

DETERMINATION OF ISOCHRONOUS FIELD USING CALCULATED MAP OF MAGNETIC FIELD IN CYCLOTRON MEDIAN PLANE

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Abstract

In this work a new scheme for calculation of a cyclotron isochronous field using the previously calculated or measured map of the cyclotron magnetic field in its median plane is adduced. The calculating map of the cyclotron magnetic field was set by the matrix having the dimensions 201×181. The flutter part of the magnetic field obtained by subtraction of the zero azimuth harmonic from the magnetic field values was calculated in all net nodes. The magnetic rigidity value in the equation for the particle radius versus the angle was replaced by product of the mean radius and mean along the closed orbit magnetic field. The flutter function was interpolated with the help of the third order Lagrange's polynomials using 16 nodes of the net. At every given radius with the help of the nonlinear simplex method of optimization one can find such value of the isochronous field when the particle path is enclosed with accuracy of 10⁻⁹. The results of the fulfilled calculations for the cyclotron DC-110 and their comparison with results of other calculations are given.

INTRODUCTION

A scheme for determination of a cyclotron isochronous field using the previously calculated or measured map of the cyclotron magnetic field in its median plane is adduced. The results of the fulfilled calculations for the cyclotron DC-110 [1] and their comparison with results of other calculations are given.

SCHEME OF CALCULATIONS

The cyclotron DC-110 has 4 “hills” and 4 “valleys”. Therefore four full periods of the magnetic field $B(r, \theta)$ are kept within the circuit ($r = const$) when $0 \leq \theta \leq 2\pi$. The calculated map of the cyclotron magnetic field in its median plane was set by the matrix $Q_{i,j}$ having the dimensions 201×181 . The values of the flutter part $F(r_i, \theta_j)$ were calculated in the net nodes (r_i, θ_j) by subtraction of the zero azimuth harmonic from the magnetic field values.

The distribution of the flutter versus radius is shown in Fig. 1. The calculated flutter versus the angle θ for $r = 80$ cm is shown in Fig. 2.

The following differential equation for a reference particle radius r [2] was used in our calculations.

$$\frac{d}{d\theta} \left(\frac{r'}{\sqrt{r^2 + (r')^2}} \right) = \frac{r}{\sqrt{r^2 + (r')^2}} + \frac{r[B(r) + F(r, \theta)]}{B\rho} \quad (1)$$

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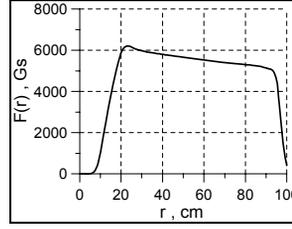


Figure 1: Flutter function $F(r)$ when $\theta = const$

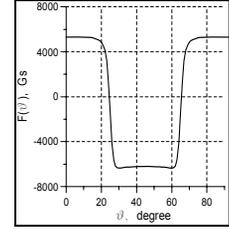


Figure 2: Flutter function $F(\theta)$ when $r = 80$ cm

The magnetic rigidity $B\rho$ value in (1) was replaced by product of the mean radius \bar{r} and mean along the closed orbit magnetic field \bar{B} :

$$B\rho = \bar{r}\bar{B}(\bar{r}) ; \bar{B}(\bar{r}) = \frac{B_0}{\sqrt{1 - (\bar{r}/r_\infty)^2}} \quad (2)$$

$$B_0 = \frac{A}{Z} \cdot \frac{M_p c^2}{e} \cdot \frac{1}{r_\infty} ; r_\infty = \frac{ch}{2\pi f}$$

Here A is the ion atomic mass, Z is the ion charge, M_p is the atomic unit mass, c is the speed of light, e is the elementary charge, h is the harmonic number and f is the frequency of the cyclotron RF generator.

As a result the isochronous magnetic field was found by solving the following system of equations:

$$\begin{cases} \frac{d}{d\theta} \left(\frac{r'}{\sqrt{r^2 + (r')^2}} \right) = \frac{r}{\sqrt{r^2 + (r')^2}} + \frac{r[B(r) + F(r, \theta)]}{r\bar{B}(\bar{r})} \\ \frac{d\rho}{d\theta} = \frac{r}{2\pi} \cdot \sqrt{1 + \left(\frac{r'}{r}\right)^2} \\ 0 \leq \theta \leq 2\pi \end{cases} \quad (3)$$

Here the function $\rho(\theta)$ defines the mean radius of the closed orbit $\bar{r} = \rho(2\pi)$. During calculation the value \bar{r} was found by iteration with specified accuracy of 10⁻⁹.

The function $B(r)$ is the varying magnetic field specified in the points w_i ($i=1, \dots, N$) along the radius where $N = R_{extr}/\Delta r$ and Δr is the value of chosen radial step. When $r = 0$ then $B(r) = B_0$.

The function $B(r)$ in the range between two points w_i and w_{i+1} was found by means of the linear interpolation:

$$B(r) = B(w_i) \cdot \frac{r - w_{i+1}}{w_i - w_{i+1}} + B(w_{i+1}) \cdot \frac{r - w_i}{w_{i+1} - w_i} \quad (4)$$

The flutter function $F(r, \theta)$ was interpolated with the help of third order Lagrange's polynomials using 16 nodes of the net. Part of this net is shown in Fig. 3.

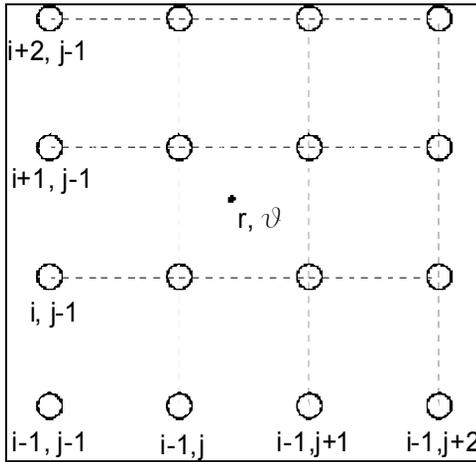


Figure 3: Part of the net

The flutter part of the magnetic field in an arbitrary point having the coordinates $\{r, \theta\}$ was found using the values of the magnetic field in 16 neighbours nodes (given in the matrix $Q_{i,j}$) in the following way. At first the value $F_k(\theta)$ was calculated on four horizontal layers of this part of the net.

$$\begin{aligned}
 F_k(\theta) = & -Q_{k,j-1} \cdot \frac{(\theta - h_\theta(j-1)) \cdot (\theta - h_\theta(j)) \cdot (\theta - h_\theta(j+1))}{6h_\theta^3} + \\
 & + Q_{k,j} \cdot \frac{(\theta - h_\theta(j-2)) \cdot (\theta - h_\theta(j)) \cdot (\theta - h_\theta(j+1))}{2h_\theta^3} - \\
 & - Q_{k,j+1} \cdot \frac{(\theta - h_\theta(j-2)) \cdot (\theta - h_\theta(j-1)) \cdot (\theta - h_\theta(j+1))}{2h_\theta^3} + \\
 & + Q_{k,j+2} \cdot \frac{(\theta - h_\theta(j-2)) \cdot (\theta - h_\theta(j-1)) \cdot (\theta - h_\theta(j))}{6h_\theta^3}
 \end{aligned} \quad (5)$$

$k = i-1, i, i+1, i+2$

Here $h_\theta = 1$ degree. It is the value of the net step. The final value of the magnetic field in the given point one can get by formula:

$$\begin{aligned}
 B(r, \theta) = & -F_{i-1}(\theta) \cdot \frac{(r - h_r(i-1)) \cdot (r - h_r(i)) \cdot (r - h_r(i+1))}{6h_r^3} + \\
 & + F_i(\theta) \cdot \frac{(r - h_r(j-2)) \cdot (r - h_r(i)) \cdot (r - h_r(i+1))}{2h_r^3} - \\
 & - F_{i+1}(\theta) \cdot \frac{(r - h_r(i-2)) \cdot (r - h_r(i-1)) \cdot (r - h_r(i+1))}{2h_r^3} + \\
 & + F_{i+2}(\theta) \cdot \frac{(r - h_r(i-2)) \cdot (r - h_r(i-1)) \cdot (r - h_r(i))}{6h_r^3}
 \end{aligned} \quad (6)$$

Here $h_r = 5$ mm is the value of the radial net step.

The system of equations (3) was solved by Runge-Kutta method with the step equal to 1/32 of degree.

In calculations the initial azimuth is chosen in the middle of the hill. Thereby the closed orbit has a maximum radius r_{\max} at this point. The radius r_{\max} is increased in our calculation from zero up to the extraction radius with the step 1 mm. At every subsequent step on the radius r_{\max} one can find with the help of optimization algorithm (simplex method, for example) such value of the isochronous field $B_{is}(r_{\max})$ when the particle orbit is enclosed with sufficient accuracy (10^{-9} in our calculation). During optimization process the values of the isochronous field at inner radii were found by formula (4) with values of isochronous magnetic field at the nodes w_i obtained in the previous steps of calculation.

To check-up the accuracy of our calculations the results were compared with ones obtained in [3] for $^{86}\text{Kr}^{13+}$ isotopes by means of Gordon's method [2]. The frequency of the RF-generator $f = 15.506$ MHz and $r_\infty = 12.31$ m. The calculated dependences of the isochronous magnetic field value versus radius, inferred in our calculations (black curve) and in [3] (red curve) are shown in Fig. 4.

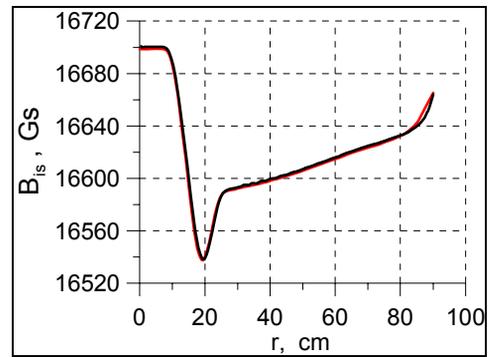


Figure 4: Isochronous magnetic field versus radius

The same curves calculated up to the radius $r = 95$ cm are shown in Fig. 5. As one can see from Fig. 4 and Fig. 5 the curves coincide practically completely except two regions of radius: $0 \leq r \leq 9$ cm and $81 \leq r \leq 90$ cm.

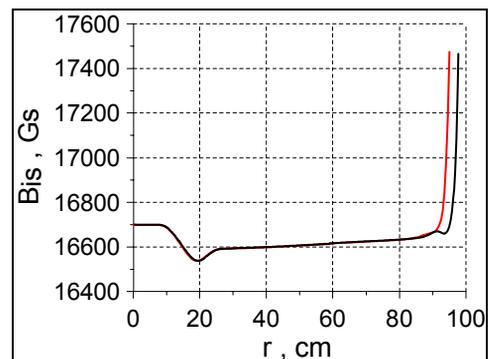


Figure 5: Isochronous magnetic field versus radius

Interpolation of the function $F(r, \theta)$ by the cubic Lagrange's polynomials in 16 nodes of the net (5 – 6) was

chosen because interpolation by linear or quadratic polynomials gives a calculation noise on some regions of the magnetic field curve connected with unstable work of the code (see Fig. 6).

This instability disappeared when we used the cubic interpolation (see Fig. 7). The calculated curves in both figures 6 and 7 are shown for the interval $25 \leq r \leq 40$ cm.

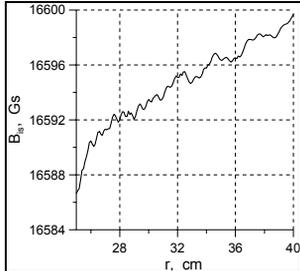


Figure 6: Noise on the calculated curve

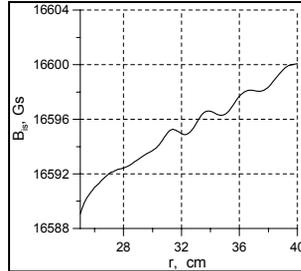


Figure 7: The curve without noise

The oscillations on the calculated curve $B_{iz}(r)$ that are shown in Fig. 7 are not already connected with unstable work of the code but they are connected with peculiar

properties of the calculated flutter magnetic field map of the cyclotron.

CONCLUSIONS

The obtained results show that the suggested scheme for calculation of the isochronous cyclotron magnetic field works and gives its correct values.

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