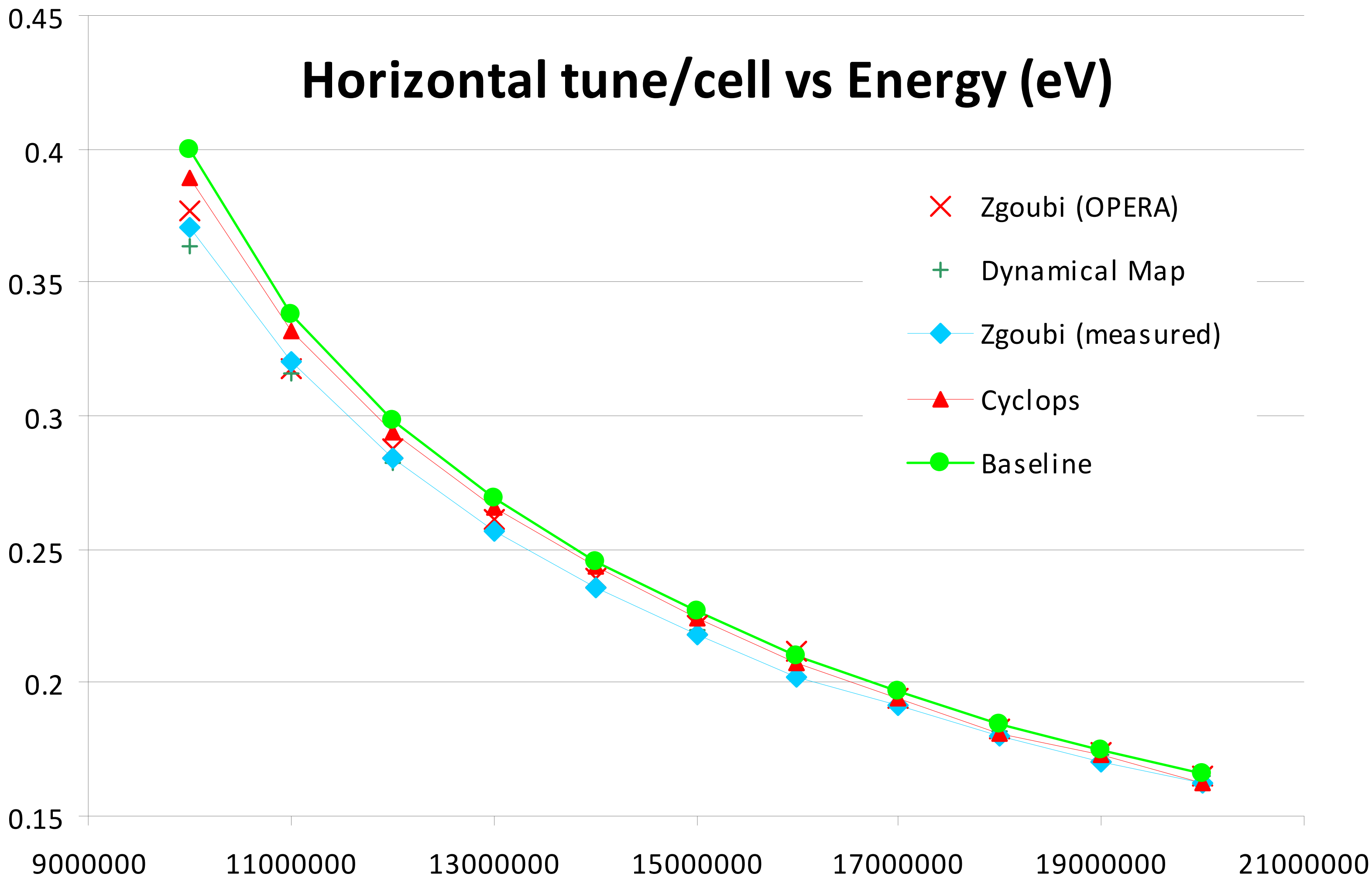
The EMMA magnets (offset quadrupoles) are very short.
Their field profiles (bottom) are therefore soft-edged
- unlike the hard-edge profiles assumed in the design (top).







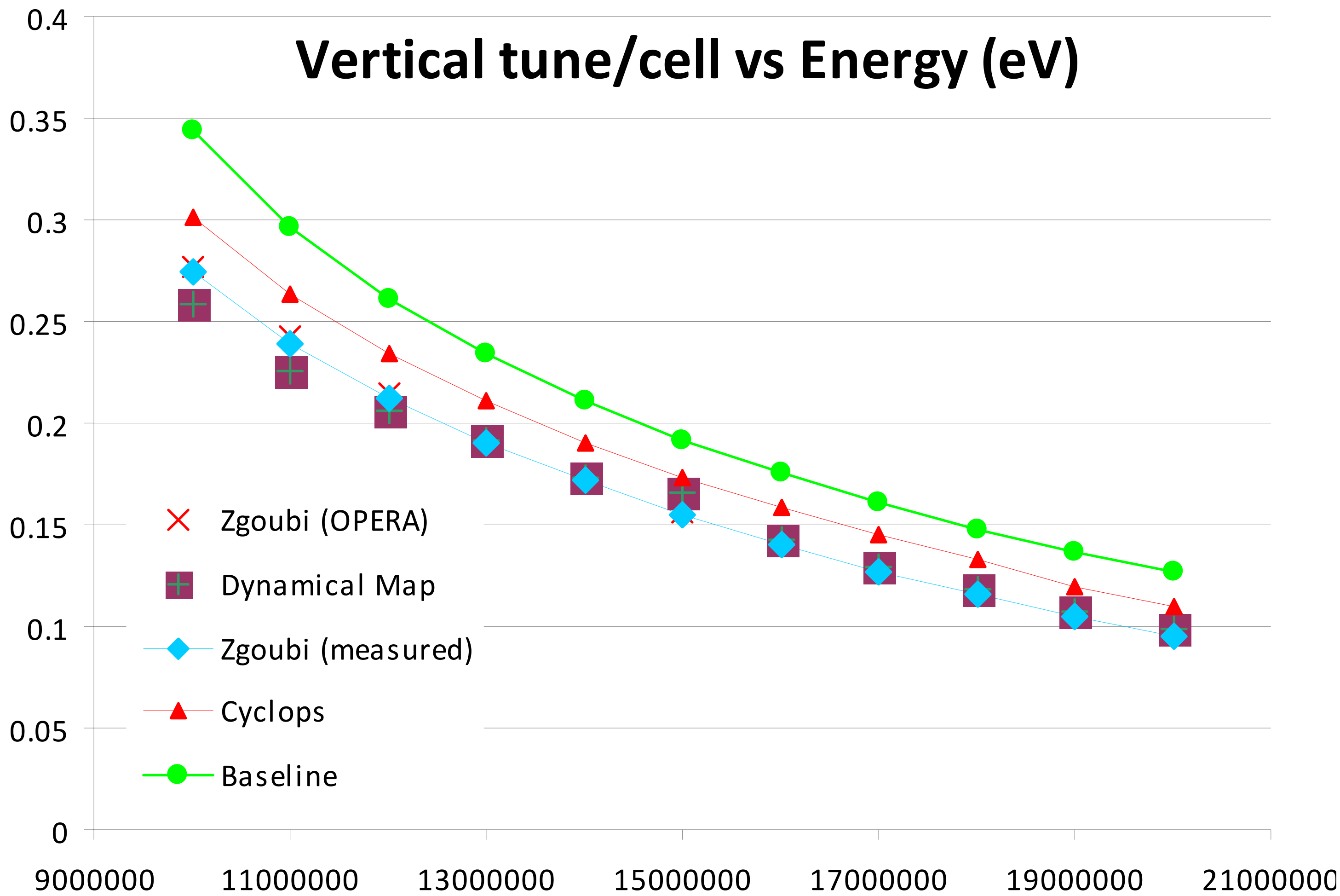
EMMA – HORIZONTAL TUNE



CYCLOPS results agree well with Scott Berg's "Baseline" data,

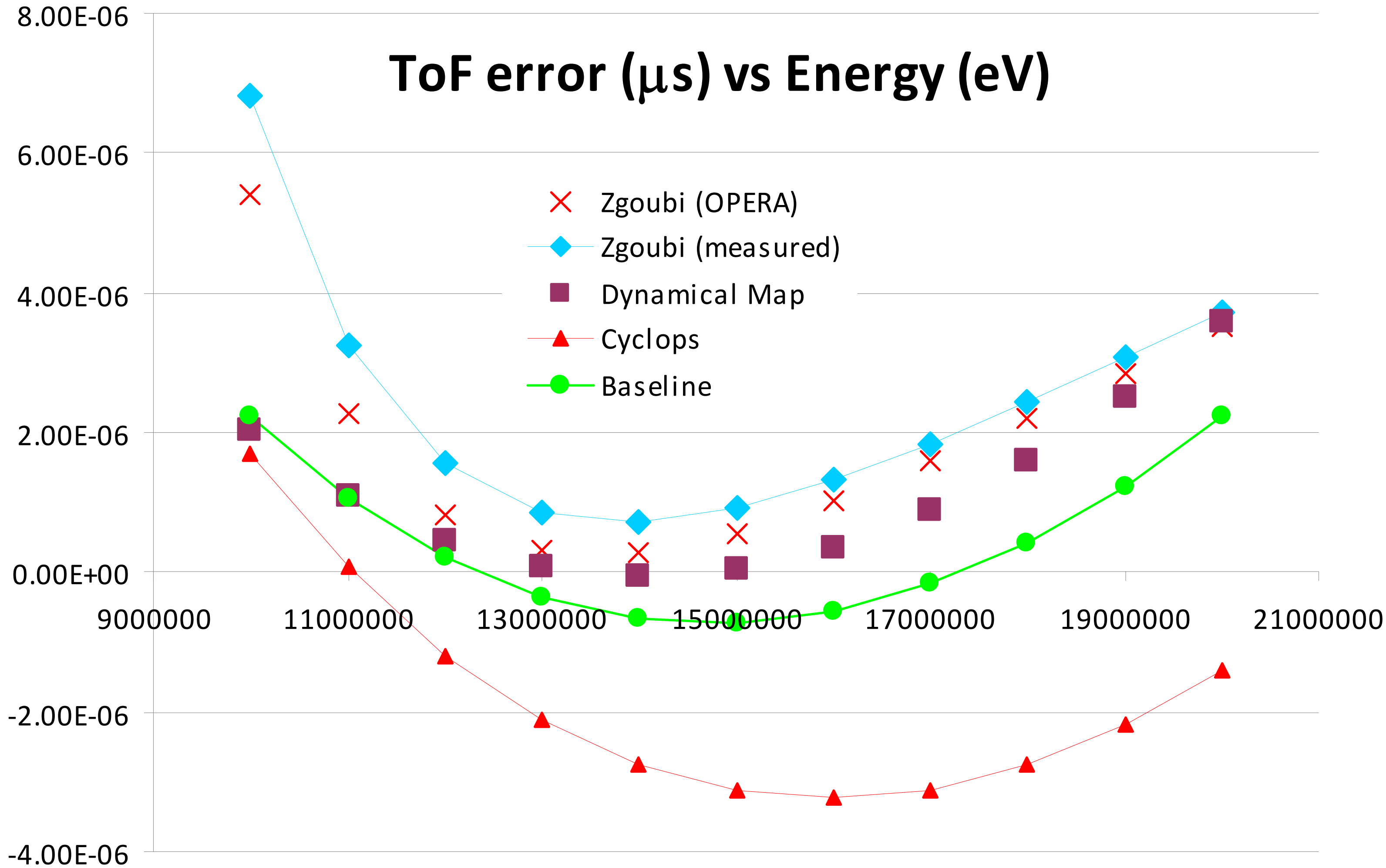
- less well with ZGOUBI runs (courtesy of Yoel Giboudot).

EMMA - VERTICAL TUNE



Here CYCLOPS, ZGOUBI and the Baseline all agree on the trend -
- but disagree on the amplitude.

EMMA – TIME OF FLIGHT



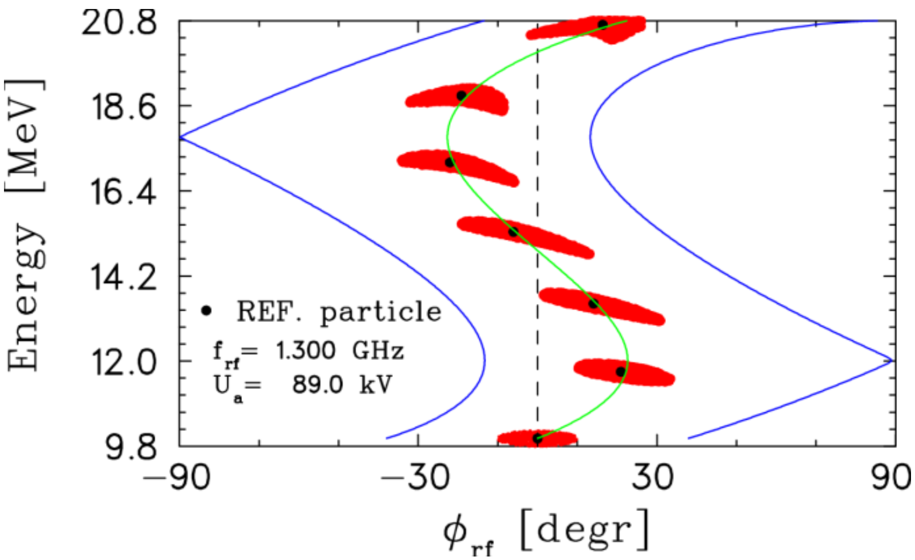
N.B. CYCLOPS, ZGOUBI & Baseline all assume different reference frequencies

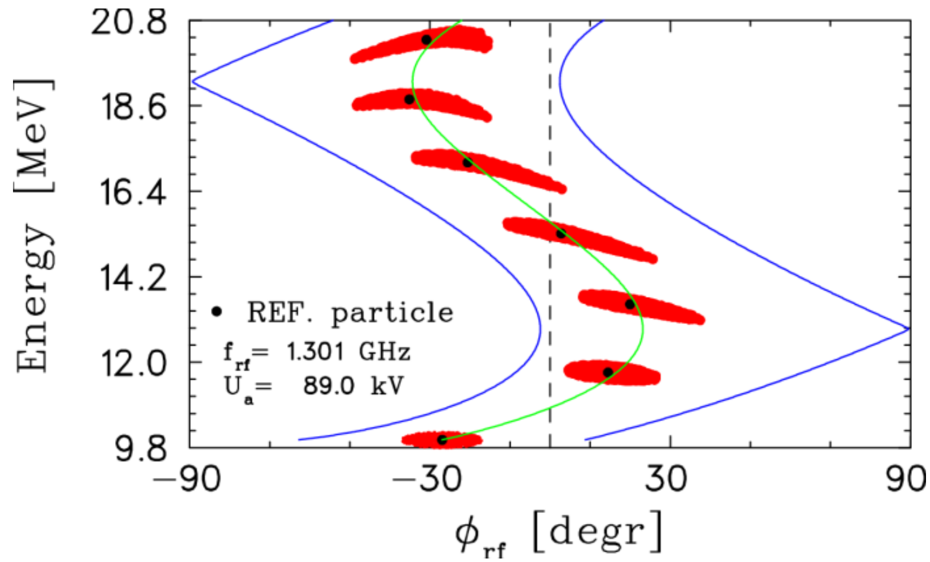
- so the vertical displacements are probably not significant
- but the displacements in energy of the minima are significant.

Accelerated Orbits in EMMA (1)

The GOBLIN code has been used to study accelerated orbits in both the Baseline and measured fields. A 4.3π eV-μs electron bunch was tracked over 5 turns through 21 evenly spaced 89-kV cavities. The initial phase was chosen midway between the two cusp trajectories (calculated by integrating the time-of-flight errors from CYCLOPS).

The plots show snapshots taken after passage through 0, 20, 41, 62, 83, 104 and 125 cavities for radial emittances of 250π μm:
(**Left**) the **Baseline field** and (**right**) the **measured field** (for which the bunch distortion is greater and the beam gains less energy).

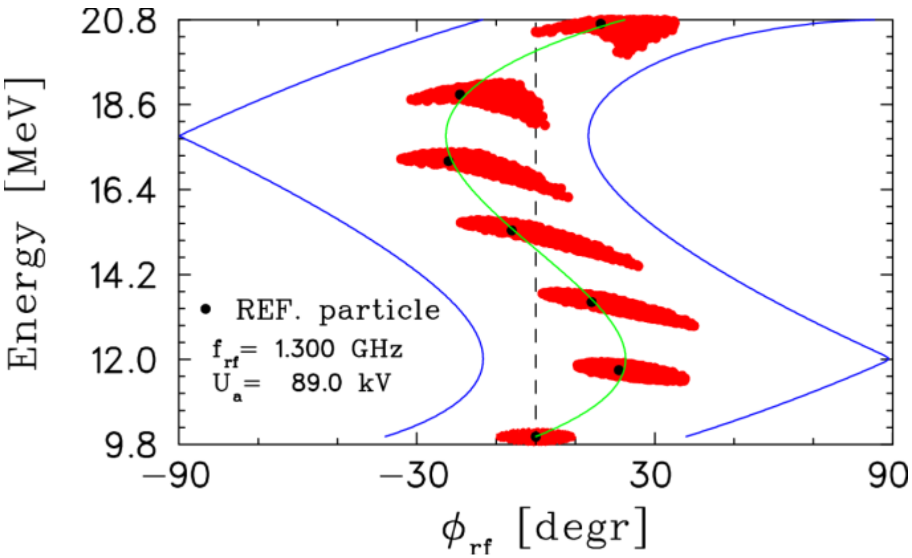


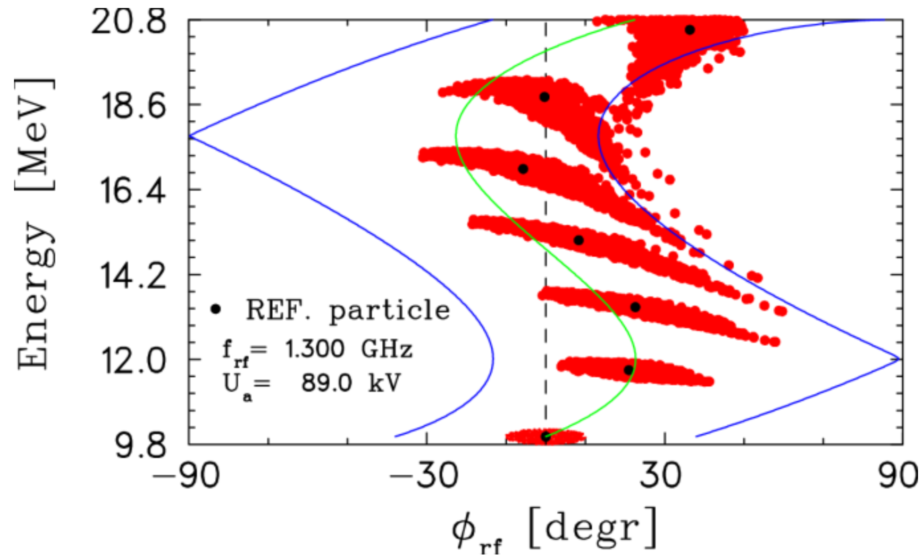


Accelerated Orbits in EMMA (2)

Studies were also carried out in the Baseline field of the effect of varying the input parameters.

- Enlarging the radial emittance from 250π μm to 1400π μm (left) increases the bunch distortion –
- Injecting off-centre (right) is even more distorting - enough to prevent some particles reaching extraction energy.





Altogether, our results for the Baseline field are very similar to those presented by Méot.

SUMMARY

The cyclotron equilibrium-orbit code CYCLOPS has been applied to:

- Reverse-bend ring cyclotrons (3 GeV @ R_c = 6.5 m, 6 GeV @ R_c = 15 m)
- LNS-FFAG EMMA for 10-20 MeV electrons
- LNS edge-focusing FODO medical FFAG for 18-400 MeV/u C ions
- NLNS edge-focusing 250-1000 MeV proton FFAG for ADSR
- NLNS isochronous pumplet IFFAG for 8-20 GeV muons.

CYCLOPS is:

- designed for energy-dependent E.O.s in wide-aperture magnets
- ideal for finding E.O. properties in **measured** magnetic fields
- uncomfortable with hard-edge magnets – data softening needed!
- in close agreement with other tracking codes,

The cyclotron accelerated orbit code GOBLIN has also been applied to EMMA

- both to the hard-edge Baseline design field
- and to a measured field.