# SIMULATION STUDY ON TRANSVERSE LASER COOLING AND CRYSTALLIZATION OF HEAVY-ION BEAMS **AT THE COOLER STORAGE RING S-LSR**

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# Abstract

Multi-dimensional laser cooling of heavy-ion beams at the cooler storage ring S-LSR in Kyoto University is studied numerically using the molecular dynamics simulation technique in which the stochastic interaction between ions and laser photons is incorporated. The purpose of the study is to find out how low-temperature we can achieve using actual experimental parameters and to verify the observation result in the experiment. In these proceedings, the characteristics of the ion beam lasercooled in S-LSR are reported. It has been confirmed that, in spite of the limitation in the experimental conditions such as a single laser beam, low power, fixed detuning and short laser-cooling section, the three-dimensionally low-temperature beam is obtained through resonant coupling at a low intensity of 10<sup>4</sup> ions in the ring, which is consistent with the experimental result. It is also demonstrated that a string crystalline state of the beam can be formed at a further lower intensity.

# **INTRODUCTION**

Laser cooling is the most promising technique for achieving an ultralow-temperature ion beam or even crystallization since its lowest attainable temperature is very low, usually on the order of milli-Kelvin in principle. Actually, the performance of laser cooling in a storage ring was proven experimentally in 1990s [1, 2]; an ultralow beam temperature of 1 mK was attained in the longitudinal direction. However, the effective transverse laser cooling of the ion beam for beam ordering or crystallization failed practically, while indirect laser cooling in the transverse directions was achieved to some extent through intra-beam scattering of an ion beam precooled by electron beams [3] and through dispersive coupling [4]. Theoretical studies have proven that the biggest reason of this failure is that the focusing lattice structures of the storage ring were not appropriate to generating a low-temperature space-charge-dominated beam [5].

It is now well-known that an operating point of a storage ring must be chosen properly for the formation of an ultralow-temperature beam including a crystalline beam [5]: The kinetic energy of the beam must be below transition energy of the ring and the average bare betatron phase advance must be less than 127 degrees per lattice period. The latter criterion "maintenance condition" is required to avoid the linear resonance at ultralow temperature. Practically, crossing linear resonances must be avoided in the cooling process of the beam: For this purpose, the betatron phase advance must be less than 90 degrees per period [6]. Note that, when the line density of the beam is very low, these criteria on betatron tunes may not always be met for the stability of ultralowtemperature beams [7].

A compact cooler storage ring S-LSR was newly built at Kyoto University in 2005 [8]. S-LSR can satisfy the above lattice conditions toward the realization of beam crystallization, and is equipped with a laser-cooling system of a <sup>24</sup>Mg<sup>+</sup> ion beam as well as with an electroncooling system. Several stable operating points can be chosen for efficient transverse cooling. Transverse laser cooling has been investigated experimentally to attain ultralow-temperature ion beams in the ring [9-13]. Recently, efficient transverse laser cooling has been accomplished for a low-current beam through the resonant coupling [14].

Molecular dynamics (MD) simulations have been performed in parallel with the experiments to clarify how the beam is laser-cooled at S-LSR. In these proceedings, MD simulation results are shown to reveal the characteristics of laser-cooled Mg beams in S-LSR. In addition, the feasibility of one-dimensional (1D) beam crystallization is explored.

# **MOLECULAR DYNAMICS SIMULATION** CODE

A dedicated MD simulation code CRYSTAL is employed for the present study [15-17]. The threedimensional (3D) equation of motion derived from the Hamiltonian in the beam rest frame is integrated in a symplectic algorithm in the code. The focusing effects from skew quadrupole and solenoid magnets as well as

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from regular dipole and quadrupole magnets and RF bunching cavities can be assumed precisely. The spacecharge force is fully incorporated based on the direct Coulomb interaction between ions in a bunch in this study.

The laser-cooling effect is introduced using the following dissipative force in the longitudinal direction:

$$F_{\pm} = \pm \frac{1}{2} \hbar k_L \Gamma \frac{S_L}{1 + S_L + \left(2\Delta_{\pm}/\Gamma\right)^2},\tag{1}$$

where  $\Gamma$  is the natural line width of the cooling transition,  $k_L$  is the wave number of the laser,  $S_L$  is the saturation parameter, and  $\Delta_{+(-)}$  is the detuning of the co-propagating (counter-propagating) laser frequency from the natural resonant frequency  $\omega_0$  of the ion. When the copropagating (counter-propagating) laser frequency in the laboratory frame is  $\omega_{+(-)}$ , we have the Doppler-shifted detuning  $\Delta_{\pm} = \omega_{\pm} \gamma \Big[ 1 \pm \beta \Big\{ 1 + (\delta p/p) / \gamma^2 \Big\} \Big] - \omega_0$ , where  $\beta$  and  $\gamma$  are Lorentz factors, and  $\delta p/p$  is the longitudinal momentum deviation of the ion. Assuming a Gaussian is given by laser. the saturation parameter  $S_L = S_0 \exp\left[-2\left(x^2 + y^2\right)/w^2\right]$ , where  $S_0$  corresponds to the peak saturation parameter on the axis of laser propagation, and w is the laser spot size that depends on the Rayleigh length.

#### SIMULATION CONDITIONS

An exact lattice structure of S-LSR has been assumed in the MD simulation so that the dynamic behaviour of the beam can be traced accurately. The lattice and beam parameters have been chosen as same as possible in the experiment. The schematic drawing of S-LSR is shown in Fig. 1.

The resonant coupling scheme is introduced for



Schematic drawing of S-LSR. Figure 1: The circumference of the ring is 22.56 m. Six bending dipole and 12 quadrupole magnets have been excited so that the six-fold lattice superperiodicity can be realized. Numbers (1 to 6) in the figure express the number of straight sections for convenience. A 40-keV  $^{24}Mg^{+}$  coasting ion beam from an ion source is injected into the ring at the first straight section and then bunched adiabatically by an RF cavity. A solenoid magnet is installed in the middle of the third straight section. A cooling laser co-propagating with an ion beam is injected along the fourth straight section.

efficient indirect cooling of the transverse direction [18]. For horizontal-longitudinal coupling, the beam is longitudinally bunched with the RF cavity placed at a dispersive position ( $D_r \sim 1$  m in the present case) in the ring. In addition, the difference resonance condition is imposed among horizontal  $v_x$ , vertical  $v_y$ , and longitudinal  $v_z$  tunes to enhance the coupling. In this study, the following two cases are investigated similarly to the experiment. Namely, one (Case-I) is  $(v_x, v_y, v_z) = (2.07, v_z)$ 1.12, 0.07), where only the horizontal-longitudinal coupling is actively introduced. The other (Case-II) is  $(v_x)$  $v_{\nu}$ ,  $v_z$ ) = (2.07, 1.07, 0.07), where 3D coupling is realized by weakly exciting the solenoid magnet for horizontalvertical coupling. In both cases, the maintenance condition has been met. The beam is bunched with a harmonic number of 100 and a sinusoidal voltage of about 40 V, which is adiabatically ramped in 0.2 s after the injection of a coasting beam from the ion source. The beam intensity reduction using a scraper, which has been performed experimentally in S-LSR [10-14], is ignored in the simulation for simplicity. The initial beam condition has been, therefore, chosen so that the beam parameters, such as the number of ions and emittance, can be equivalent with the experimental observation after beam scraping. The initial normalized root-mean-square (rms) emittance of the beam is horizontally  $2 \times 10^{-9}$  mm.rad and vertically  $4 \times 10^{-10}$  m.rad right after adiabatic capture. The rms momentum spread has been set at  $3 \times 10^{-4}$ .

Laser cooling is started after the RF adiabatic capture of the beam. A co-propagating laser with a wavelength of 280 nm is turned on at the third straight section. Similarly to the experiment, the total power of the laser has been set to be low (8 mW). However, a high peak saturation parameter of  $S_0 \sim 5$  has been achieved since the radius of the laser spot is as small as 0.33 mm. The frequency of the laser is not scanned but fixed at a given value during cooling (unless otherwise noted). Note that the filling factor, defined as a ratio of the length of the laser-cooling section to the ring circumference, is only 12%, which is smaller than those in the previous experiments [1-4].

#### SIMULATION RESULTS

### *Case-I (Longitudinal-Horizontal Coupling)*

The transverse real-space intensity distributions of the beam with an intensity of  $1.5 \times 10^4$  ions in the ring  $(1.5 \times 10^2 \text{ ions per bunch})$  are shown in Fig. 2 during laser cooling. The initial distribution after adiabatic capture (before cooling) is well Gaussian-like. In 40 ms after switching on the laser, a sharp peak in the distribution can appear in both horizontal and longitudinal directions, while nothing happens in the vertical direction. In 0.12 s, a sharp peak can be seen also in the vertical direction. This indicates that the vertical direction is cooled slowly through Coulomb interaction between ions after cooling in the longitudinal and horizontal directions since no artificial coupling source is applied in the vertical motion. The longitudinal distribution behaved similarly to the



Figure 2: Cooling-time dependence of the transverse beam intensity distribution during laser cooling. The distributions are monitored in the middle of the first straight section.

horizontal one except that the longitudinal beam size is about one order of magnitude larger than the horizontal one.

The distribution of the cooled beam in the equilibrium state is shown in Fig. 3(a) where no ordered configuration of the beam can be seen. Since the laser detuning is fixed, only a part of the beam can be three-dimensionally cooled by the laser. The cooling efficiency is about 70% during 3-s cooling in the present case. The normalized rms emittances of the cooled part of the beam are  $5 \times 10^{-11}$  $\pi$ m.rad in the horizontal direction and  $3 \times 10^{-11}$   $\pi$ m.rad in the vertical direction, which respectively correspond to the average temperature of 20 K and 4 K using the emittance-based temperature [19]. The rms beam radius is 0.2 mm in the equilibrium for both transverse directions. These simulation results are close to the experimental observations [12-14]. In the longitudinal direction, the rms momentum spread of the beam is  $2 \times 10^{-5}$ , corresponding to the longitudinal temperature of 0.5 K. In the smaller-detuning case of -41 MHz, which is approximately equal to the natural line width of the <sup>24</sup>Mg<sup>+</sup> ion's transition, the beam size has been shrunk further as shown in Fig. 3(b) and the beam emittance and momentum spread become about five times lower. The longitudinal centroid of the beam is positioned slightly forward because the beam is accelerated by the copropagating laser.

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Figure 3: Real-space distributions of the beams after 3-s laser cooling. The number of laser-cooled ions is  $1 \times 10^4$  in the ring  $(1 \times 10^2$  ions in a bunch). The laser detuning is fixed at (a) -200 MHz and (b) -41 MHz. The laser power is 8 mW.



Figure 4: Real-space distribution of the laser-cooled ultralow-current beam. Ten ions are laser-cooled among initial 15 ions. The laser detuning is -41 MHz.

The single-particle tunes of the beam in Fig. 3(a) are analyzed through Fourier transformation of betatron and synchrotron orbits of arbitrary ions in the cooled beam. The betatron tunes are shifted horizontally from 2.07 to 2.05, and vertically from 1.12 to 1.09, while the synchrotron oscillation is almost frozen out. The transverse tunes are changing rapidly since random intrabeam scattering is still dominant in the beam. For the beam in Fig.3 (b), the depressed betatron tunes are fixed at a nearest round number (2.0 horizontally and 1.0 vertically in the present case) [15, 16].

In order to reveal the feasibility of a low-dimensional crystalline state, laser cooling of a lower-current beam is investigated. Any ordered structure cannot be formed at the large detuning (-200 MHz). At a detuning smaller than -100 MHz, on the other hand, a string crystalline ordered state has been attained when the number of cooled ions is about 10 or less in a bunch, as demonstrated in Fig. 4. Note that indirect cooling in the vertical direction through the space-charge force cannot work any longer when the initial intensity of the injected beam is further reduced below  $10^3$  ions in the ring.



Figure 5: Real-space distribution of the beam cooled by two lasers. The number of cooled ions is  $9.3 \times 10^2$  ions in a bunch. The final detuning is -41 MHz.

## Case-II (3D Coupling)

The vertical tune has been changed to meet the resonant difference condition three-dimensionally. The solenoid magnet is also turned on at a weak field of 80 G so that the high lattice periodicity can be maintained.

In this case, the beam is cooled simultaneously in all three directions, unlikely Case-I (Fig. 2). The equilibrium transverse normalized rms emittance is  $4 \times 10^{-11} \pi$ m.rad and  $1 \times 10^{-11} \pi$ m.rad at a fixed detuning of -200 GHz, and -41 MHz, respectively. The temperature of the cooled beam for -41 MHz is 4 K, 2 K, and 22 mK in the horizontal, vertical, and longitudinal directions, respectively. The transverse temperature is approximately the same as the latest experimental result [13]. The real-space distributions in the equilibrium state are similar to Fig. 3. Similarly to Fig. 4, a 1D crystalline beam has been generated when the detuning is small and the beam intensity is sufficiently low.

## Ideal Case (3D Coupling & 2 Lasers)

It is worthy to demonstrate the performance of 3D laser cooling of a high-intensity beam under an ideal laser condition in S-LSR. Here, we consider two lasers with a sufficiently high power, i.e., one co-propagating laser in the fourth straight section and one counter-propagating laser in the fifth straight section. The total power of each laser is 100 mW, which corresponds to  $S_0=1.0$  with a  $1\sigma$ spot radius of 2.5 mm. The frequencies of the lasers are scanned from an initial detuning of -4.5 GHz to a final detuning of -41 MHz in 1 sec. The tunes in Case-II are chosen for 3D cooling. Figure 5 shows the equilibrium spatial distribution of a high-current beam. More than 90% among injected ions of  $1 \times 10^5$  in the ring are lasercooled. A highly space-charge-dominated bunched beam has been obtained that has almost uniform and parabolic distributions in the transverse and longitudinal directions, respectively. The normalized transverse rms emittance of the beam is  $5 \times 10^{-11}$  mm.rad. The longitudinal momentum spread of the beam is  $1 \times 10^{-5}$ . Such a high-quality intense beam has been not ever achieved in heavy-ion storage rings. Even in such an ideal case, 3D ordered configuration cannot be seen because the laser cooling force is not tapered, which is required for the formation of 3D crystals [5].

#### SUMMARY

We numerically explored the feasibility of generating an ultralow-temperature beam and of beam crystallization considering actual experimental parameters at S-LSR. Systematic MD simulations clarified that, even under the limited experimental conditions, such as low laser power, (large) fixed detuning, and short cooling section, the ultralow-temperature three-dimensionally heavy-ion beams could be formed using resonant coupling. The simulation results were not inconsistent with the observation results in the experiment at S-LSR. When the beam intensity was sufficiently low, 1D crystalline beams could be formed at small detuning. The present study strongly suggests that a heavy-ion beam with the highest quality will be realized at S-LSR.

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