

# Fundamentals of Logarithmic Amplifiers and Recent Advances

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ANALOG  
DEVICES

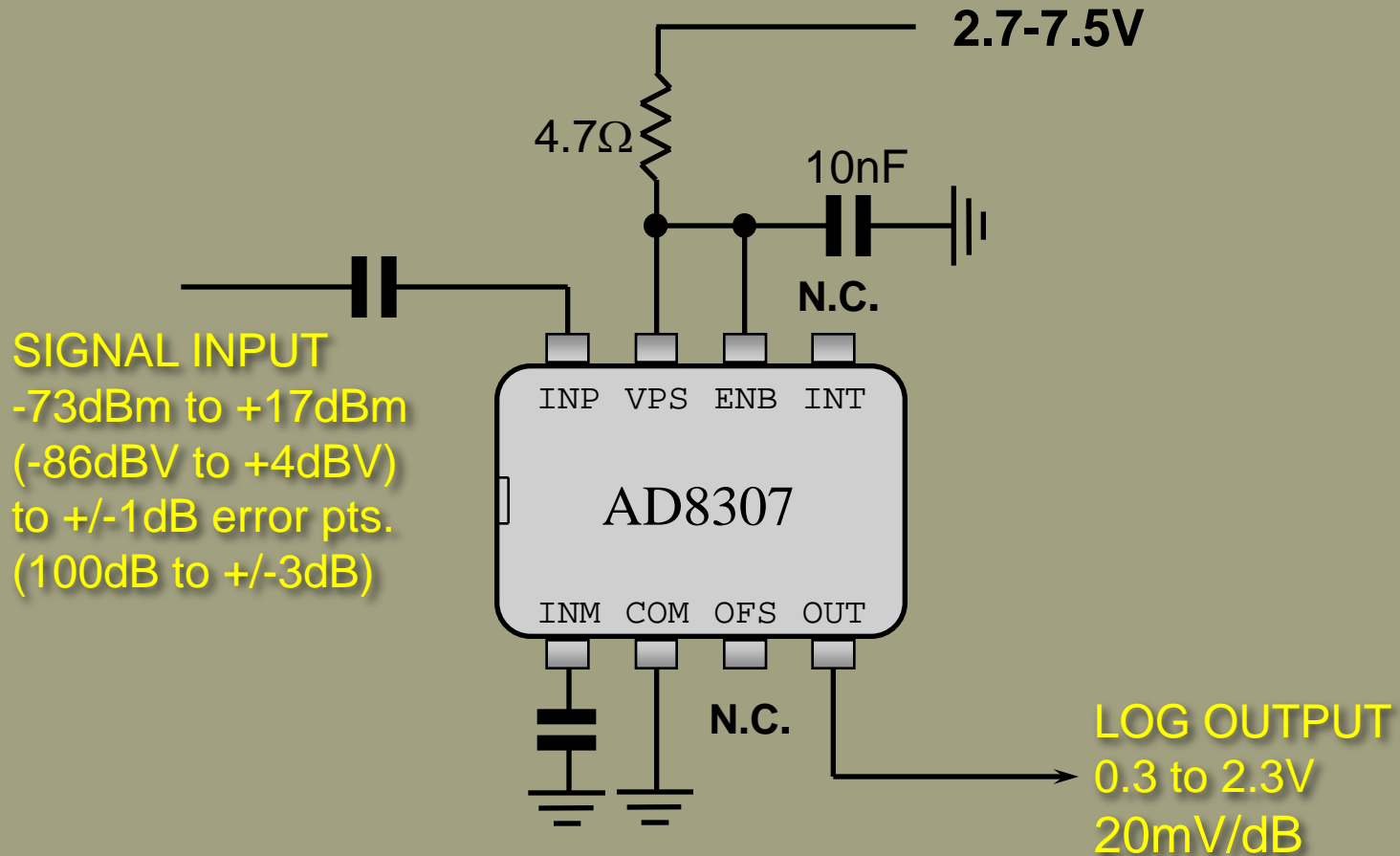
# LOGARITHMIC AMPLIFIERS



- Unique Nonlinear Function
- Integrated Multistage Systems
- Calibrated Slope and Intercept
- Provide Complete Solutions
  - *easy to use*
- Up to 100 dB Dynamic Range
- Now covering DC - 30 GHz
- Limiter Versions for PSK, FSK
- Low Cost, Small Packages
- Numerous types available

AD606	AD8313
AD607	AD8314
AD608	AD8315
AD640	AD8316
AD641	AD8317
AD8302	AD8318
AD8306	AD8319
AD8307	AD8362
AD8309	AD8363
AD8310	AD8364
AD8311	more.....

# A Personal Goal: Make Log Amps as Cheap, and Easy to Use, as Op Amps



# SO HERE'S THE PLAN....

- BEGIN WITH A BRIEF OVERVIEW OF THE VARIOUS TYPES, TO SET THE STAGE
- MOVE ON AS QUICKLY AS POSSIBLE TO PROGRESSIVE COMPRESSING TYPES as likely to be of most value in beam instr.
- DEVELOP THEIR FUNDAMENTAL THEORY starting from the most basic of foundations
- SHOW SOME PRACTICAL EMBODIMENTS

# WHAT DO LOG AMPS DO?

- Convert signals of high dynamic range (HDR) to a substantially smaller dynamic range
- The output is readily scaled to represent the *decibel value* of the input, in simple units
- This is a *fundamental nonlinear conversion* of the signal representation - with important consequences
- Some types may be used to simply *compress* a HDR signal, thereby achieving high observational or measurement-sensitivity near a nominal null

# FUNDAMENTAL FUNCTION

$$W = Y \log \frac{X}{Z}$$

*where*

$W$	<i>is the</i>	Output variable
$X$	.....	Input variable
$Y$	.....	Slope parameter
$Z$	.....	Log Intercept

# FUNDAMENTAL FUNCTION

$$V_W = V_Y \log \frac{V_X}{V_Z}$$

*where*

$V_W$  *is the* Output voltage

$V_X$  . . . . . Input voltage

$V_Y$  . . . . . Slope voltage

$V_Z$  . . . . . Intercept voltage

# FUNDAMENTAL FUNCTION

$$I_W = I_Y \log \frac{I_X}{I_Z}$$

*where*

$I_W$	<i>is the</i>	Output current
$I_X$	<i>.....</i>	Input current
$I_Y$	<i>.....</i>	Slope current
$I_Z$	<i>.....</i>	Intercept current

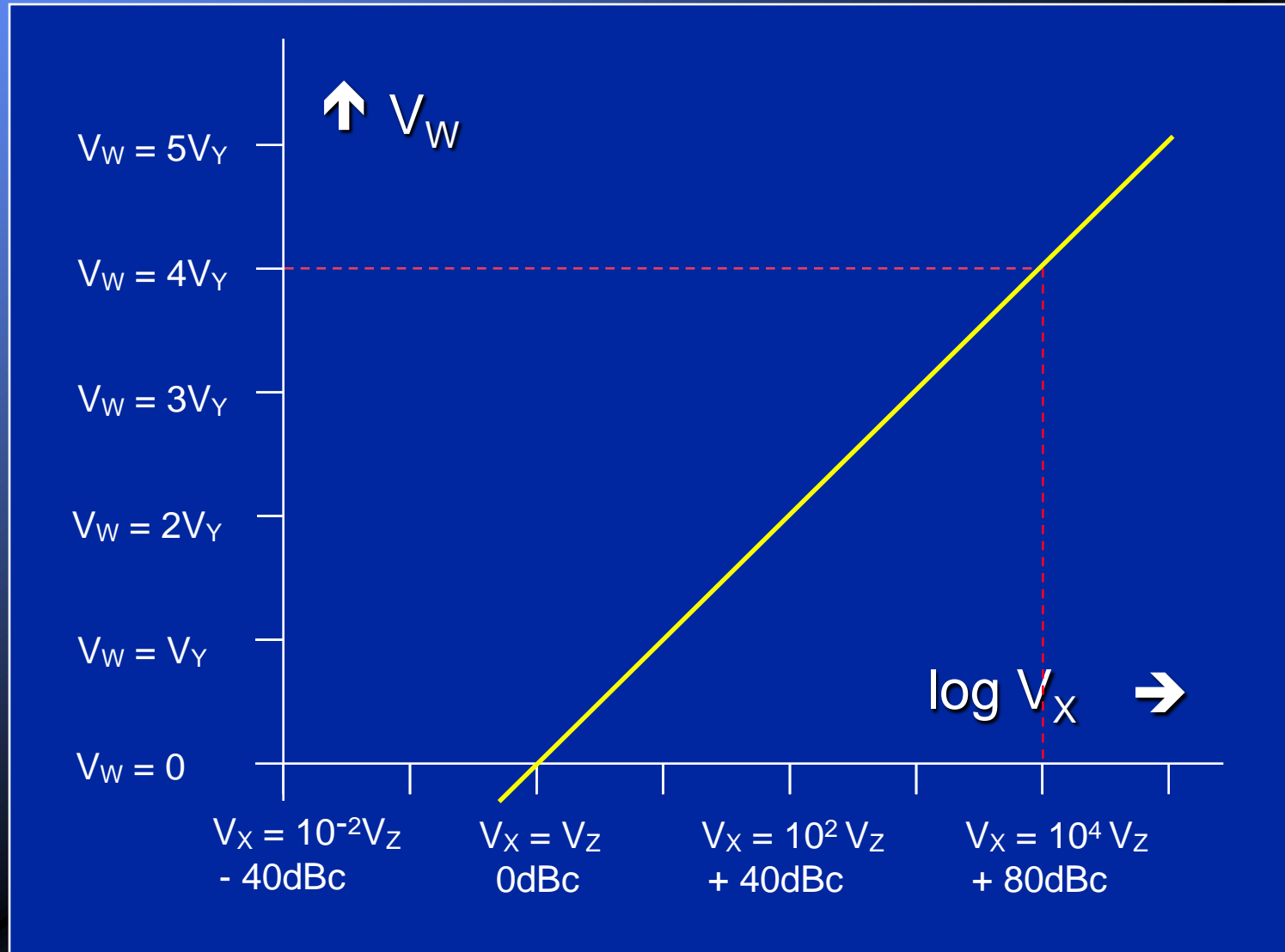
# FUNDAMENTAL FUNCTION

$$V_W = V_Y \log \frac{I_X}{I_Z}$$

*where*

$V_W$	<i>is the</i>	Output voltage
$I_X$	.....	Input current
$V_Y$	.....	Slope voltage
$I_Z$	.....	Intercept current

# THE BASIC LOGARITHMIC RELATIONSHIP



# REGION NEAR ZERO

$$\begin{aligned}\frac{\partial V_W}{\partial V_X} &= \frac{\partial}{\partial V_X} V_Y ( \log V_X + \log V_Z ) \\ &= \frac{V_Y}{V_X}\end{aligned}$$

THE INCREMENTAL GAIN OF A LOG-AMP  
SHOULD APPROACH INFINITY AS  $V_X \rightarrow 0$

# WHAT HAPPENS WHEN $V_X < 0$ ?

Formally,  $\log (-X)$  IS COMPLEX

One might consider using

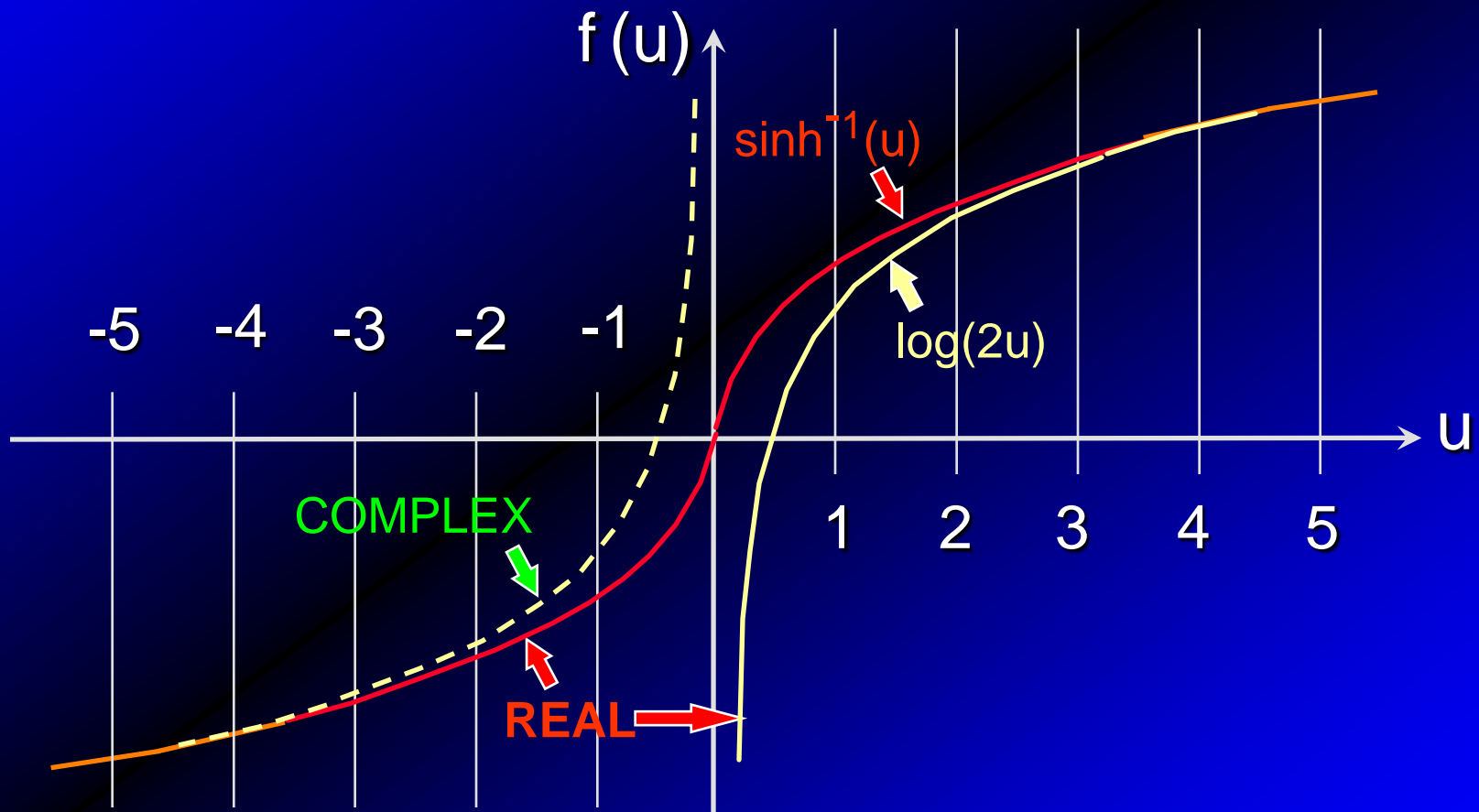
$$V_W = \text{sgn}(V_X) V_Y \log(|V_X| / V_Z)$$

The inverse hyperbolic sine is useful here:

$$\begin{aligned} \sinh^{-1}(u) &= \log \{ u + \sqrt{u^2 + 1} \} \\ &\rightarrow \log(2u) \quad \text{for } u > \text{than } \sim 3 \end{aligned}$$

Of course,  $\sinh^{-1}(u) = -\sinh^{-1}(-u)$

# $\sinh^{-1}(u)$ & $\log(2u)$



# PRACTICAL LOG AMP TYPES

- TRANSLINEAR (inc. DC/LF log-ratio)
- EXPONENTIAL-AGC
- PROGRESSIVE COMPRESSION
  - Baseband
  - Demodulating
  - “True-log” ( $\sinh^{-1}$ )
  - Synchronous
  - Dual (RF log-ratio)

# DIRECT TRANSLINEAR

- BASED ON THE LOG-EXPONENTIAL PROPERTIES OF THE BJT
- CURRENT-MODE DYNAMIC RANGE CAN BE VERY HIGH (e.g., 200dB, 1pA to 1mA)
- INVALUABLE IN MANY TYPES OF LF INSTRUMENTATION (DC to about 10MHz)
- THE MOST ADVANCED FORMS ACTUALLY PROVIDE FULL LOG-RATIO OPERATION

# THE TOUCHSTONE:

$V_{BE}$

A widely used formulation is

$$V_{BE}(T, I_C) = \frac{kT}{q} \log \frac{I_C}{I_S(T)}$$

.. which is very nearly exact in most respects, over a temperature range of -250°C to 250°C and an  $I_C$  range from (typically) 1pA to 1mA; at high currents the junction resistances and other effects cause  $V_{BE}$  to exceed this value. But what is this peculiar quantity  $I_S(T)$ ?



# The Saturation Current $I_S(T)$

If the wondrous  $V_{BE}$  can be called the heart of a bipolar transistor,  $I_S(T)$  must surely be its soul! Although very tiny, it is also a marvel to behold, and derives from the expression for the intrinsic carrier concentration – the number of free holes and electrons per unit volume generated by the *thermal energy* in an unbiased semiconductor:

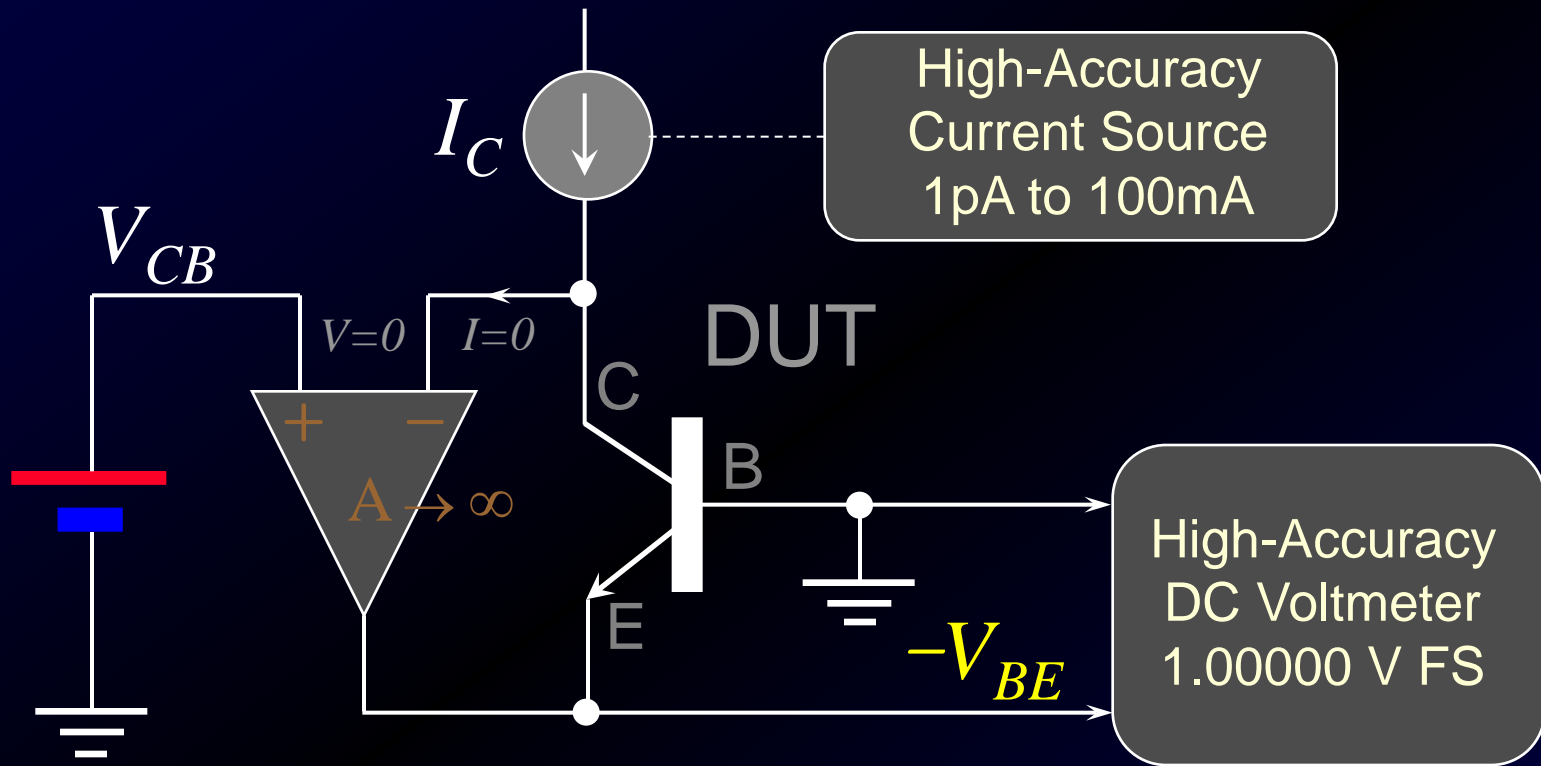
$$n_i^2(T) = 32\left(\pi \frac{k}{\hbar} m_e m_h\right)^3 T^3 \exp \frac{\alpha}{k} \exp \frac{-E_{GO}}{kT}$$

# The Saturation Current $I_S(T)$

In practice  $I_S(T)$  cannot be accurately calculated -or even measured directly - for use in the basic expression  $V_{BE} = (kT/q) \log I_C / I_S(T)$ .

Instead, the  $V_{BE}$  of a representative transistor is measured at a known temperature and current; then a different formulation for  $V_{BE}$  is used for calculating its value at other operating points.

# Measurement of $V_{BE}(T, I_C)$

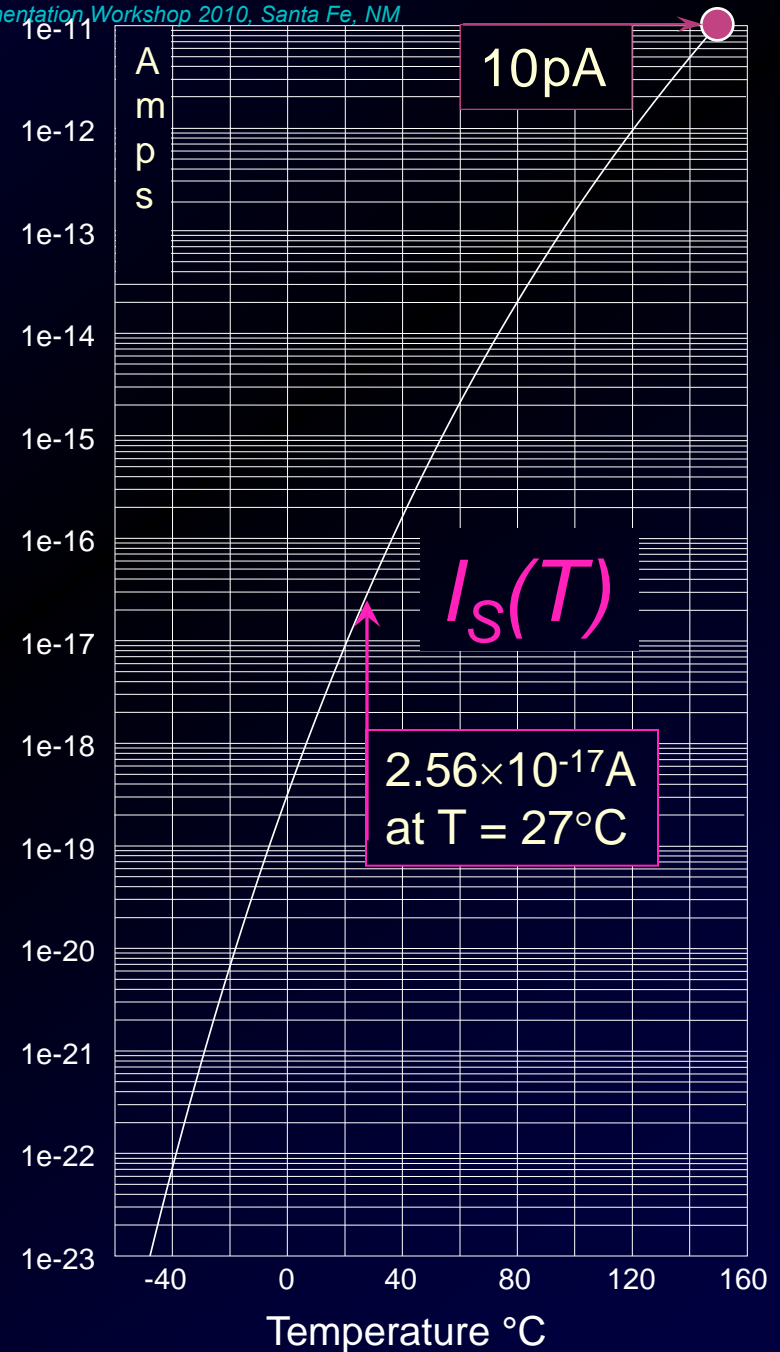


In BJT modeling, the default collector bias is  $V_{CB} = 0$ , that is,  $V_{CE} = V_{BE}$ . Collector current  $I_C$  is forced by an electrometer-grade op amp

The form

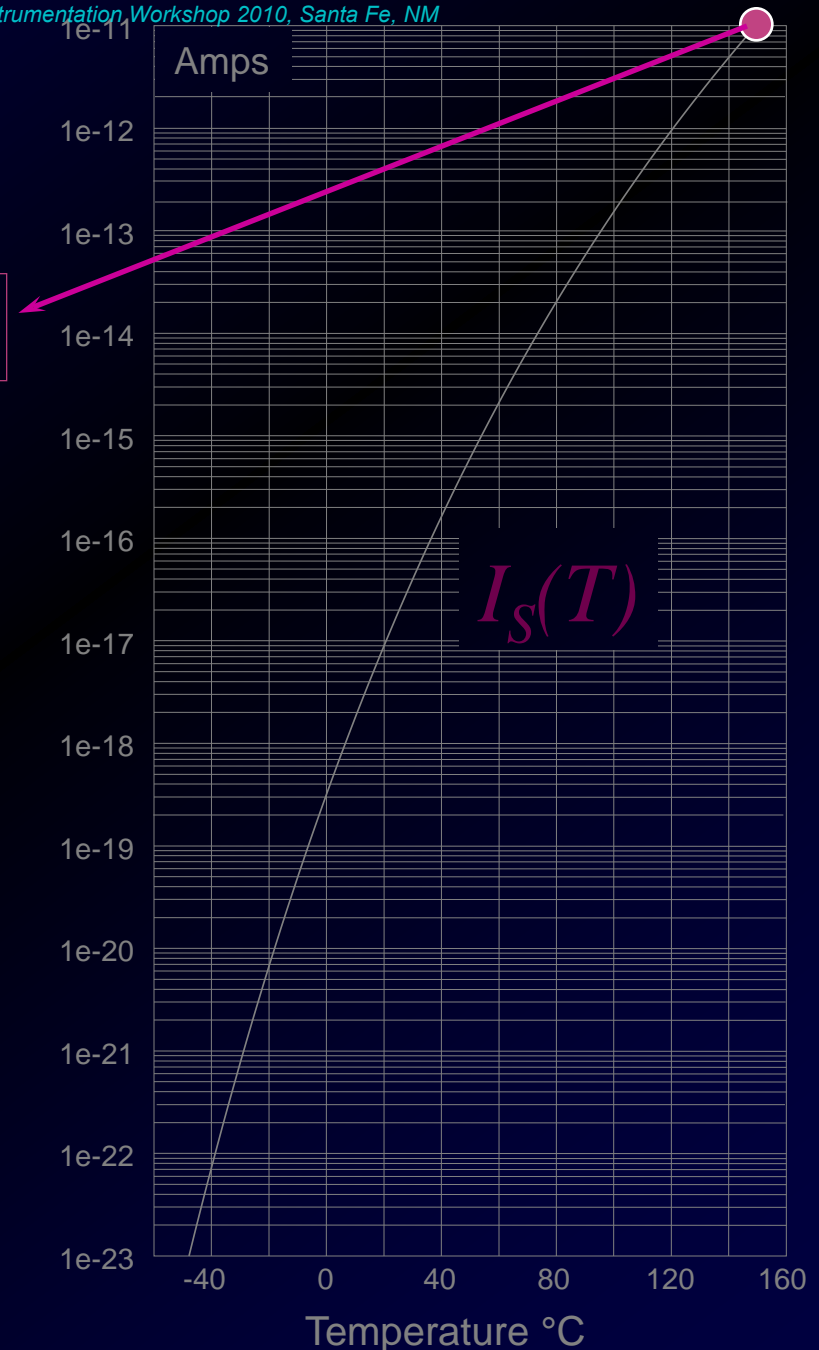
$$V_{BE}(T, I_C) = \frac{kT}{q} \log \frac{I_C}{I_S(T)}$$

might suggest that  $V_{BE}$  has a strong positive coefficient of temperature - because of the factor  $kT/q$ . However, it is the saturation current  $I_S(T)$  which dominates, climbing by a ratio of about a trillion from  $-50^\circ\text{C}$  to  $+150^\circ\text{C}$ . Graph is for a  $V_{BE}$  of  $0.75\text{V}$  at  $I_C = 100\mu\text{A}$  &  $300\text{K}$

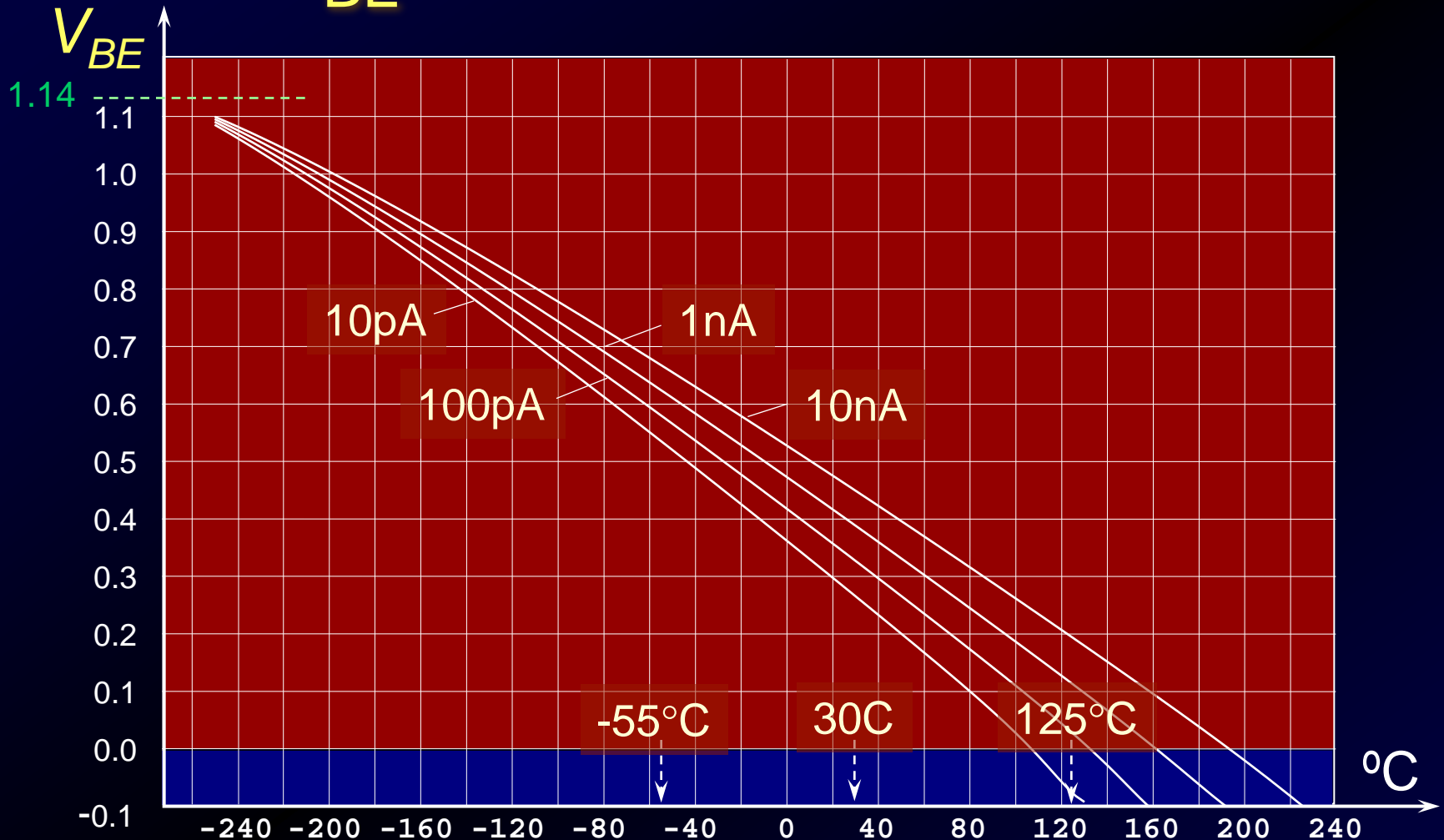


$$V_{BE}(T, I_C) = \frac{kT}{q} \log \frac{10\text{pA}}{10\text{pA}}$$

also suggests that the  $V_{BE}$  of this transistor would become ZERO at 150°C for  $I_C = 10\text{pA}$  .... which indeed really does happen, because all of  $I_C$  is then supplied entirely by  $I_S$ ! In fact, at high temperatures and picoamp current levels,  $V_{BE}$  may become negative !!



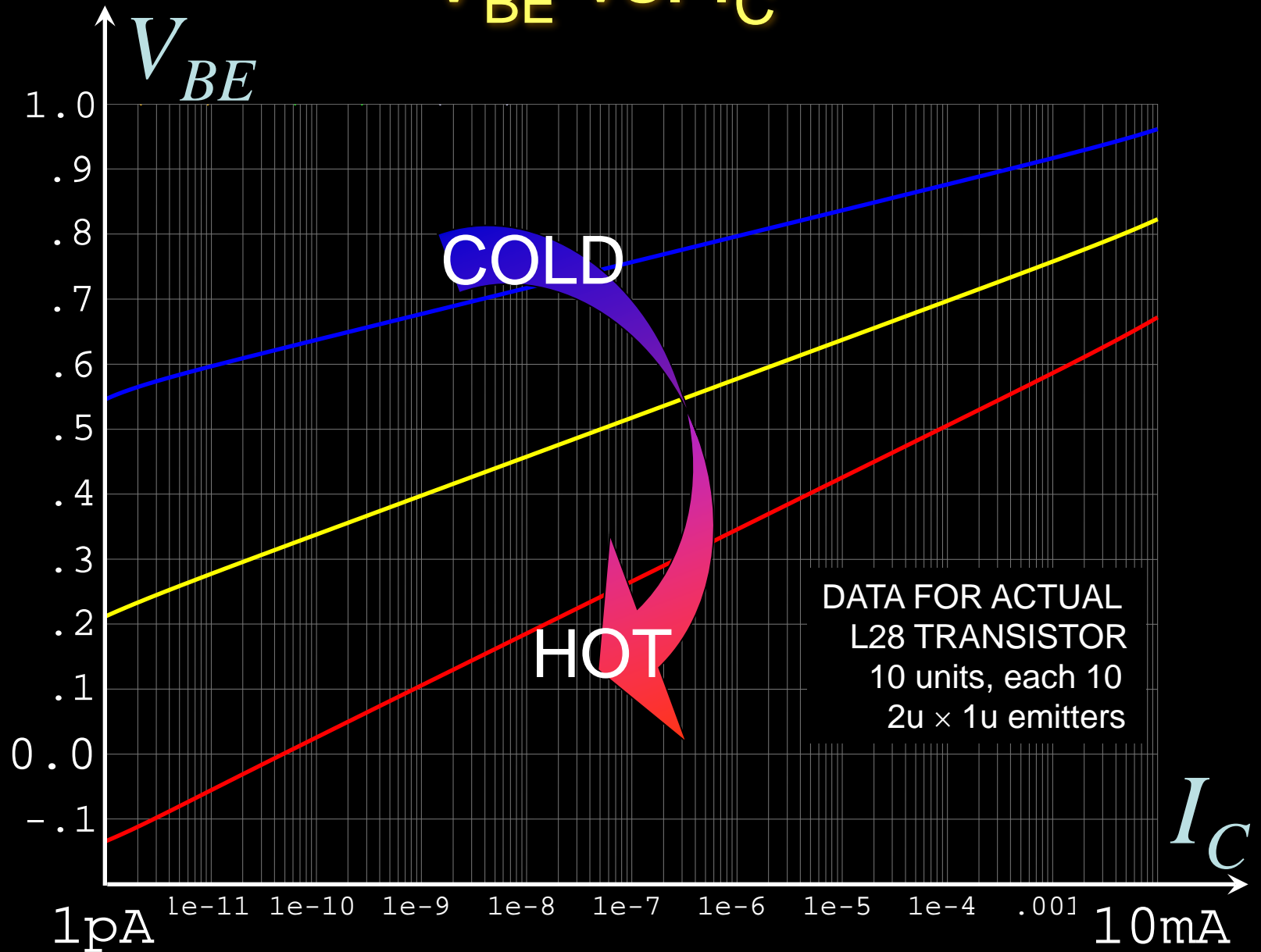
# $V_{BE}$ vs. TEMPERATURE

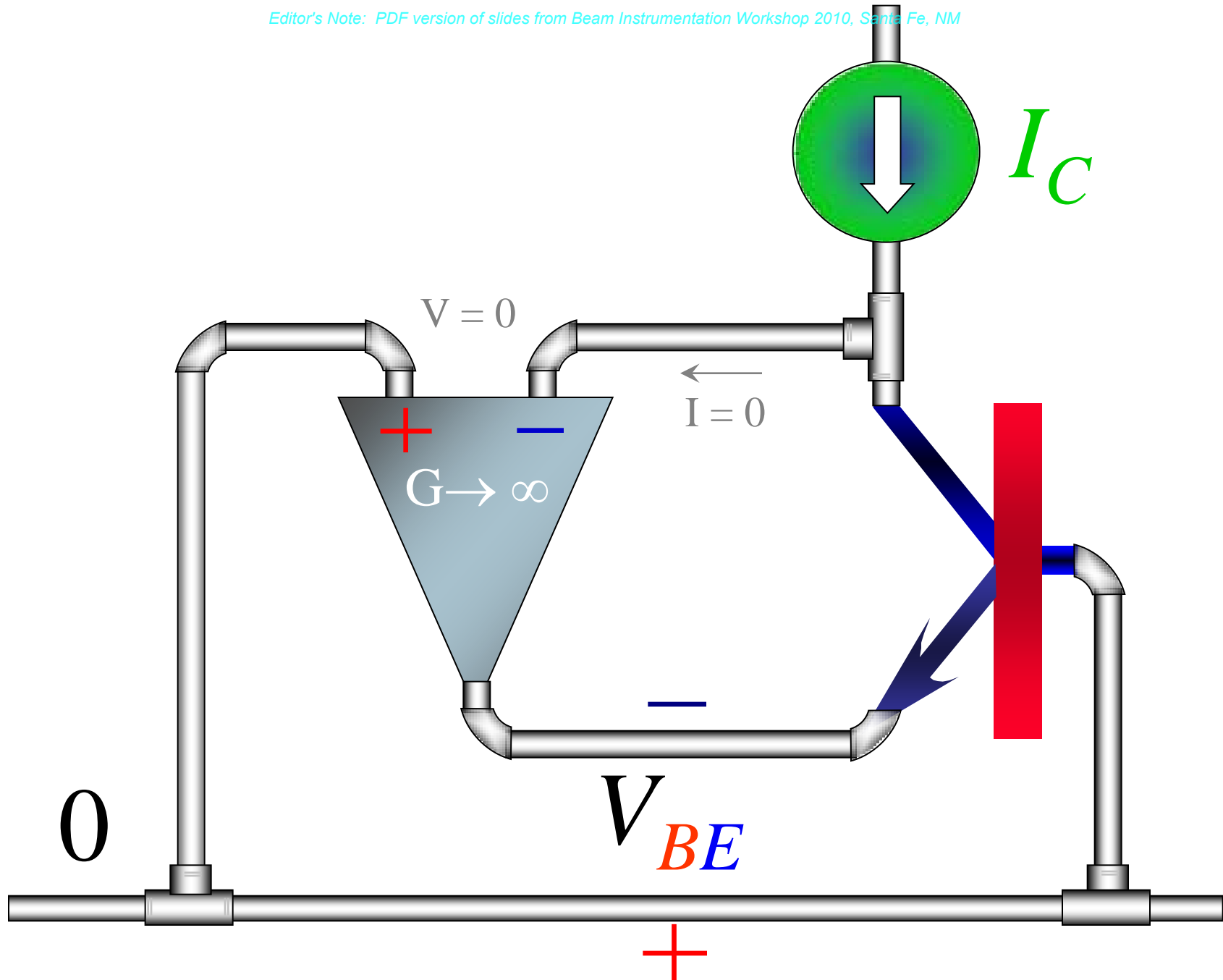


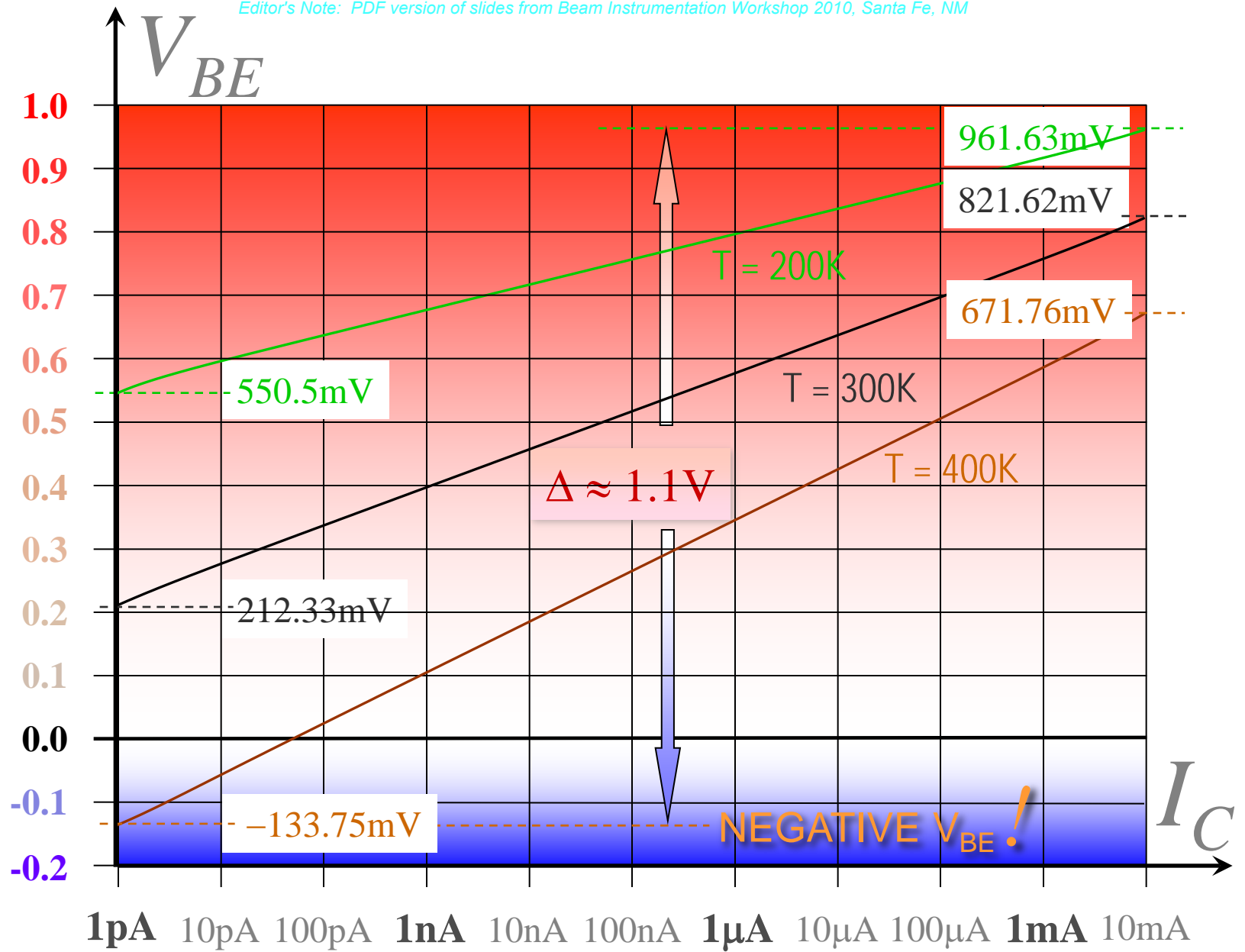
Actual PNP transistor having  $LE=100\mu\text{m}$ ,  $WE=2\mu\text{m}$ ,  $EG=1.13$ ,  $XTI=4.03$ ,  $IS = 2.644\text{e-}16$

$I_C = 10\text{pA}$ ,  $100\text{pA}$ ,  $1\text{nA}$  and  $10\text{nA}$ . To optimally illustrate the effect,  $V_{BC}$  was adjusted to 52mV (10pA), 83mV (100pA), 110mV (1nA) and 150mV (10nA) - all values ZTAT

# $V_{BE}$ vs. $I_C$







## ELIMINATING THE VERTICAL SHIFT

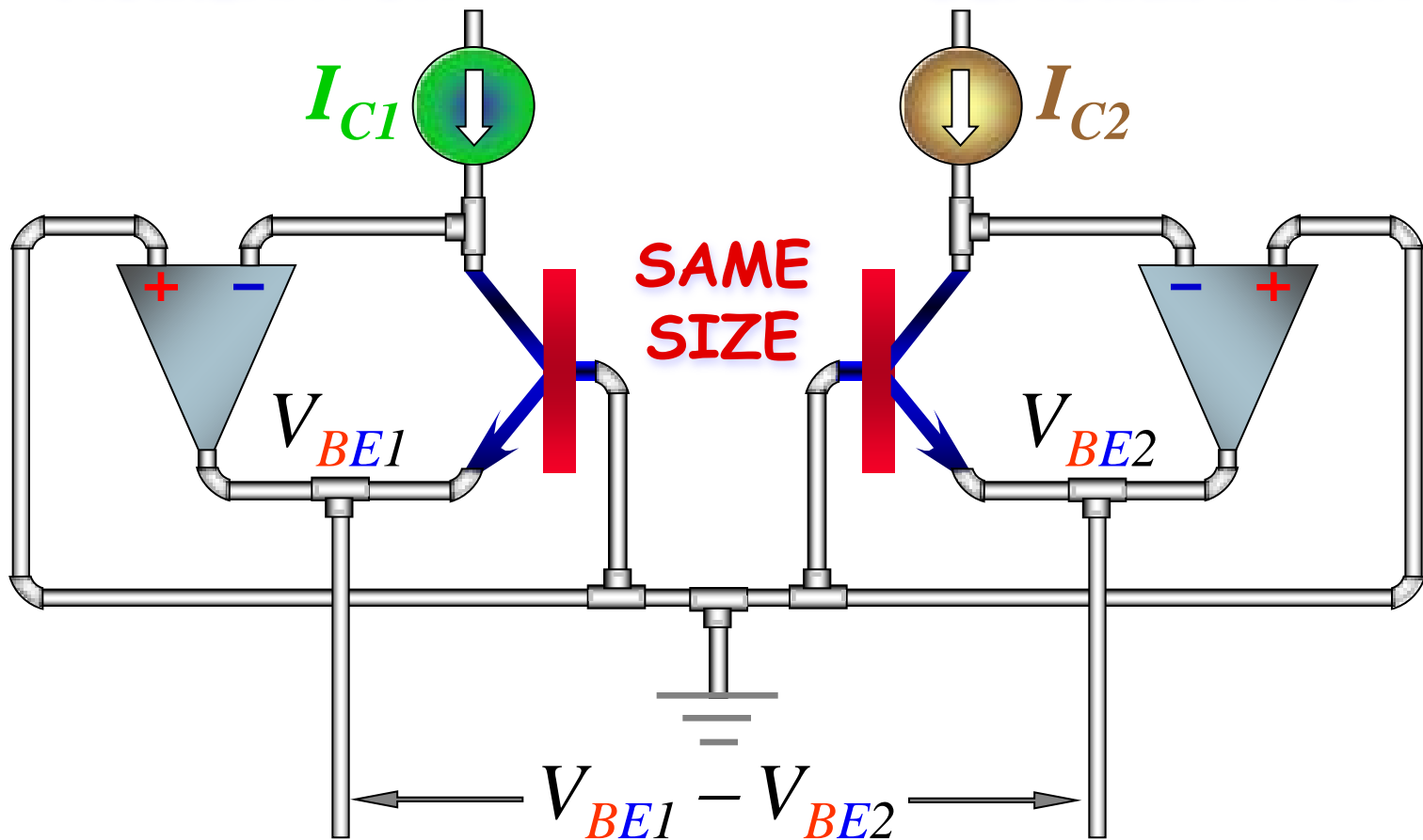
The vertical shift in  $V_{BE}(T)$  is due to the extreme change in  $I_S(T)$  over temperature (the ratio can be as high as a trillion-to-one from  $-55^{\circ}\text{C}$  to  $+125^{\circ}\text{C}$ ).

This shift is readily eliminated by using a second transistor, operating at a fixed reference current; this current and the size of this device determines the raw intercept of the logarithmic conversion.

# NUMERATOR

Editor's Note: PDF version of slides from Beam Instrumentation Workshop 2010, Santa Fe, NM

# DENOMINATOR



$$\begin{aligned}
 V_{BE1} - V_{BE2} &= V_K \log I_{C1} / I_S(T) - V_K \log I_{C2} / I_S(T) \\
 &= \textcircled{V_K} \log I_{C1} / I_{C2}
 \end{aligned}$$

## ELIMINATING THE CHANGE IN SLOPE

The variation in the slope of  $V_{BE}(T)$  is due simply to the inherent PTAT scaling factor of  $V_K = kT/q$ :

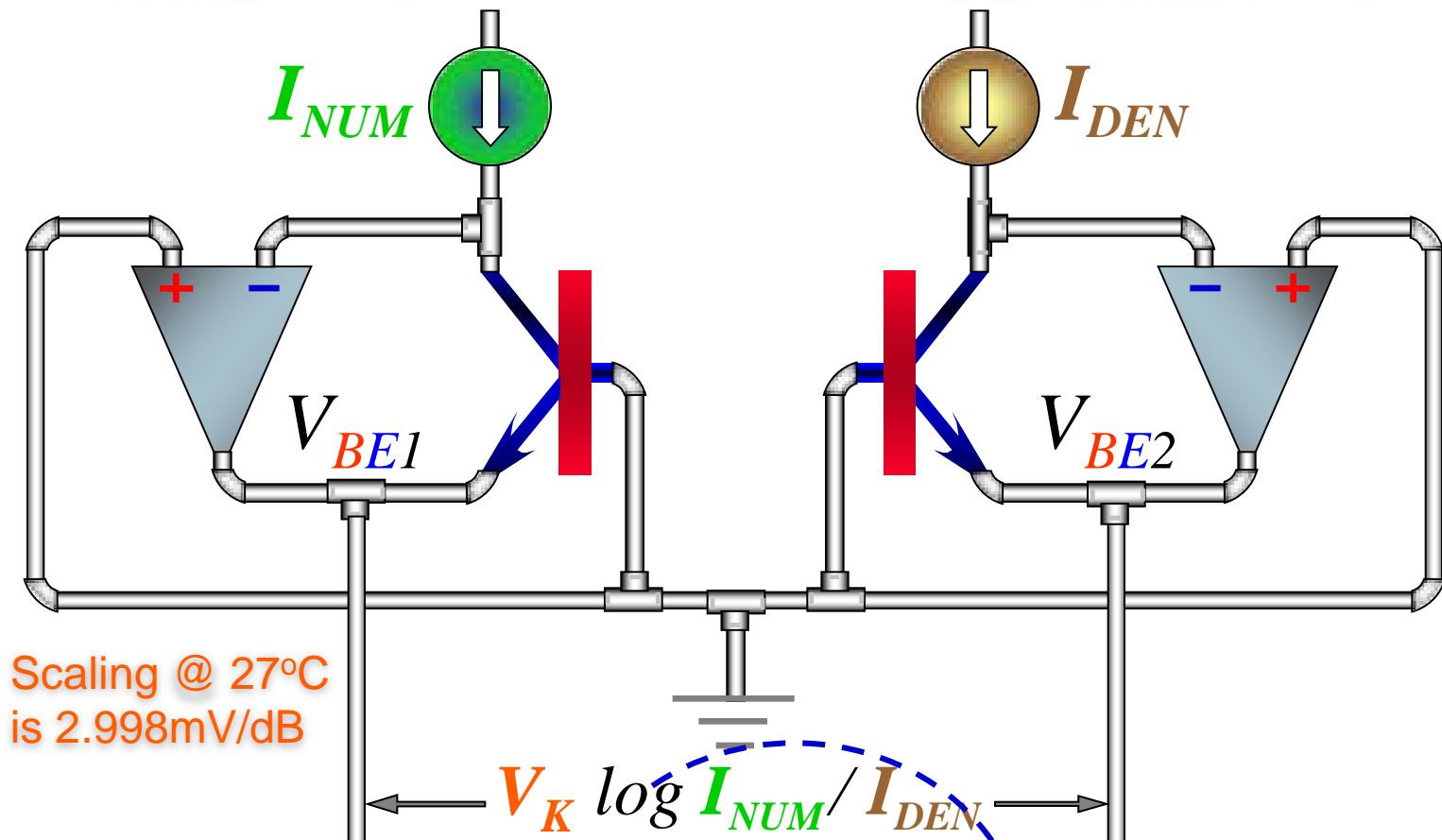
$$\Delta V_{BE} = V_K \log I_{C1}/I_{C2}$$

We need to convert this factor from PTAT to ZTAT, that is, a slope that is invariant with temperature, re-evaluate it, now using the decibel basis of  $\log_{10}$  and rename the two currents:

$$V_{LOG} = V_Y \log_{10} I_{NUM}/I_{DEN}$$

# NUMERATOR

# DENOMINATOR



**Ergo: This bit is very important**  
(PTAT-to-ZTAT CONVERTER)

Scaling here is  
10mV/dB at all  
temperatures



$V_Y \log_{10} \frac{I_{NUM}}{I_{DEN}}$

FLT1

PDBS

INUM

VNUM

FLT2

1  $\mu$ A SRC

PDBS

LOG  
Q'S

JFET  
OP AMP

SUMP  
PUMP

2.8V LDO

HYPERTANH

V-MIRROR

OUTPUT AMP

REFERENCES

VOUT

SCL1

SCL2

SCL3

SCL4

VREF

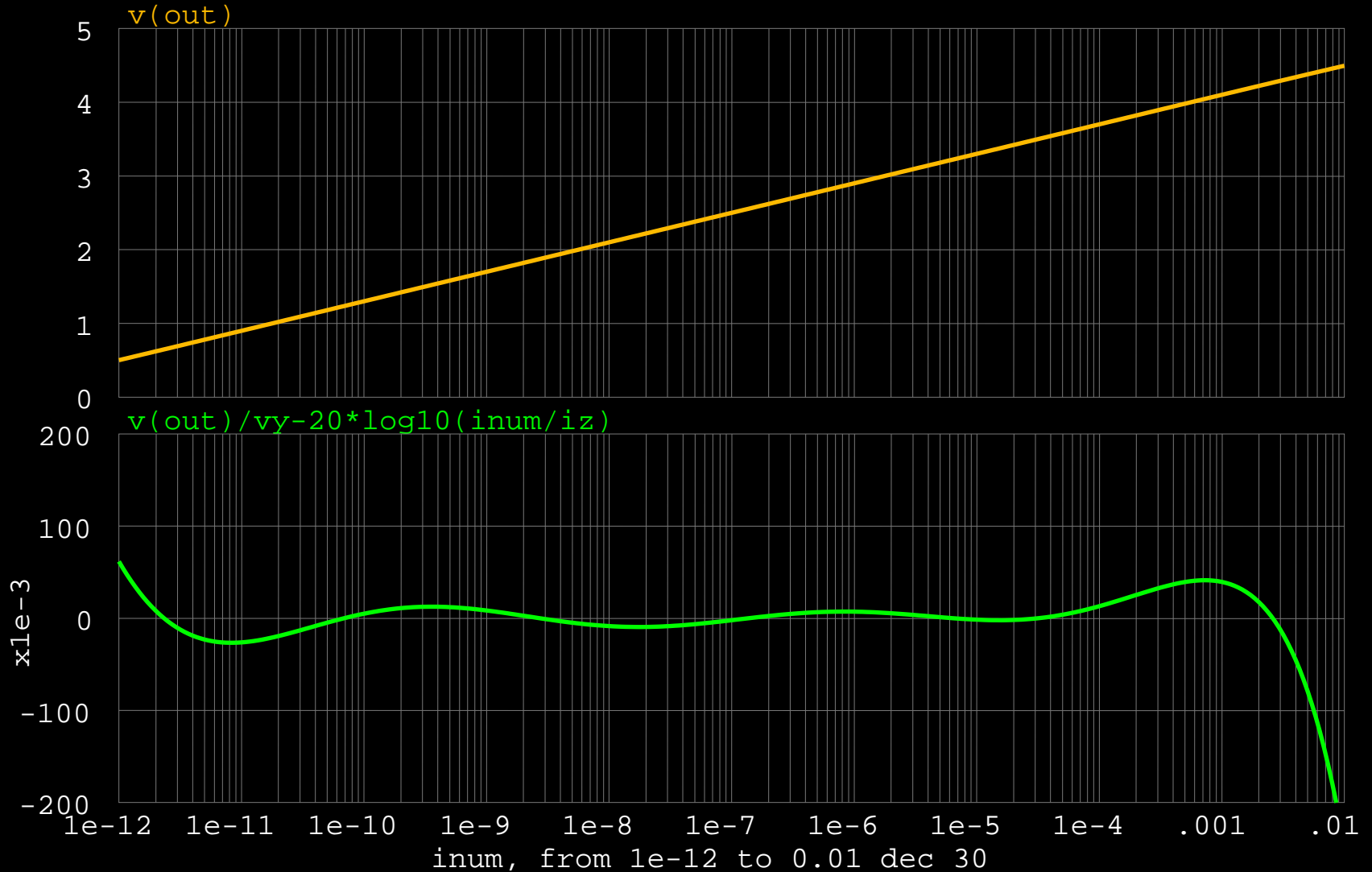
COMM

ACOM

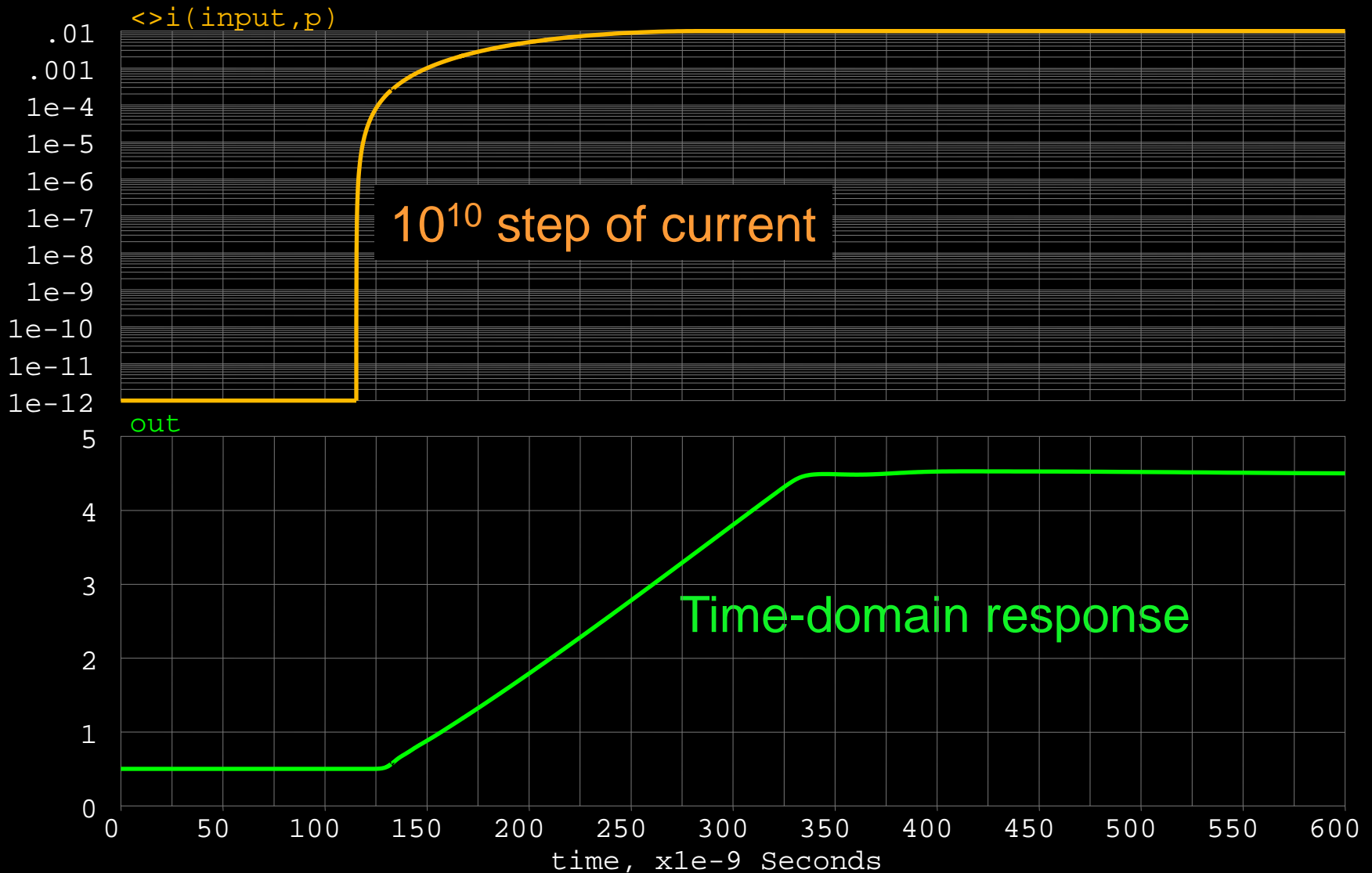
SHF1

SHF2

A 200-dB sweep of DC input current, from 1pA to 10mA, showing the scaling of 20mV/dB and very low log-conformance error ( $\pm 0.05\text{dB}$ )

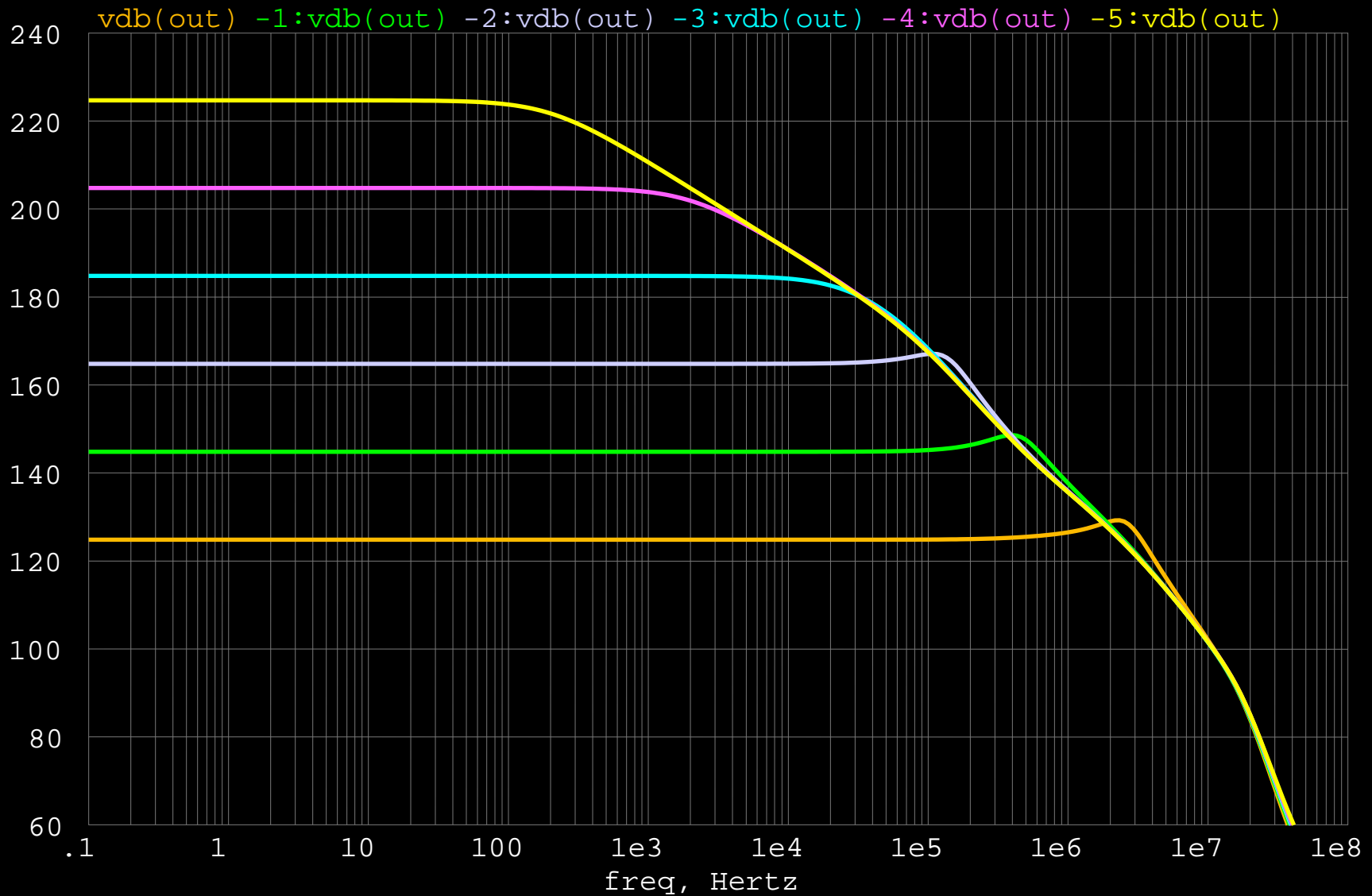


A 200-dB step of input current, from 1pA to 10mA, with a sine-squared rise-time of 100ns, is accommodated within a slewing time of 200ns



AC response for DC inputs of 1pA, 10pA, 100pA, 1nA, 10n and 100nA

Note the LF gain at 1pA is 225dB, corresponding to  $R_T = 178 \text{ G-ohm}$

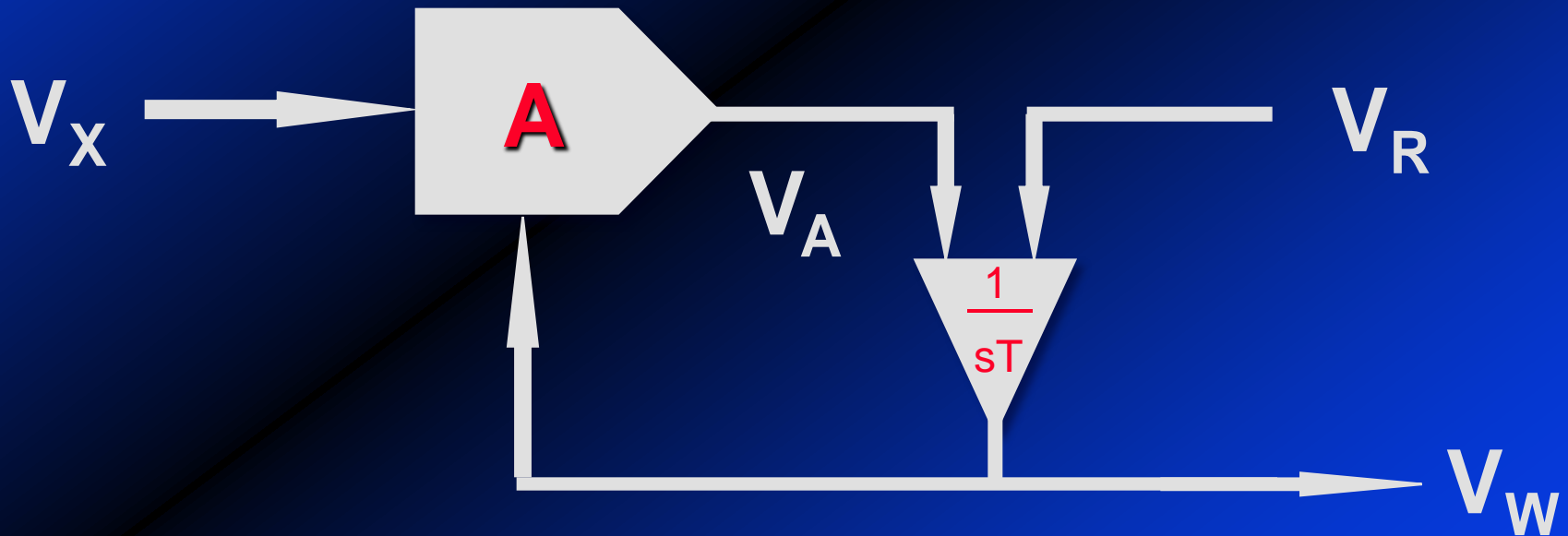


# EXPONENTIAL AGC

- A VARIABLE-GAIN AMPLIFIER HAVING INVERSE EXPONENTIAL CONTROL OF GAIN ( $e^{-x}$ , that is, 'Linear-in-dB') .....
- .... DRIVES A DETECTOR CELL TO A FIXED SET-POINT AT ITS OUTPUT
- THIS SCHEME CAN OPERATE OVER A LARGE DYNAMIC RANGE (noise-limited)...
- ... AND IS CAPABLE OF PROVIDING AN EXACT RMS (TRUE POWER) RESPONSE

# LOG-AMP BASED ON 'EXPONENTIAL AGC'

$$A = A_0 \exp(-V_w/V_Y)$$



# LOG-AMP BASED ON 'EXPONENTIAL AGC'

The amplifier output settles to equal  $V_R$  by the action of the loop: the mean input to the integrator must be forced to zero. Thus

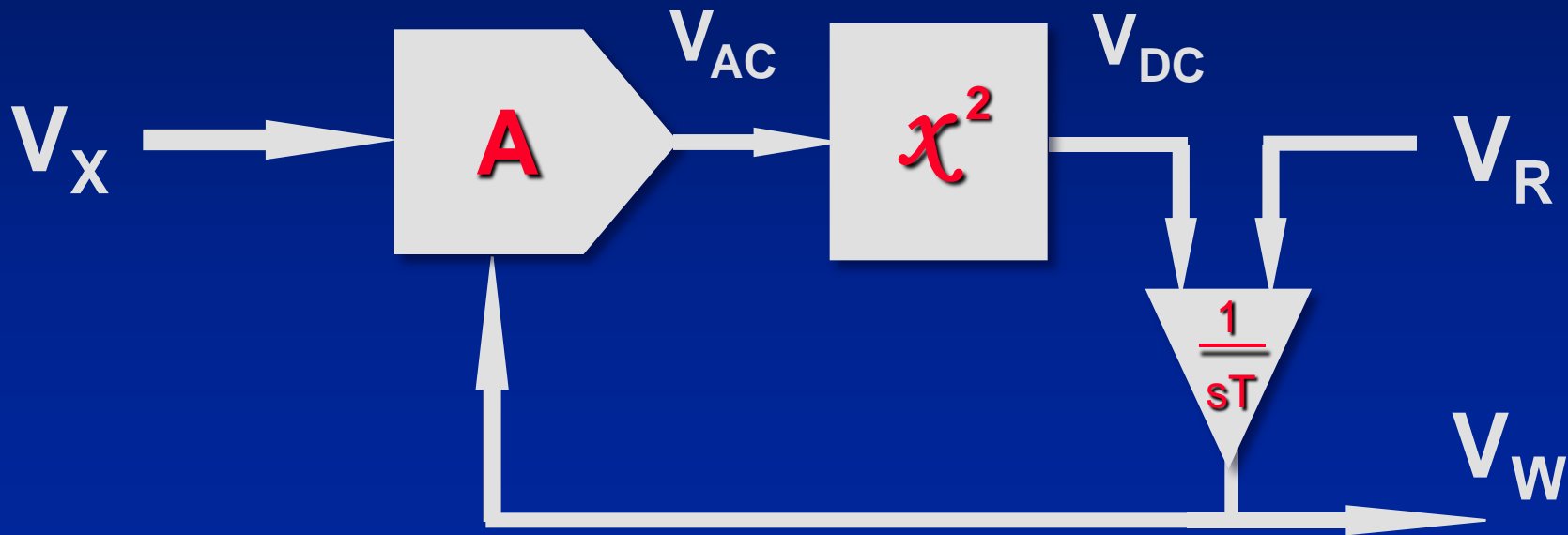
$$V_A = V_X A_0 \exp(-V_W/V_Y) \rightarrow V_R$$

Solving for  $V_W$ :

$$V_W = V_Y \log(V_X/V_Z) \quad V_Z = V_R/A_0$$

# LOG-AMP BASED ON 'EXPONENTIAL AGC'

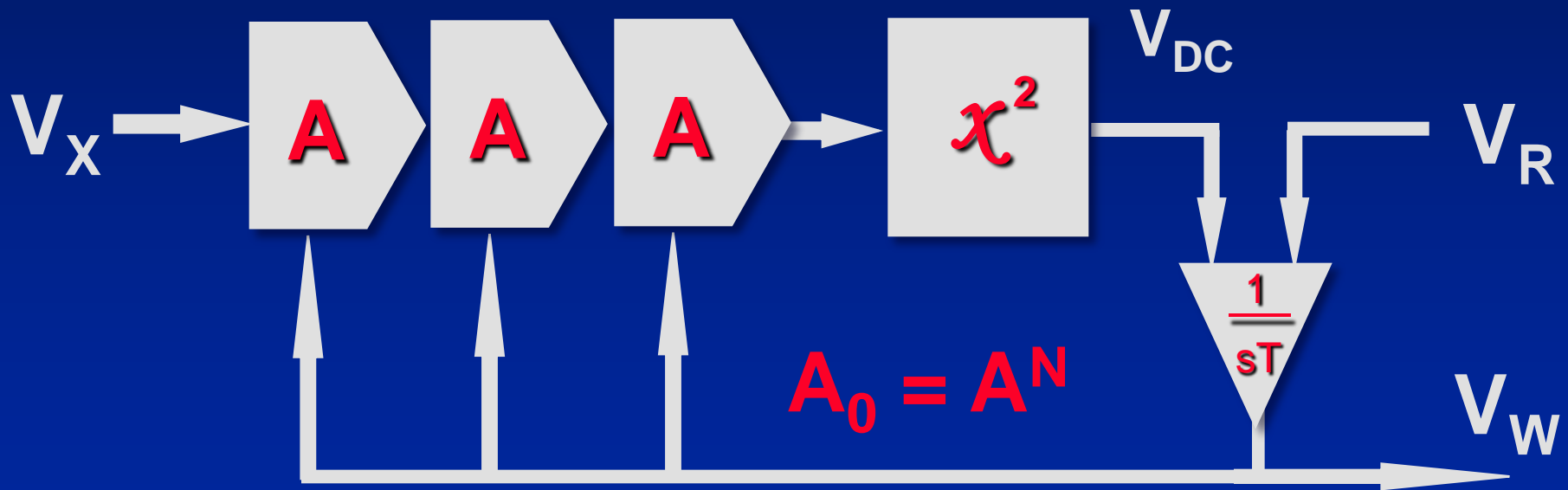
For AC signals, add a detector:



# LOG-AMP BASED ON 'EXPONENTIAL AGC'

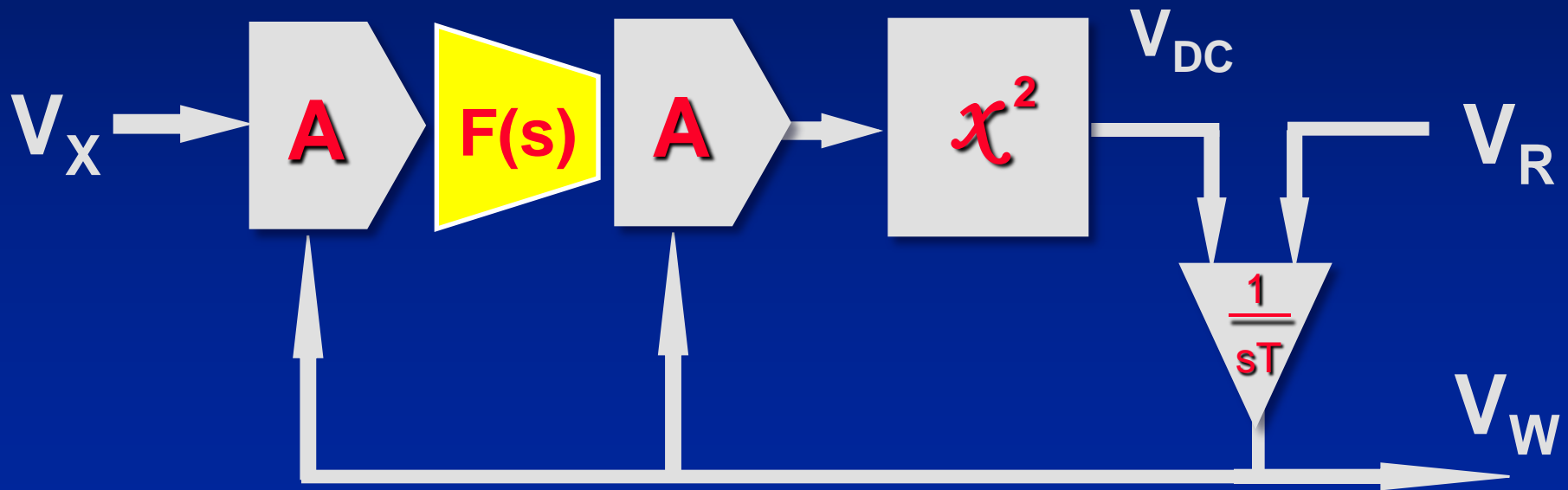
For higher dynamic range and higher bandwidth, cascade several stages:

N stages



# LOG-AMP BASED ON 'EXPONENTIAL AGC'

For even higher dynamic range, add filter(s):



120dB of range is readily obtainable

# The X-AMP



A PROPRIETARY VGA PRINCIPLE



FUNDAMENTALLY “LINEAR-in-dB”



USES FEEDBACK IN ORDER TO:  
*ACCURATELY DETERMINE GAIN  
& MINIMIZE HF NONLINEARITIES*

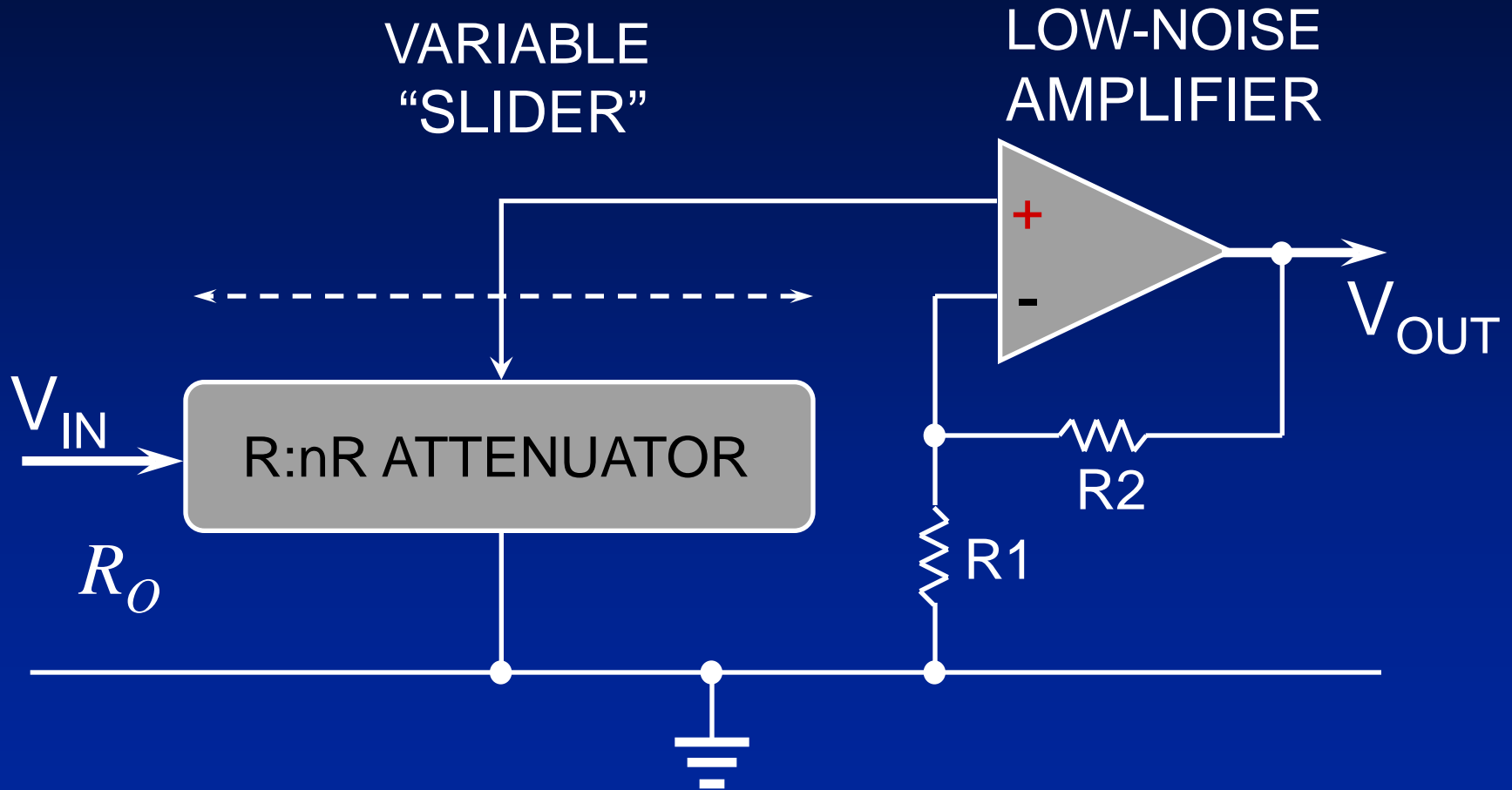


GUARANTEES ULTRA-LOW NOISE

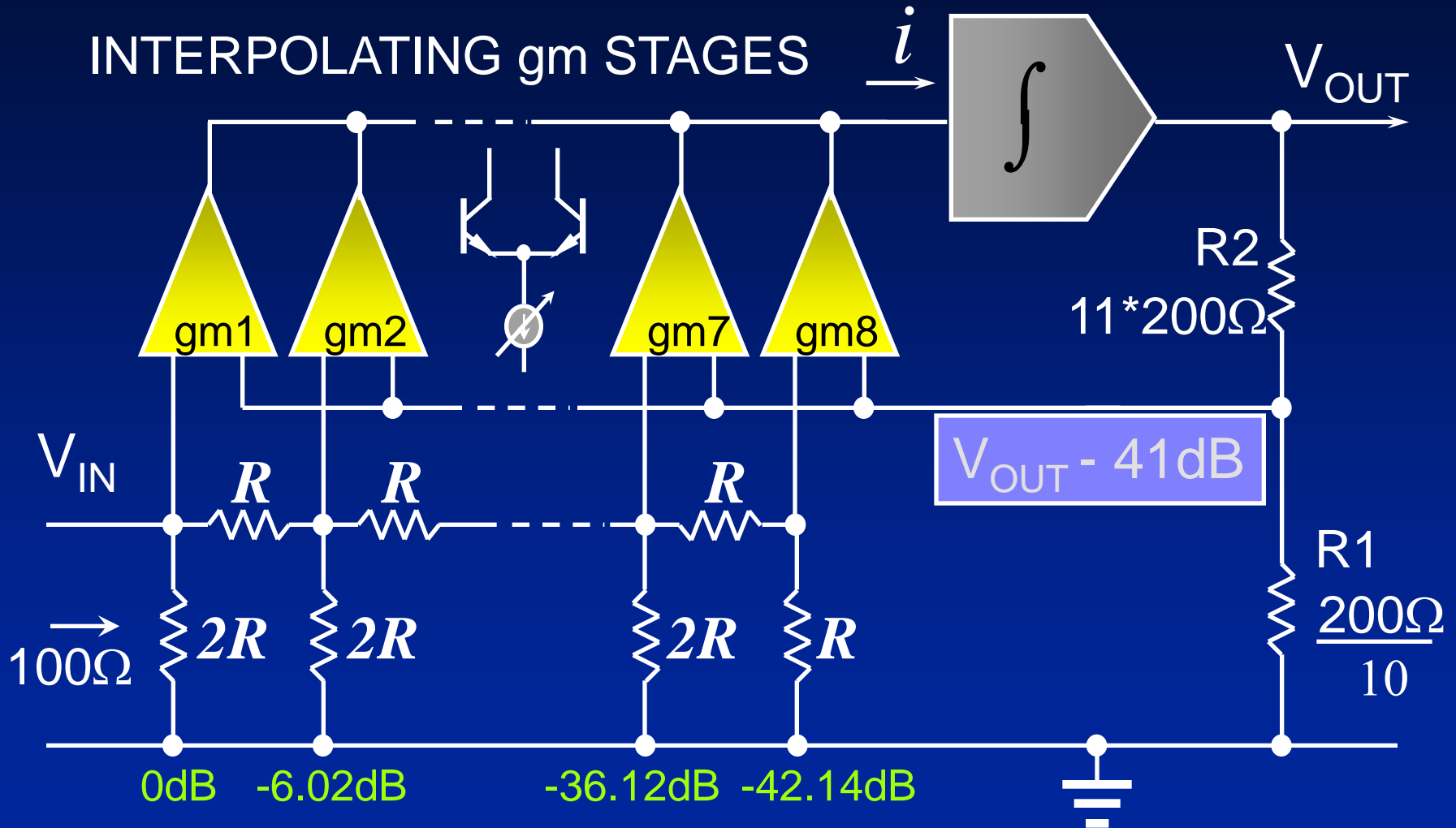


EXHIBITS WIDE DYNAMIC RANGE  
*FROM NOISE FLOOR ( $0.7\mu\text{V RMS}$ )  
TO TYPICALLY  $1.4\text{V RMS}$  (106dB)*

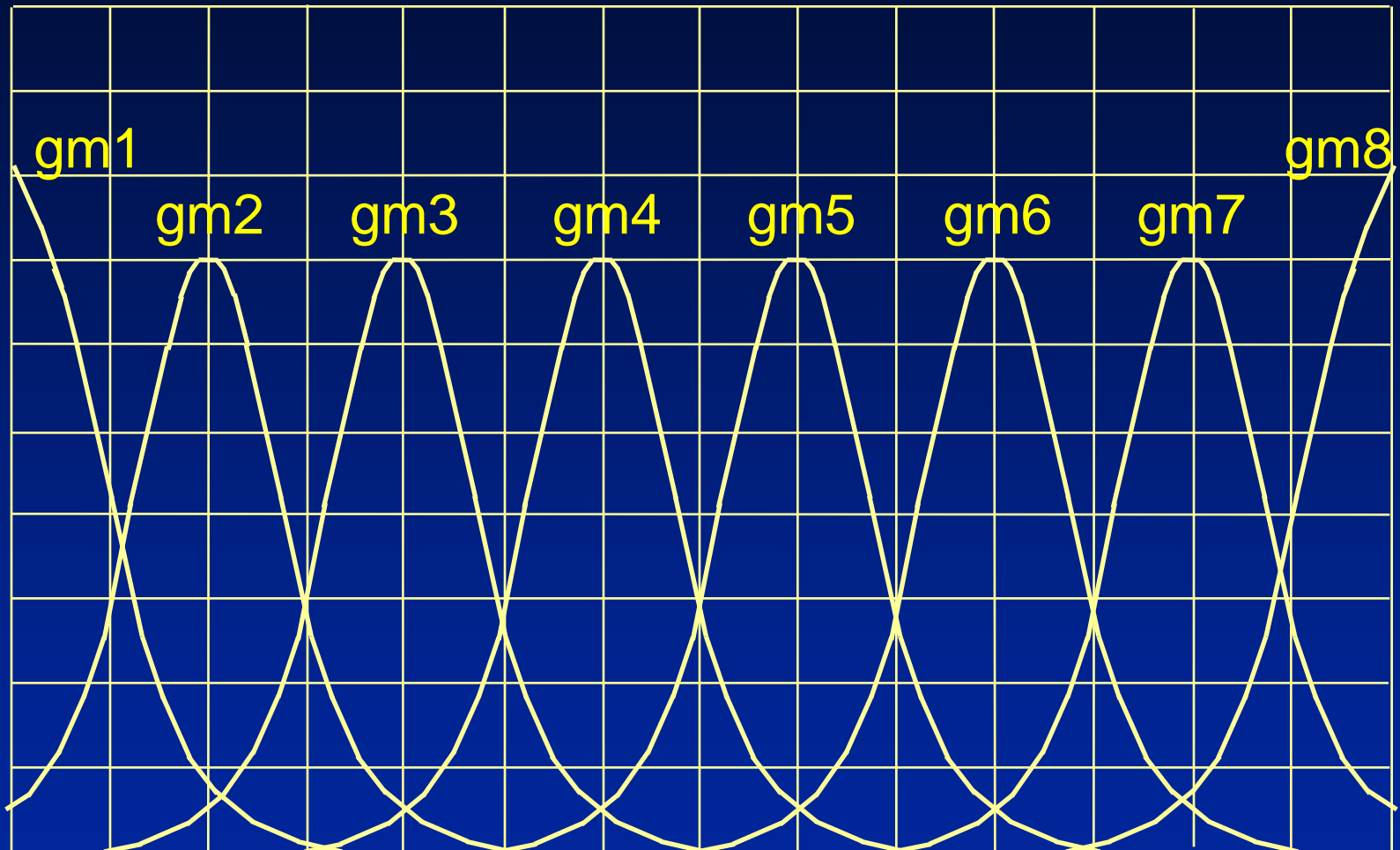
# A BASIC X-AMP



# TYPICAL 8-STAGE X-AMP



# CURRENTS IN THE gm STAGES



**INCREASING GAIN**

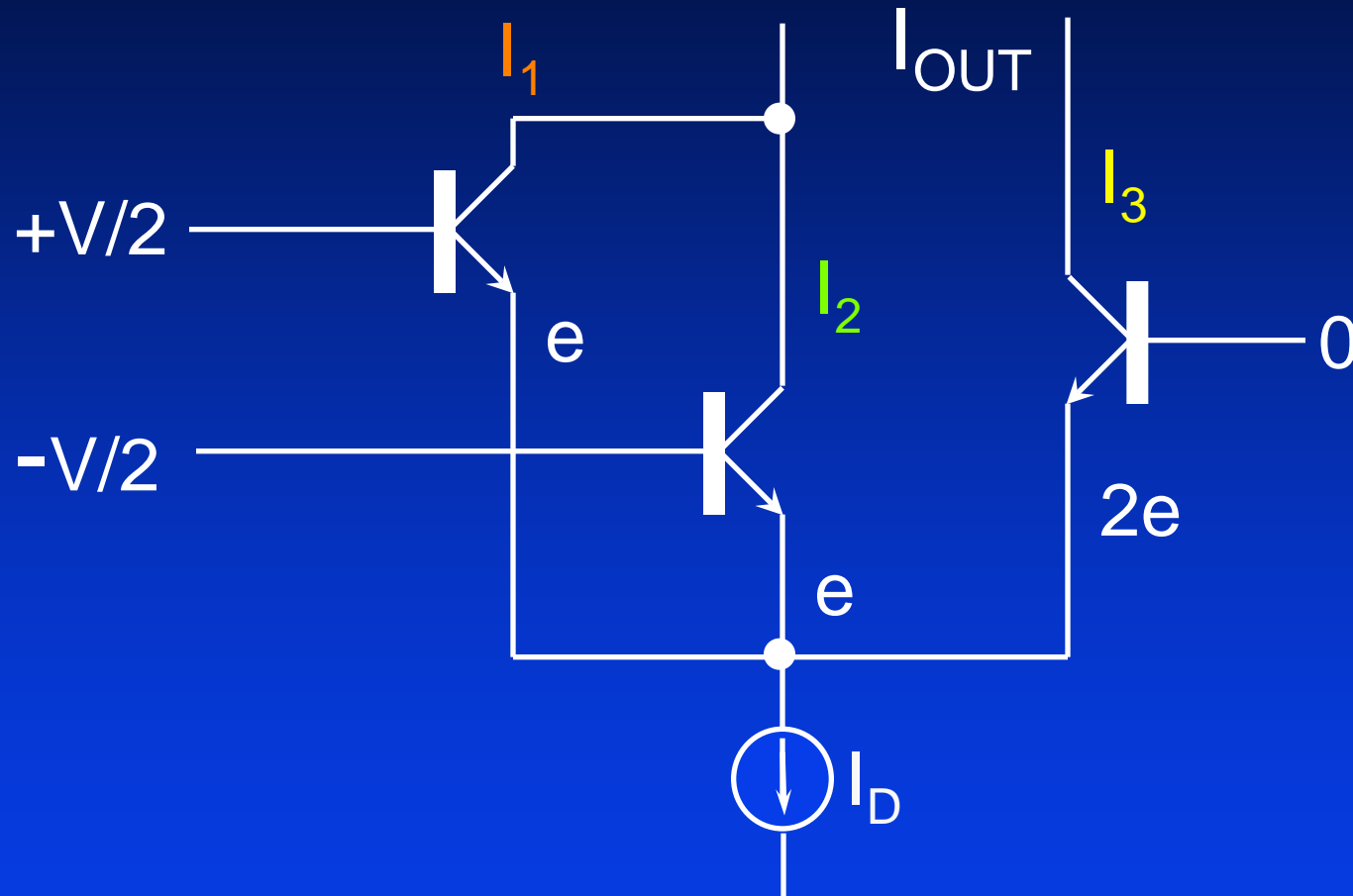
(MOVES ACTION TOWARDS FRONT)

# AD8363

is an example of a log-amp that is also RMS-responding.

It can operate from a few Hz up to a specified 6 GHz, providing a 50-dB range and laser-trimmed calibration to absolute standards.

# SQUARE-LAW DETECTOR



# SQUARE-LAW DETECTOR

$$I_1 = I_D e^u / (e^u + e^{-u} + 2)$$

$$I_2 = I_D e^{-u} / (e^u + e^{-u} + 2)$$

where

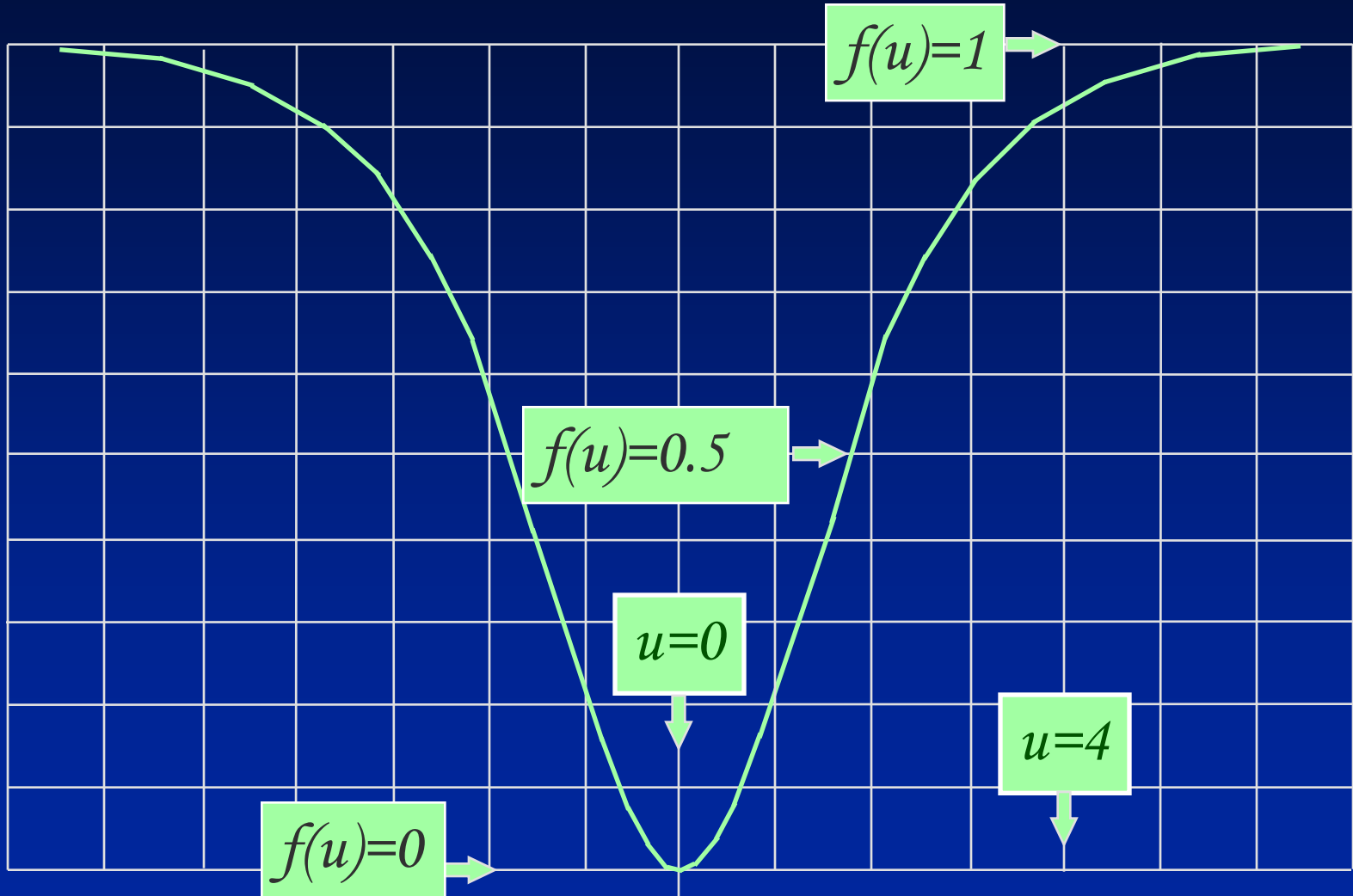
$$u = V/2V_T$$

$$I_3 = 2I_D / (e^u + e^{-u} + 2)$$

$$I_{OUT} = I_1 + I_2 - I_3 = \frac{e^u + e^{-u} - 2}{e^u + e^{-u} + 2} I_D$$

Error is  $< \pm 2.7\%$ ,  $-2V_T < V < +2V_T$

$$f(u) = \frac{e^u + e^{-u} - 2}{e^u + e^{-u} + 2} = 1 - \operatorname{sech}^2(u/2)$$



# END OF OVERVIEW

# PROGRESSIVE COMPRESSION

- A MAJOR CLASS, with many sub-types
- THE METHOD USED FOR PRACTICALLY ALL WIDEBAND LOG-AMPS, UP TO SHF
- THE BACKBONE IS A CHAIN OF SIMPLE AMPLIFIER CELLS
- FUNCTION IS A TYPE OF PIECEWISE LINEAR APPROXIMATION, but the “law conformance error” may be as low as 0.1dB

# PROGRESSIVE COMPRESSION

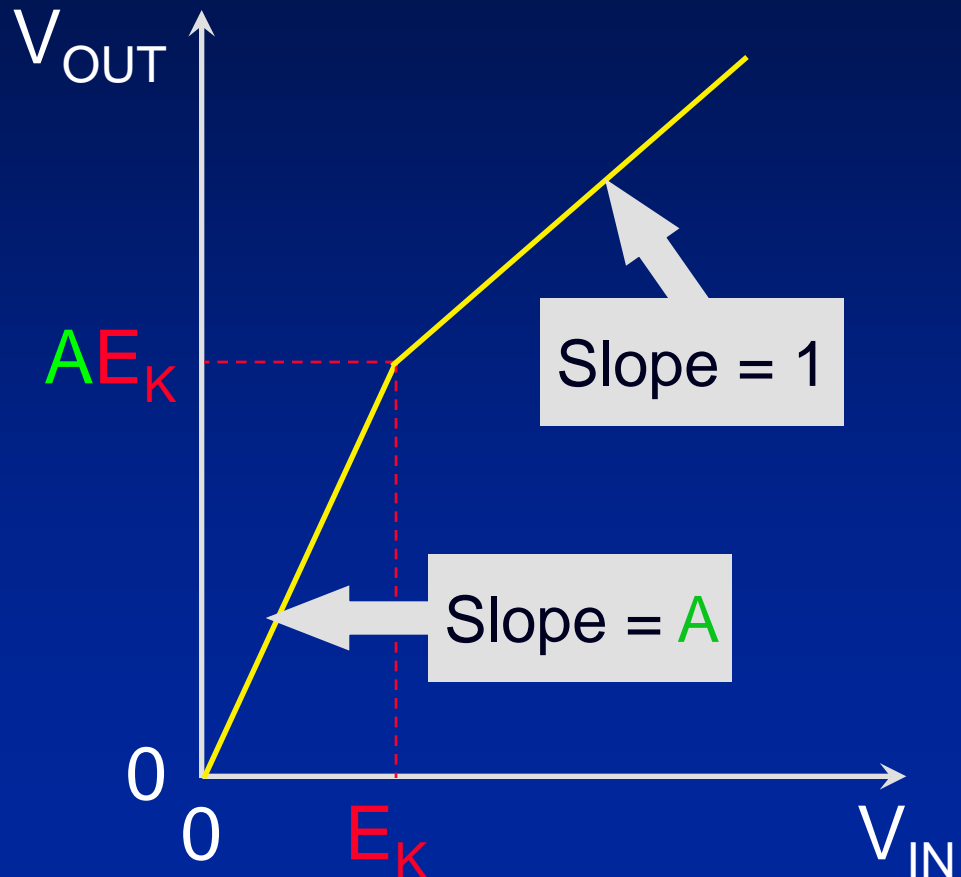
THE LITERATURE ON LOG AMP DESIGN IS SCANT, EMPIRICAL, AND SURPRISINGLY QUIET ON THE CRUCIAL MATTER OF SCALING.

THE ANALYSES NOW PRESENTED FLOW DIRECTLY FROM A SIMPLE, FUNDAMENTAL STARTING POINT. NO GUESSING NEEDED!

# THE A/1 AMPLIFIER



SYMBOL



# THE A/1 AMPLIFIER

$$V_{\text{OUT}} = AV_{\text{IN}}$$

$$\text{for } V_{\text{IN}} < E_K$$

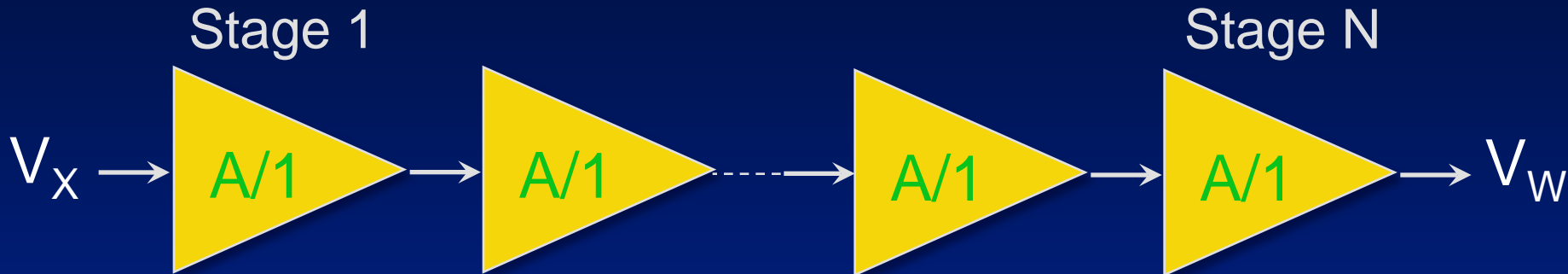
$$V_{\text{OUT}} = (A-1)E_K + V_{\text{IN}}$$

$$\text{for } V_{\text{IN}} > E_K$$

Note:

The amplifier function is usually odd-order symmetric

# A CASCADE OF $N$ , $A/1$ CELLS



- WE'LL IMPLEMENT A BASEBAND LOG-AMP
- USING PIECEWISE-LINEAR APPROXIMATION
- CASCADE HAS VERY HIGH OVERALL GAIN ( $A^N$ )

# A CASCADE OF N, A/1 CELLS

THE FUNCTION WILL BE

$$V_W = V_Y \log \frac{V_X}{V_Z}$$

but.....

WHERE DO  $V_Y$  &  $V_Z$  “COME FROM”?

# A CASCADE OF N, A/1 CELLS

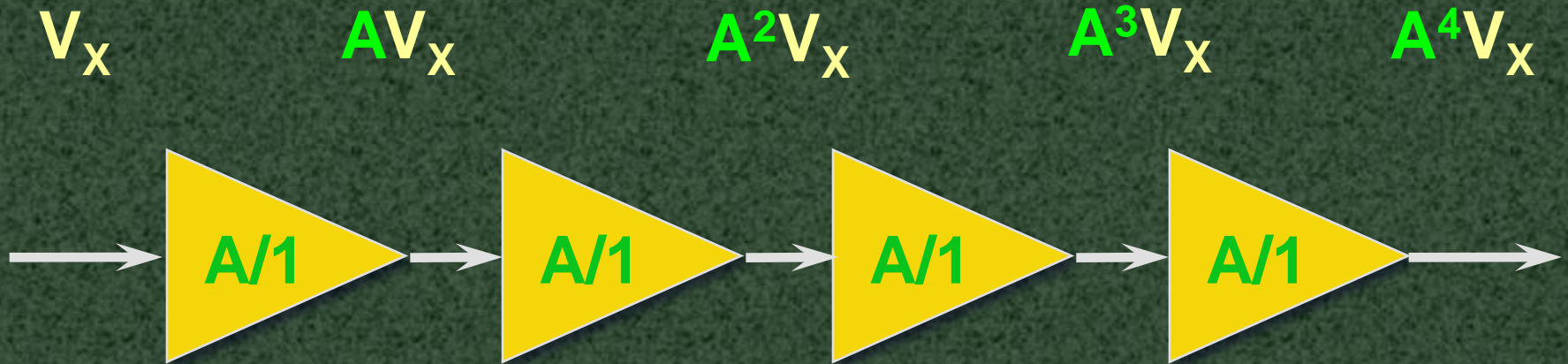
WE MUST CONCLUDE THAT

$$V_Y = y E_K$$

$$V_Z = z E_K$$

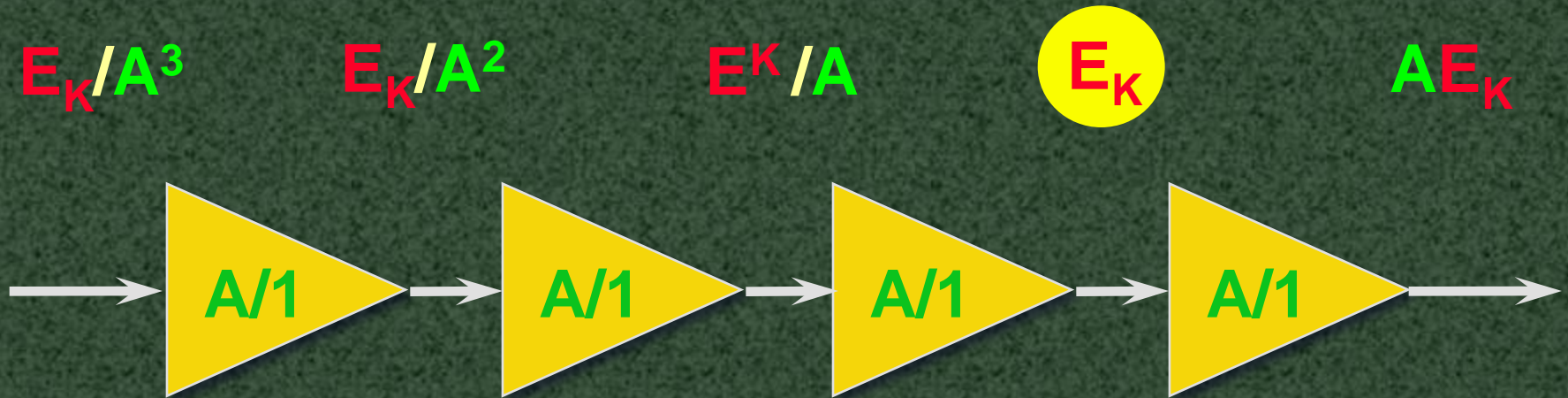
where  $y$  and  $z$  must be linear functions of  $A$  and  $N$

# FOUR-STAGE EXAMPLE



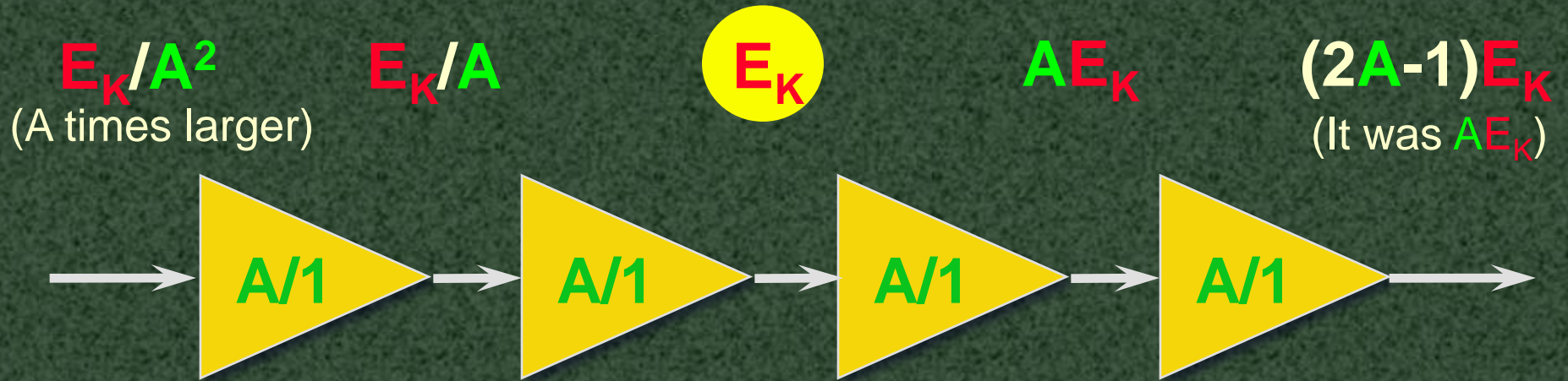
- INPUT VOLTAGE VERY SMALL
- ALL STAGES REMAIN LINEAR
- INTERNAL VOLTAGE  $E_K$  IS HIDING

# FOUR-STAGE EXAMPLE



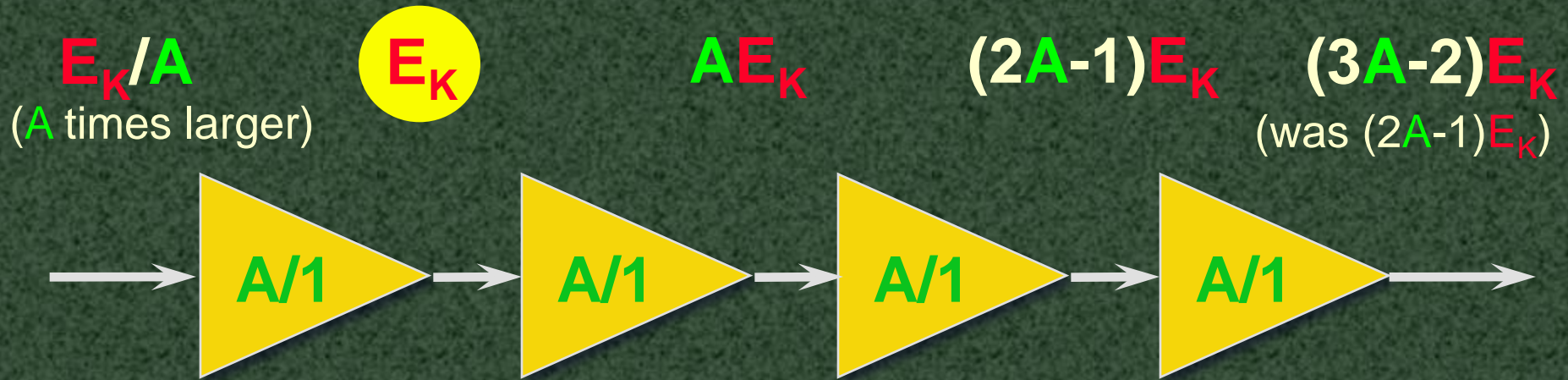
- FIRST TRANSITION POINT REACHED
- BOTH OUTPUT AND INPUT DEFINED
- VOLTAGE  $E_K$  NOW IN EVIDENCE

# FOUR-STAGE EXAMPLE



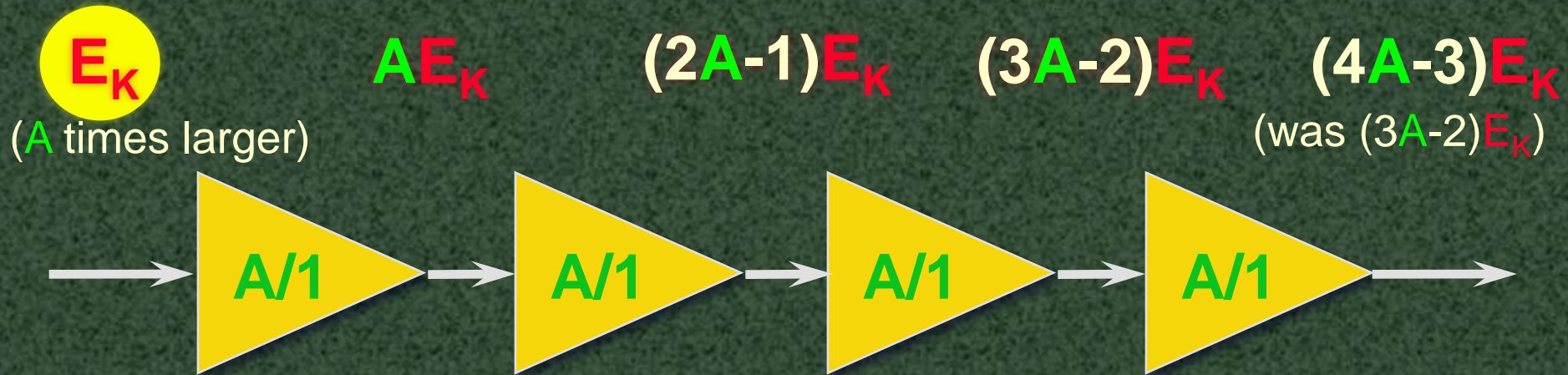
- SECOND TRANSITION POINT
- OUTPUT HAS INCREASED BY  $(A-1)E_K$
- INPUT INCREASED BY THE RATIO  $A$

# FOUR-STAGE EXAMPLE



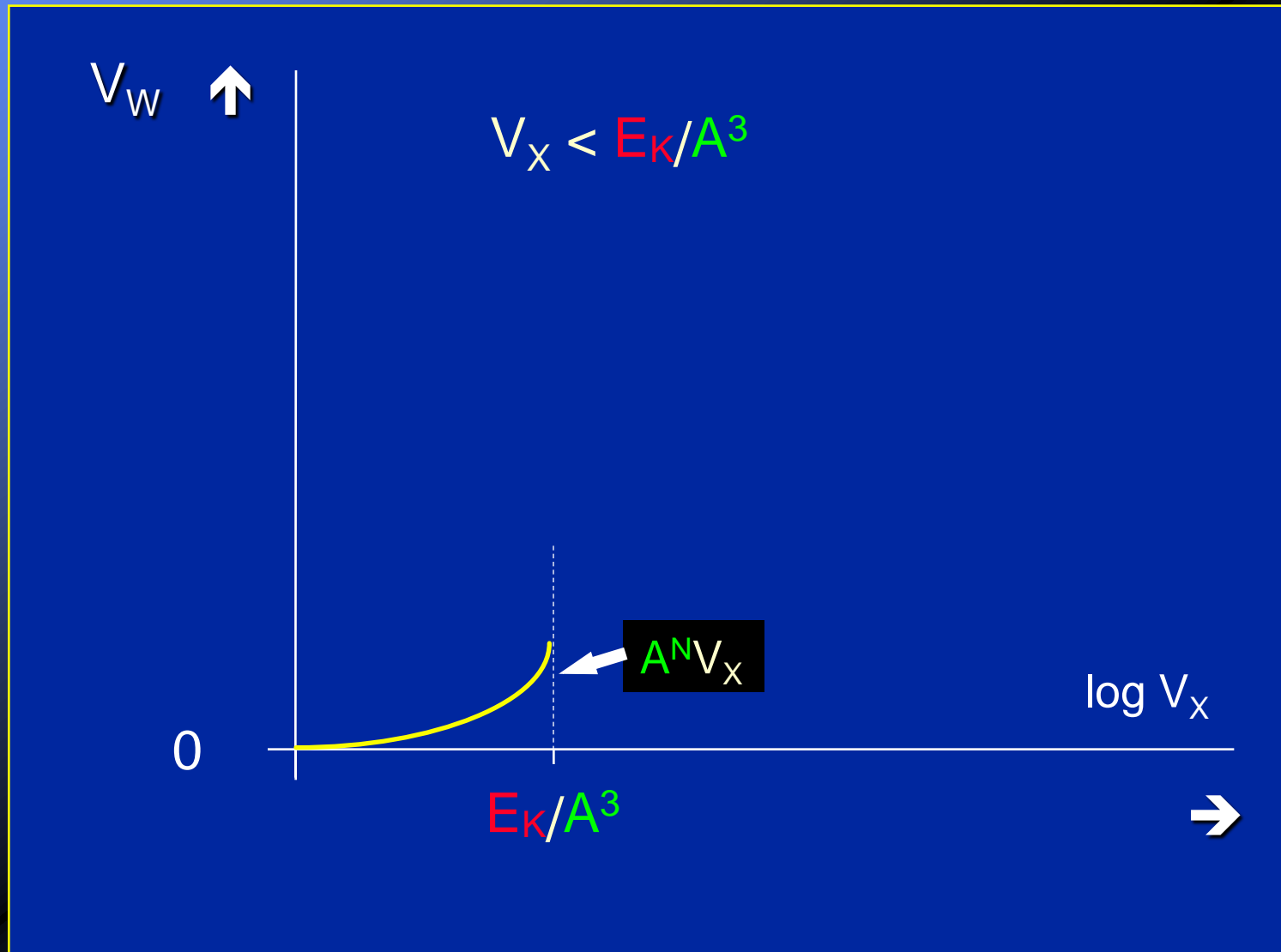
- THIRD TRANSITION POINT
- OUTPUT HAS INCREASED BY  $(A-1)E_K$
- INPUT INCREASED BY THE RATIO  $A$

# FOUR-STAGE EXAMPLE

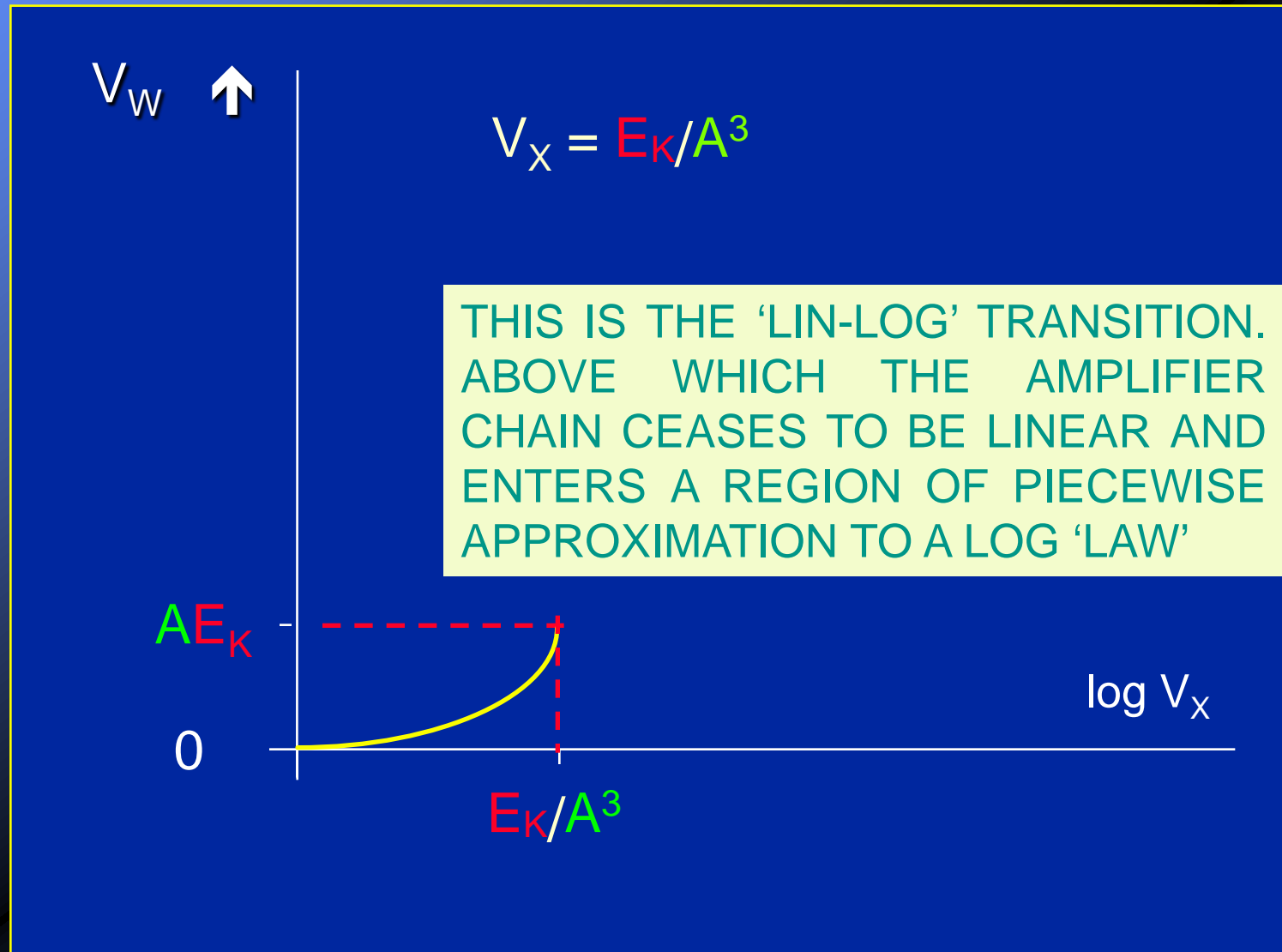


- FOURTH & FINAL TRANSITION POINT
- OUTPUT HAS INCREASED BY  $(A-1)E_K$
- INPUT HAS INCREASED BY RATIO  $A$

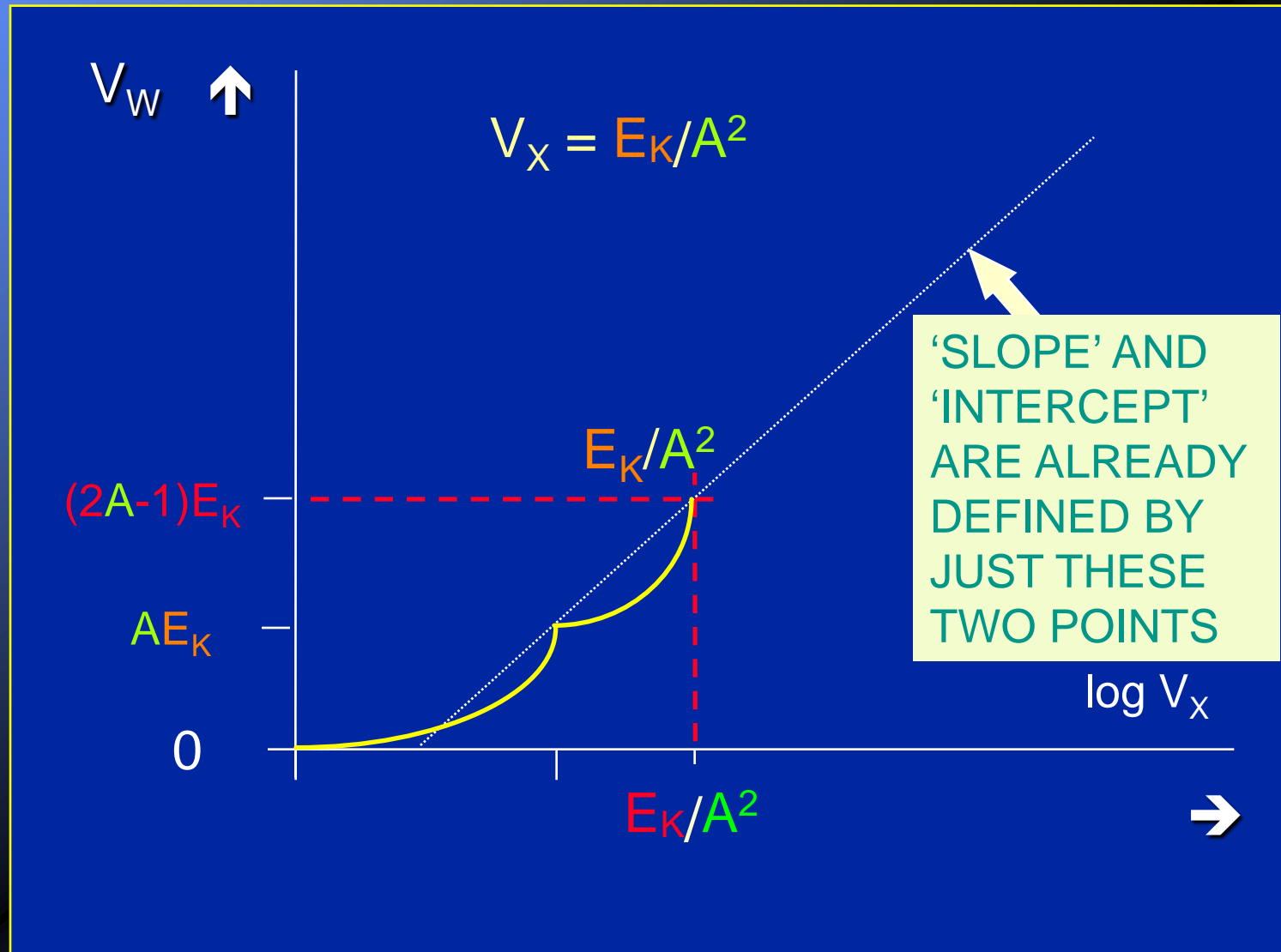
# BUILDING THE LOGARITHMIC FUNCTION



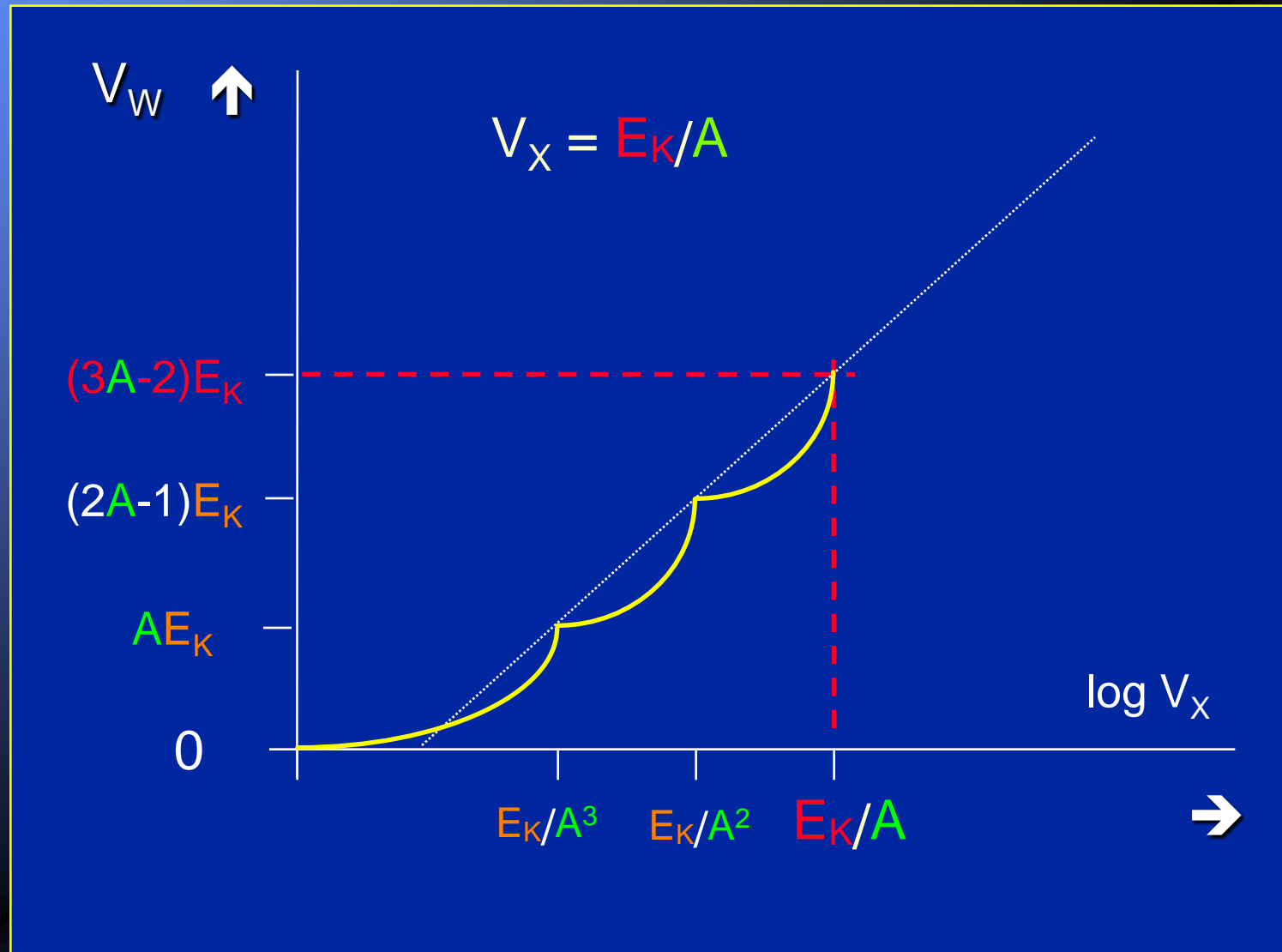
# BUILDING THE LOGARITHMIC FUNCTION



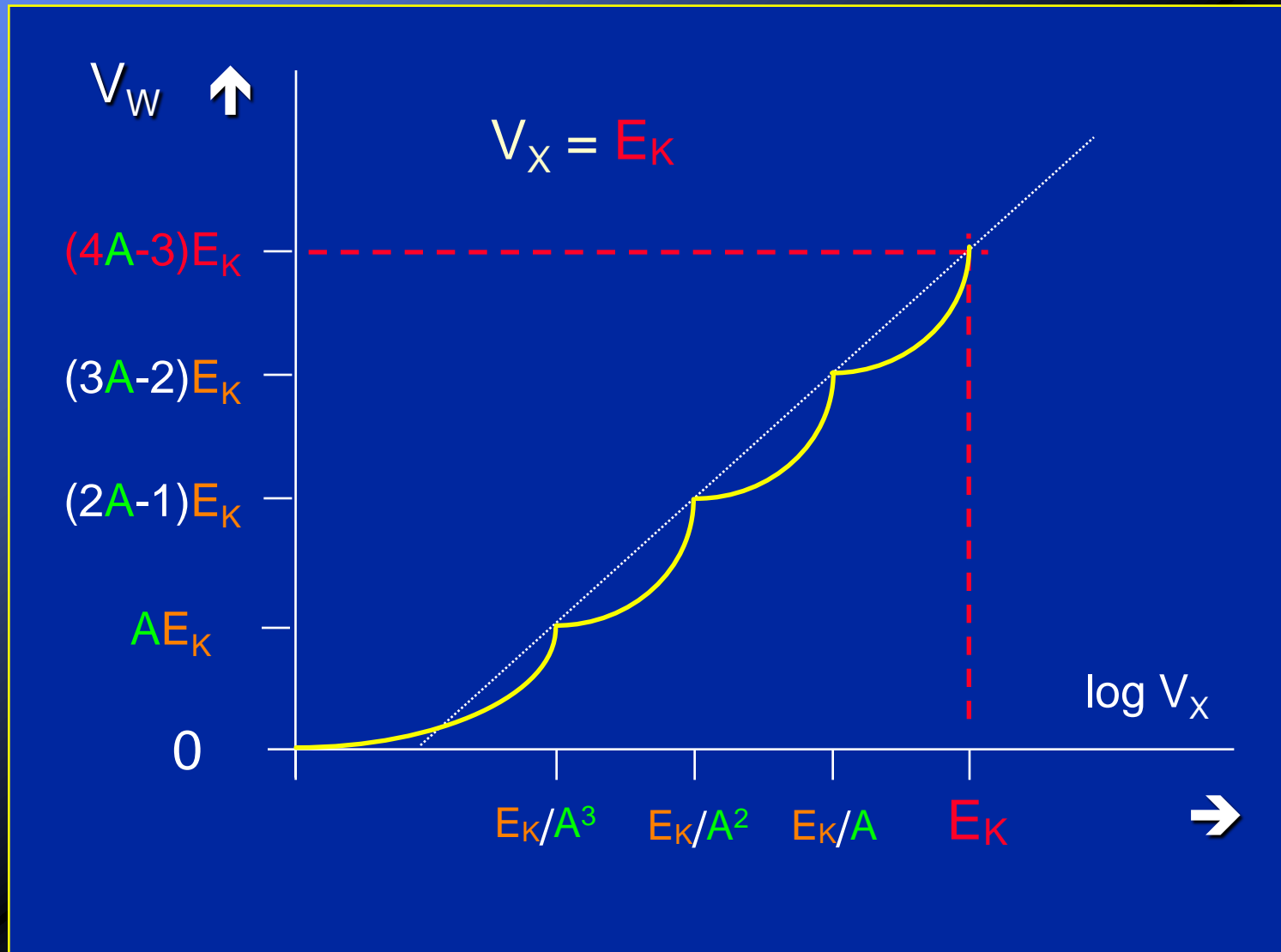
# BUILDING THE LOGARITHMIC FUNCTION



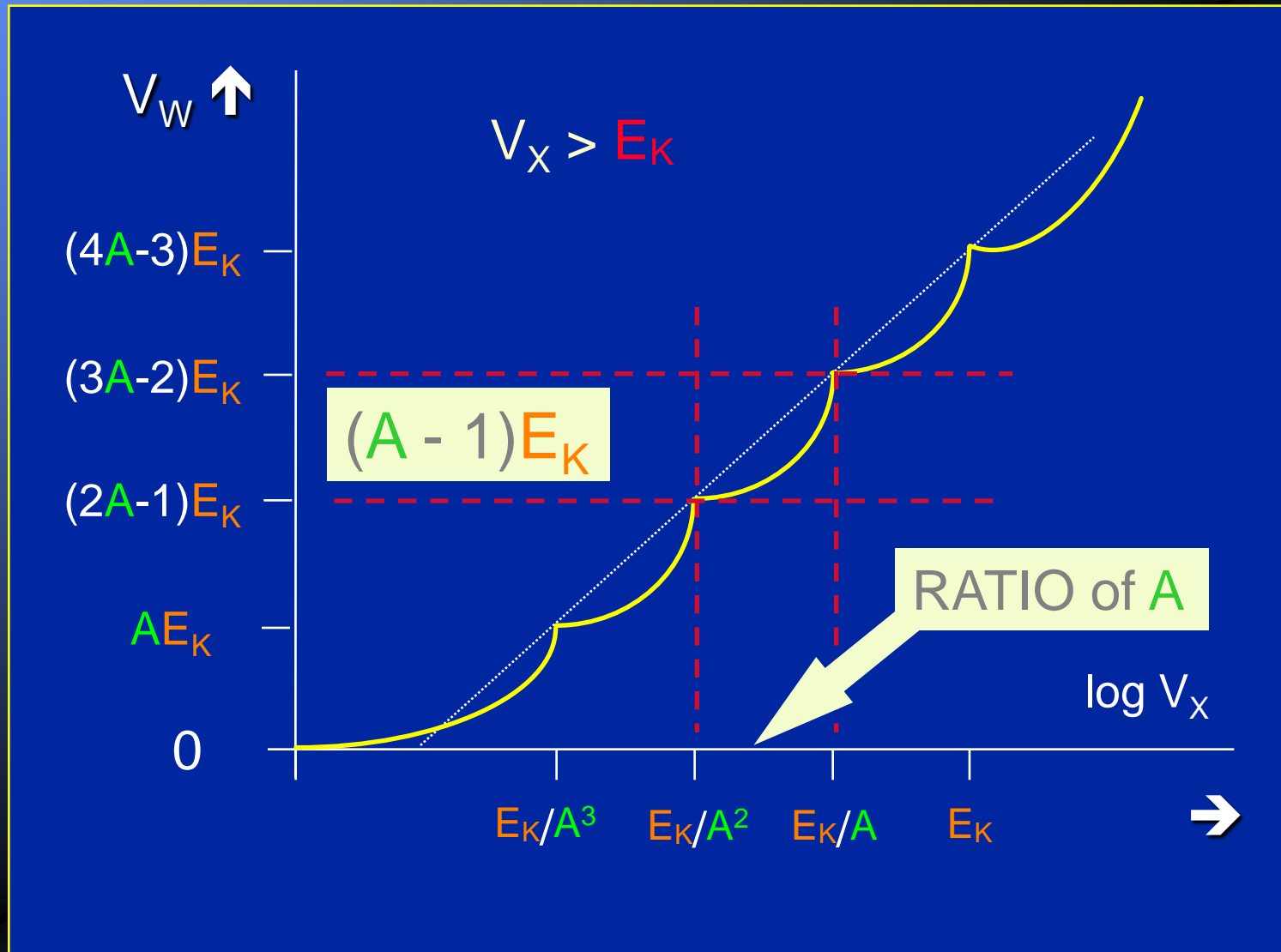
# BUILDING THE LOGARITHMIC FUNCTION



# BUILDING THE LOGARITHMIC FUNCTION



# BUILDING THE LOGARITHMIC FUNCTION



# SLOPE CALCULATION

THE OUTPUT CHANGES BY  
(2A-1)E<sub>K</sub> WHEN THE INPUT  
CHANGES BY THE *RATIO* A

$$V_Y = \frac{(2A-1)E_K}{\lg(A)} \quad (\text{volts/decade})$$

# INTERCEPT CALCULATION

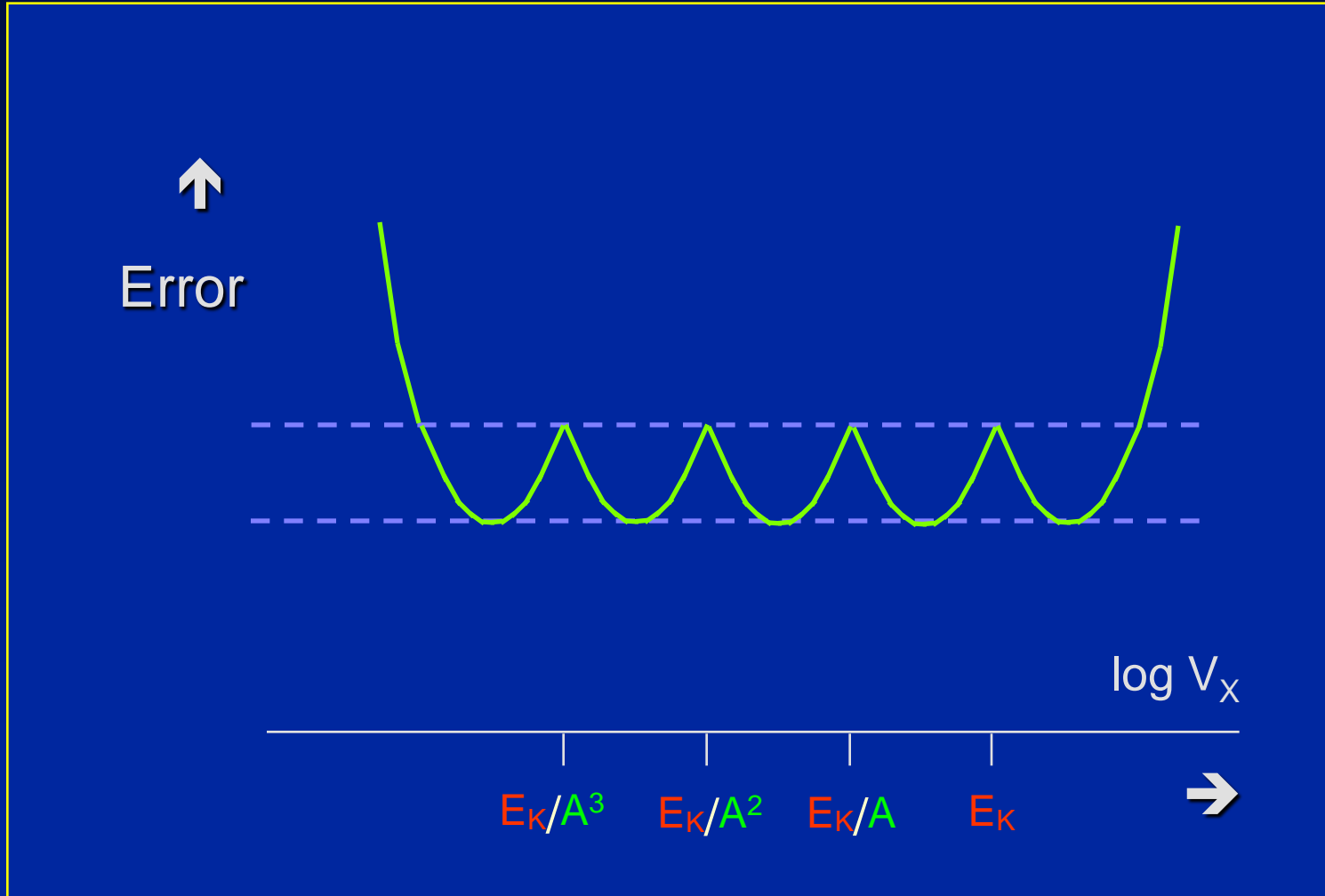
$$V_W = V_Y \lg \frac{V_X}{V_Z}$$

SOLVE FOR  $V_Z$  BY SUBSTITUTING

$$V_Y = \frac{(2A-1)E_K}{\lg(A)}, \quad V_X = E_K/(N-1), \quad V_W = AE_K$$

$$V_Z = \frac{E_K}{AN + 1/(A-1)}$$

# APPROXIMATION ERROR



# LAW CONFORMANCE

dB Error = Actual Output (in dB)  
minus Ideal Output (in dB)

Since there are 20 decibels per decade  
the ideal output in dB is

$$\text{IDEAL}_{\text{dB}} = 20 \lg(V_X/V_Z)$$

# LAW CONFORMANCE

The Actual Output in dB is

$$\text{ACTUAL}_{\text{dB}} = 20 \frac{\text{ACTUAL OUTPUT } V_W}{\text{VOLTS/DECADE } (V_Y)}$$

$$\text{ERROR}_{\text{dB}} = 20 \left\{ V_W/V_Y - \lg(V_X/V_Z) \right\}$$

# LAW CONFORMANCE

For a log-amp based on the  $A/1$  gain stages, the peak deviation from an ideal logarithmic response is

$$R_{\text{dB}} = 10 \{ (A + 1 - 2\sqrt{A}) \lg A \} / (A - 1)$$

Examples:

$$A = 2 \text{ (6dB)}, \quad R = 0.52\text{dB};$$

$$A = \sqrt{10} \text{ (10dB)}, \quad R = 1.4\text{dB}$$

$$A = 4 \text{ (12dB)}, \quad R = 2.01\text{dB};$$

$$A = 5 \text{ (14dB)}, \quad R = 2.67\text{dB}$$

# DYNAMIC RANGE

THE FUNCTION FIT CONTINUES TO BE USEFULLY ACCURATE EVEN AT INPUTS BELOW THE FIRST TRANSITION AND ABOVE LAST ONE. IN PRACTICE, THE DYNAMIC RANGE IS SLIGHTLY MORE THAN  $A^N$ . THUS, USING  $A=4$ ,  $N=10$  WE COULD EXPECT A 120dB RANGE .....

# DYNAMIC RANGE

... EXCEPT FOR THE LITTLE MATTER OF NOISE. FOR EXAMPLE, A FIRST-STAGE NOISE-SPECTRAL-DENSITY OF  $1\text{nV}/\sqrt{\text{Hz}}$  IN A 500-MHZ BANDWIDTH IS EQUIVALENT TO AN INPUT-REFERRED NOISE OF  $22.4\mu\text{V}$  RMS, OR  $-80\text{dBm}/50\Omega$

# DYNAMIC RANGE

THE INPUT NEEDS TO BE AT LEAST  
6dB ABOVE THE NOISE FLOOR TO  
MAKE AN ACCURATE MEASUREMENT,  
THUS AT -74dBm IN THIS EXAMPLE.

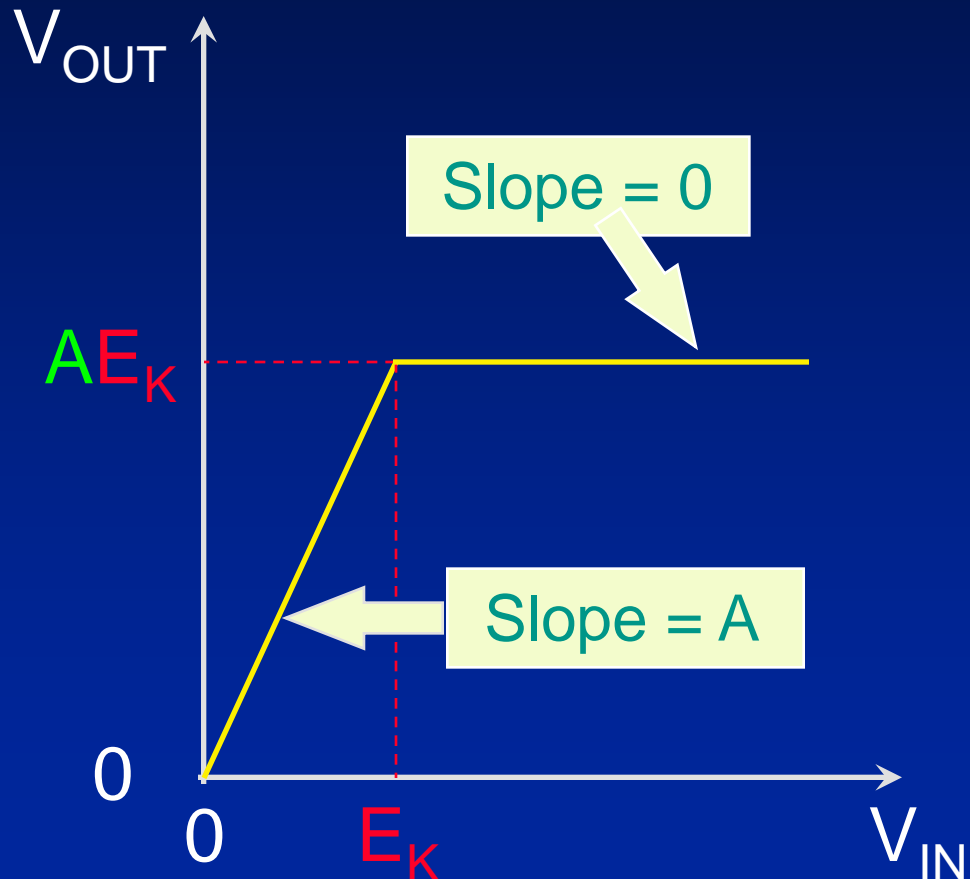
120dB ABOVE THIS NOISE FLOOR IS  
AT +46dBm (63V SINE AMPLITUDE!)



# THE A/0 AMPLIFIER



SYMBOL



# THE A/O AMPLIFIER

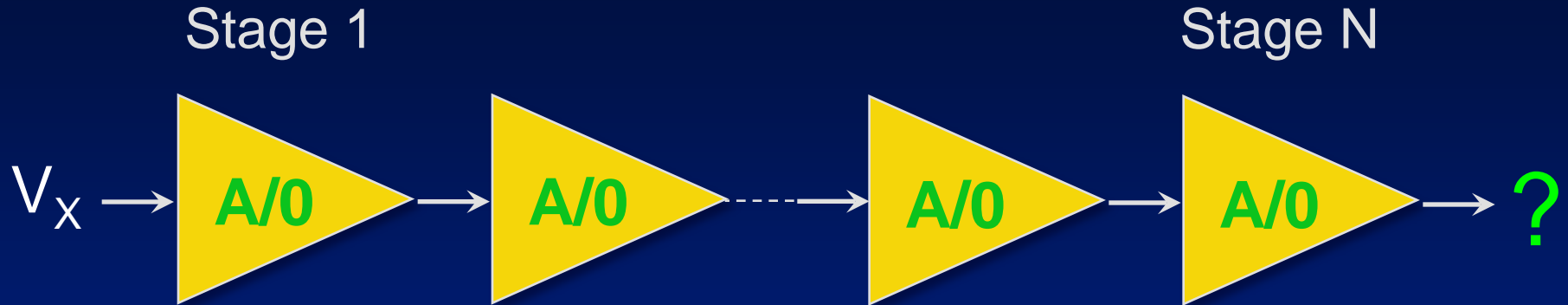
$$V_{\text{OUT}} = AV_{\text{IN}}$$

for  $V_{\text{IN}} < E_K$

$$V_{\text{OUT}} = AE_K$$

for  $V_{\text{IN}} > E_K$

# N-CASCADE OF A/O CELLS



WE CAN NO LONGER TAKE THE OUTPUT FROM THE LAST STAGE: IT LIMITS TO  $AE_k$  AS SOON AS ITS INPUT IS  $E_k$  AND THEREAFTER *IT DOES NOT INCREASE*

# N-CASCADE OF A/0 CELLS

WHILE WE MUST STILL FIND

$$V_Y = y E_K$$

$$V_Z = z E_K$$

we should expect  $y$  and  $z$  to now be different functions A and N than for the cascade of A/1 stages

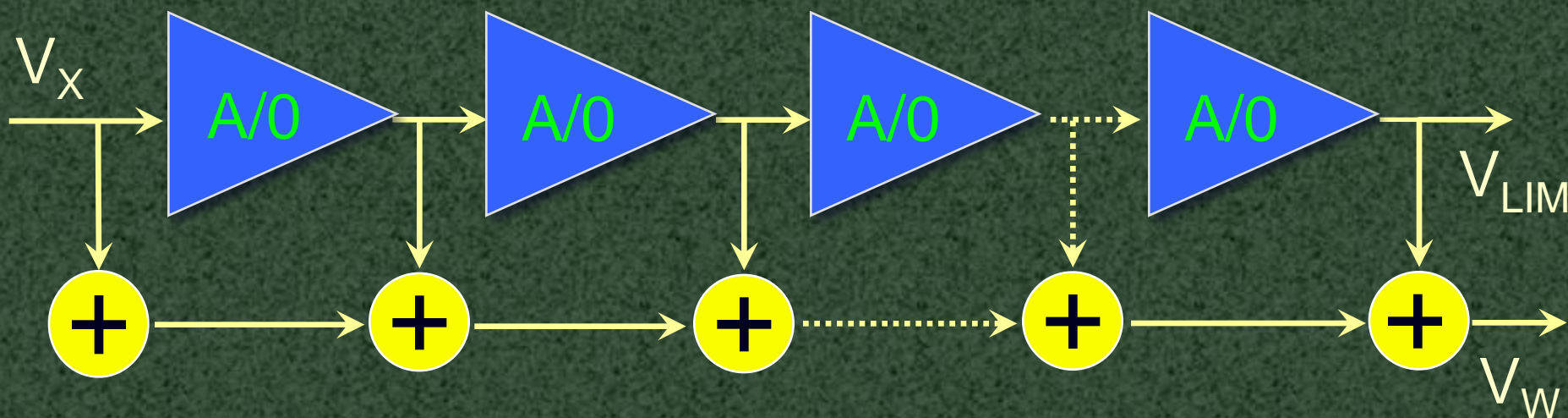
Editor's Note: PDF version of slides from Beam Instrumentation Workshop 2010, Santa Fe, NM

# THE SOLUTION: ADD ALL THE OUTPUTS TOGETHER

STAGE 1

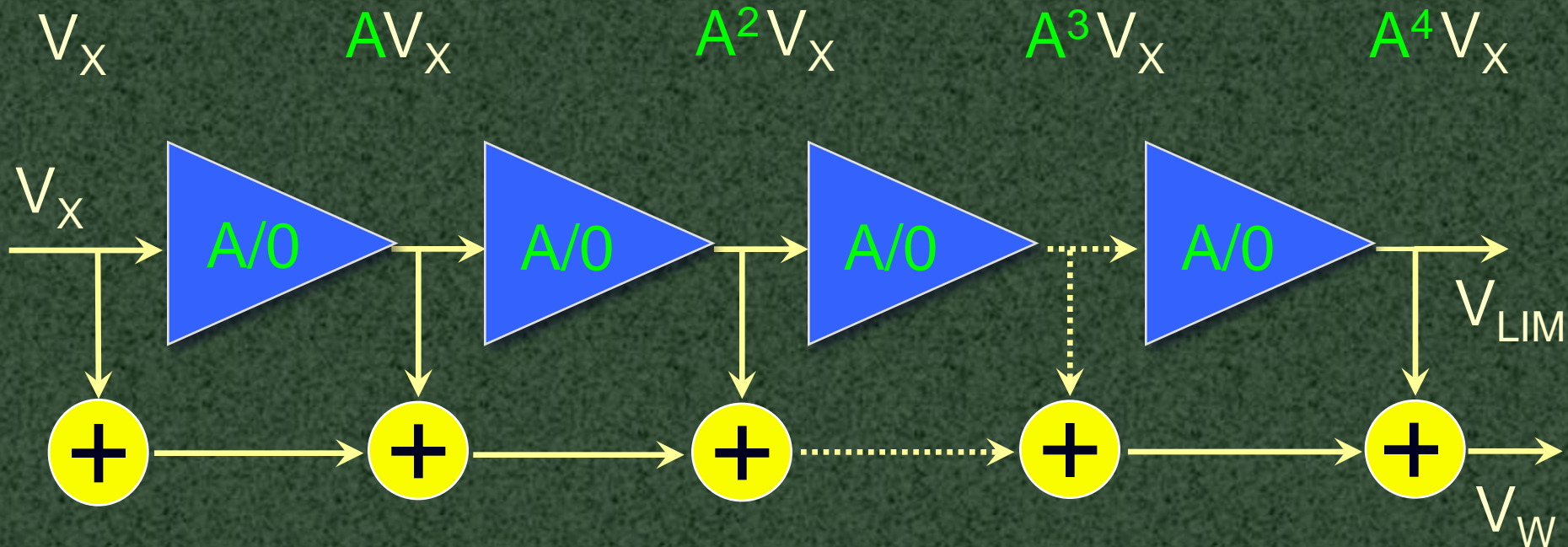
STAGE 2

STAGE 3 . . . . . STAGE N



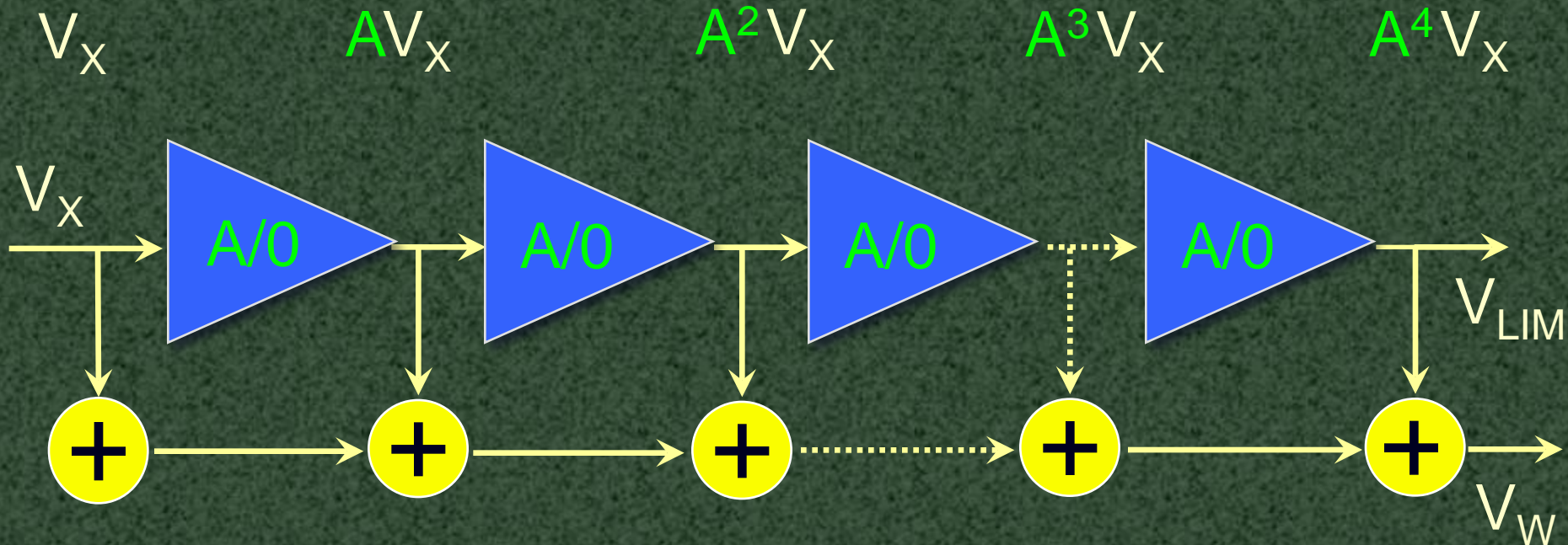
THIS IS THE BASIC BACKBONE OF PRACTICALLY ALL MONOLITHIC PROGRESSIVE-COMPRESSION LOG-AMPS; NOTE THAT LAST STAGE OUTPUT IS NOW CALLED  $V_{LIM}$

# FOUR-STAGE EXAMPLE



- INPUT VOLTAGE VERY SMALL
- ALL STAGES REMAIN LINEAR
- INTERNAL VOLTAGE  $E_K$  HIDDEN

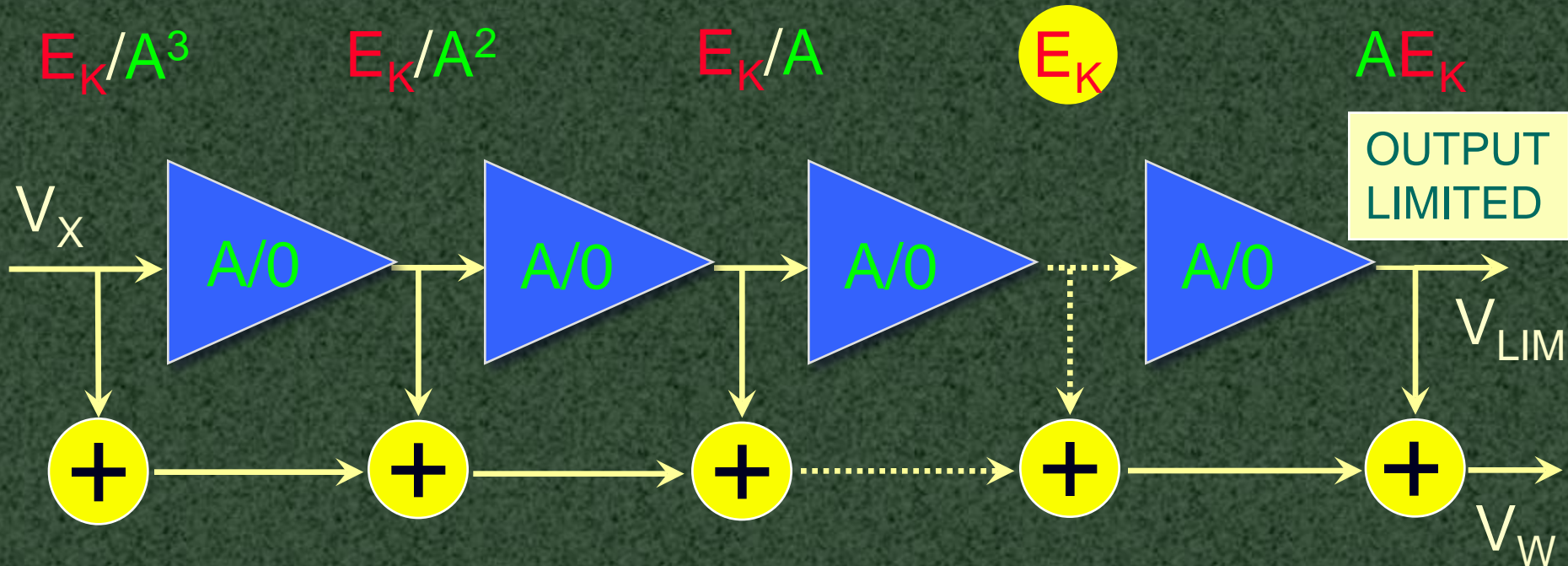
# FOUR-STAGE EXAMPLE



FOR SMALL INPUTS

$$V_W = (1 + A + A^2 + A^3 + A^4) V_X$$

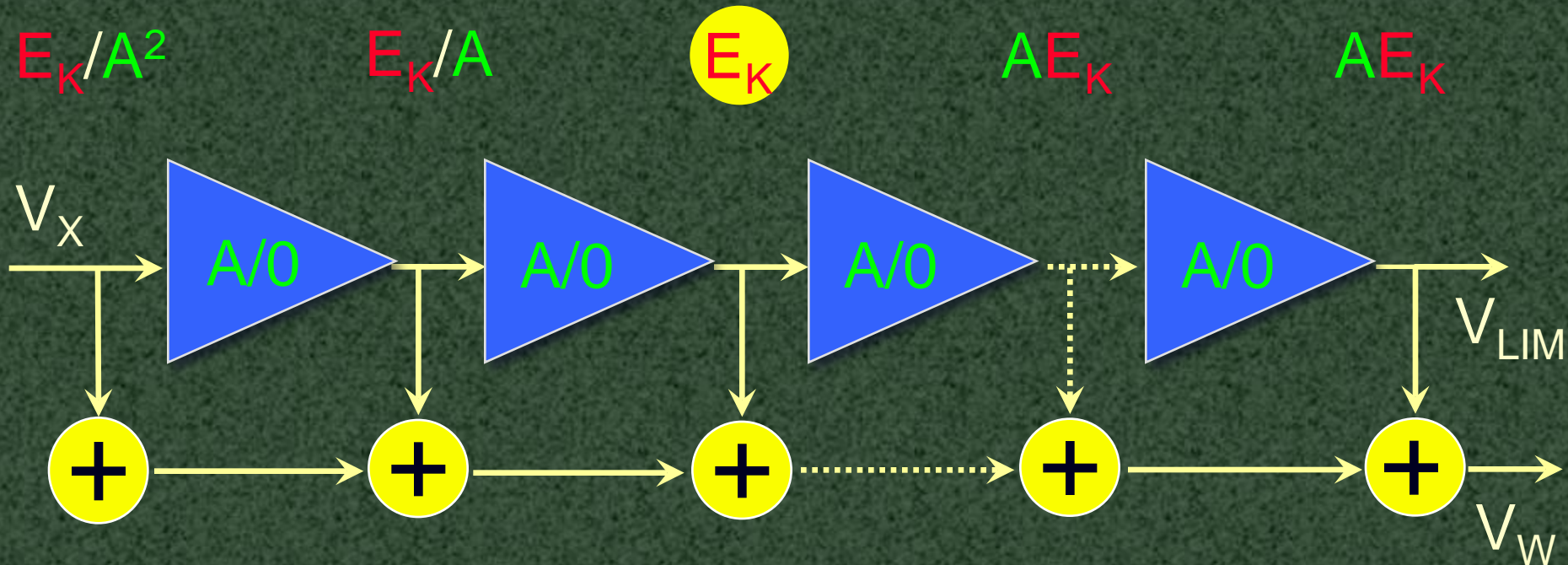
# FOUR-STAGE EXAMPLE



FOR AN INPUT OF EXACTLY  $E_K/A^3$

$$V_W = (A + 1 + A^{-1} + A^{-2} + A^{-3}) E_K$$

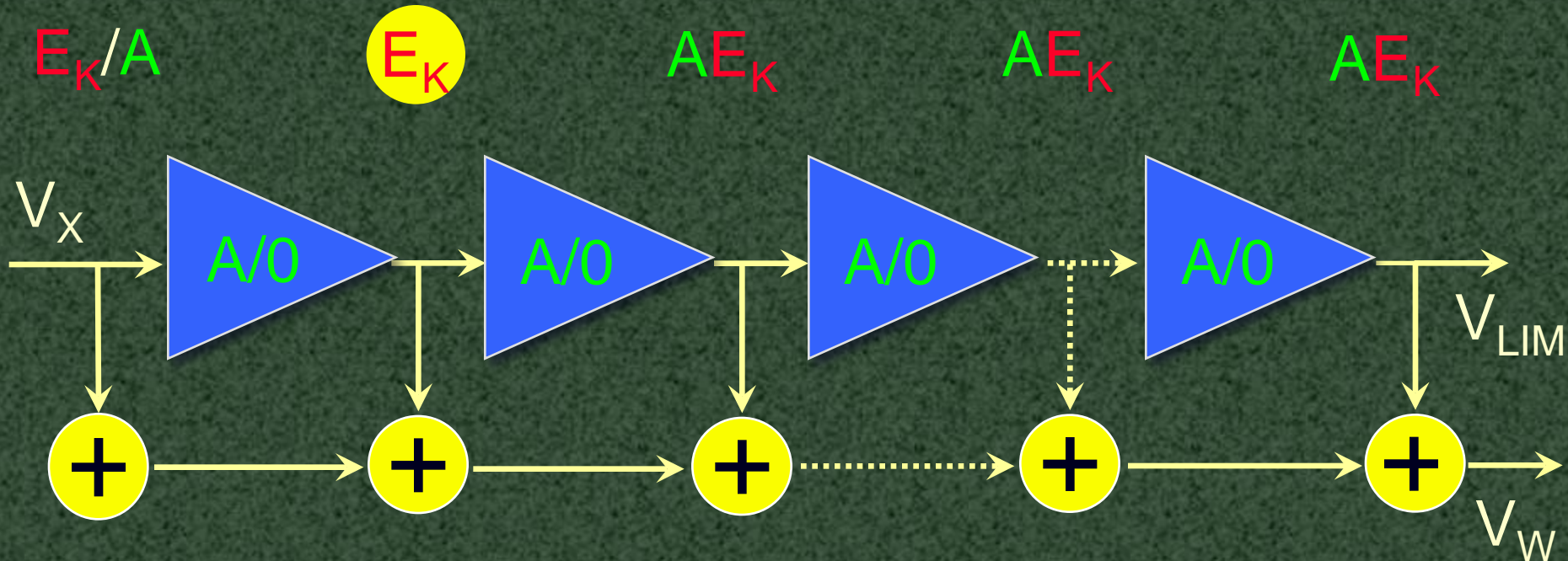
# FOUR-STAGE EXAMPLE



FOR AN INPUT OF EXACTLY  $E_K/A^2$

$$V_W = (2A + 1 + A^{-1} + A^{-2})E_K$$

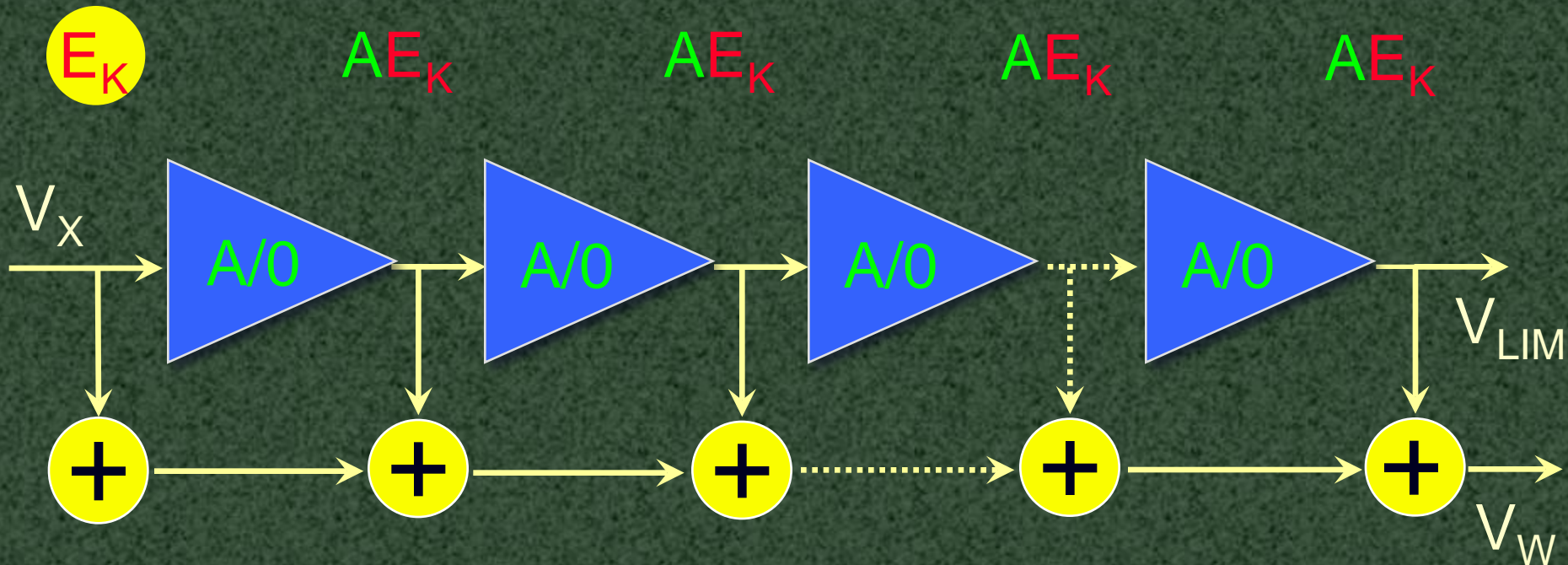
# FOUR-STAGE EXAMPLE



FOR AN INPUT OF EXACTLY  $E_K/A$

$$V_w = (3A + 1 + A^{-1})E_K$$

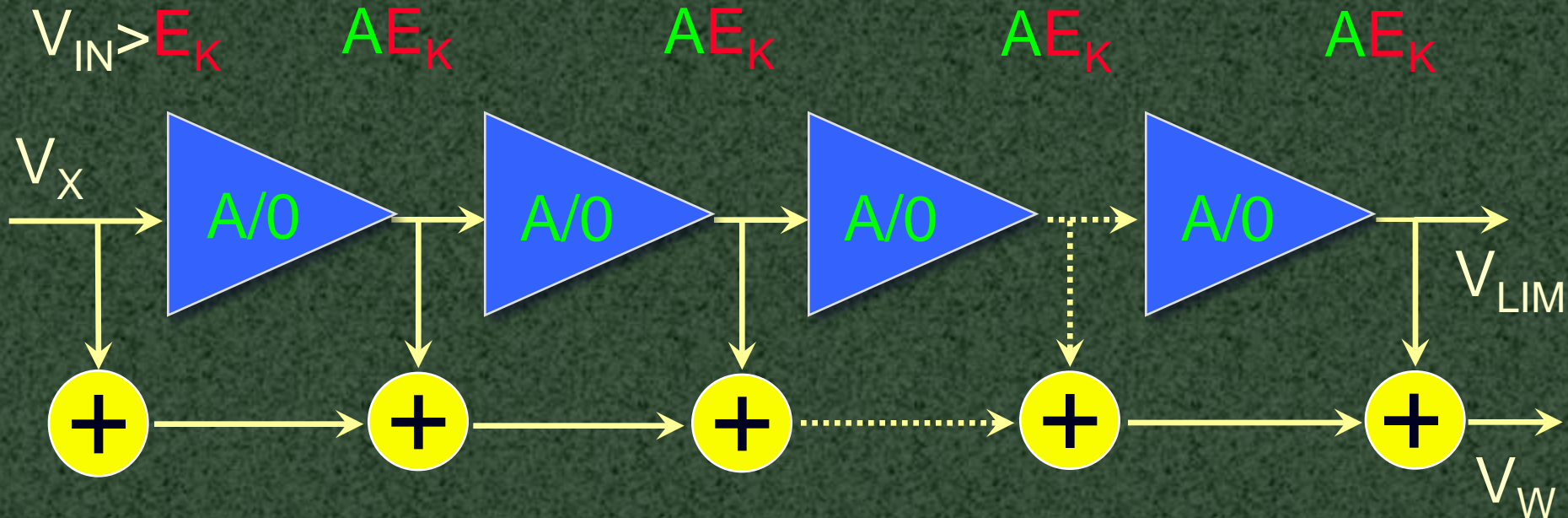
# FOUR-STAGE EXAMPLE



FOR AN INPUT OF EXACTLY  $E_K$

$$V_W = (4A + 1) E_K$$

# FOUR-STAGE EXAMPLE



FOR ANY INPUT ABOVE  $E_K$

$$V_W = 4AE_K + V_{IN}$$

# SLOPE CALCULATION

THE OUTPUT CHANGES BY  
essentially\*  $AE_K$  as the INPUT  
CHANGES BY THE *RATIO*  $A$

$$V_Y = \frac{AE_K}{\lg(A)} \quad (\text{volts/decade})$$

\* it's actually  $A(1 - A^{-N})E_K$

# SLOPE CALCULATION

IT IS INTERESTING TO NOTE  
THAT THE FUNCTION  $A/\ln(A)$   
VARIES BY ONLY 6% OVER  
 $2 < A < 4$  ( $6.64 \rightarrow 6.26 \rightarrow 6.64$ )  
WITH ITS MINIMUM AT  $A = e$

# SLOPE CALCULATION

SO, BY CHOOSING TO USE A  
GAIN OF  $A = e$  (2.72, or 8.7dB)  
THE LOG SLOPE WILL EXHIBIT  
*ZERO SENSITIVITY* TO SMALL  
VARIATIONS IN ACTUAL GAIN:  
IT WOULD BE SIMPLY  $6.26E_K$

# INTERCEPT FOR A/0 SYSTEM

USING THE SAME APPROACH AS FOR THE  $A/1$  CASE, IT IS FOUND THAT THE INTERCEPT IS POSITIONED AT

$$V_z = \frac{E_k}{A^N + 1/(A-1)}$$

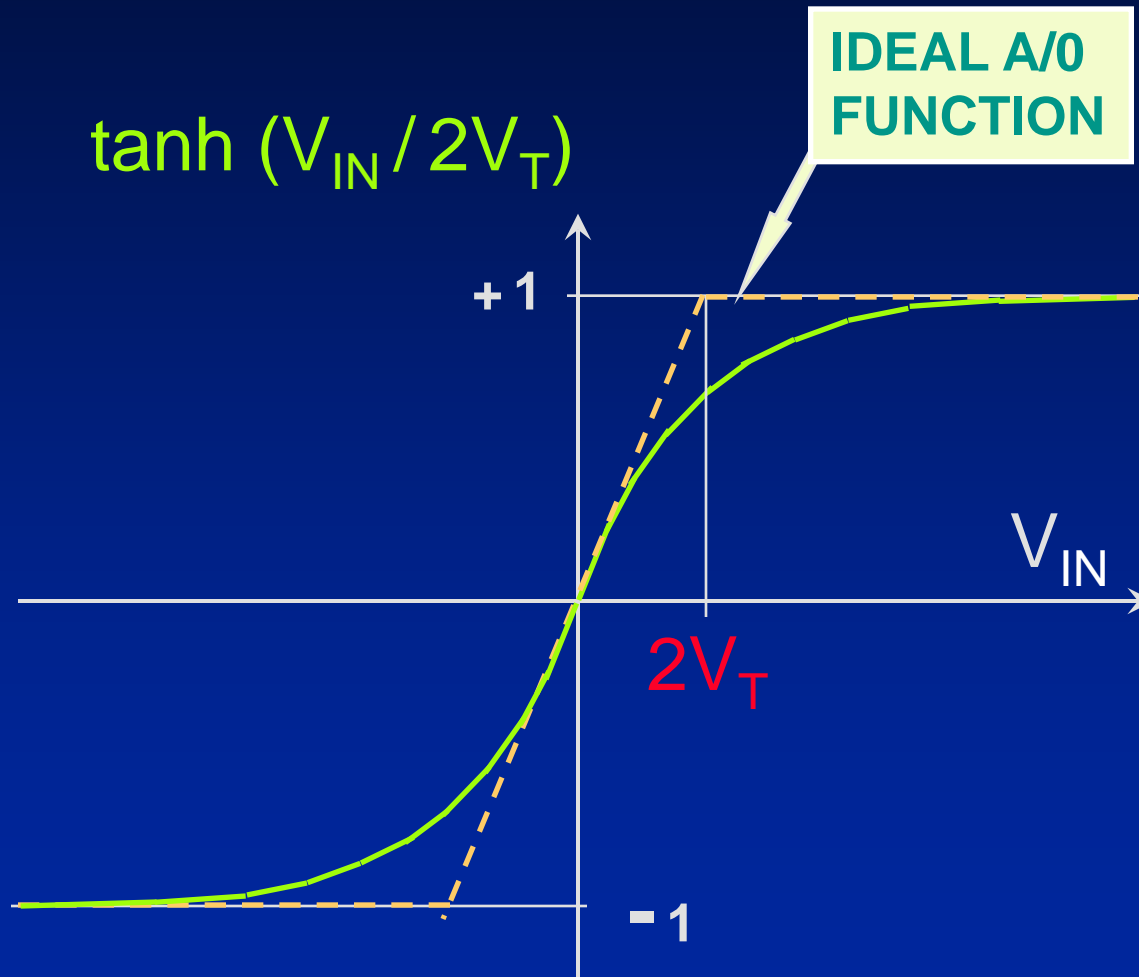
# PRACTICAL REALIZATION

Progressive-compression log amps have been implemented in many different technologies. In the past this included discrete and monolithic BJT and GaAs embodiments.

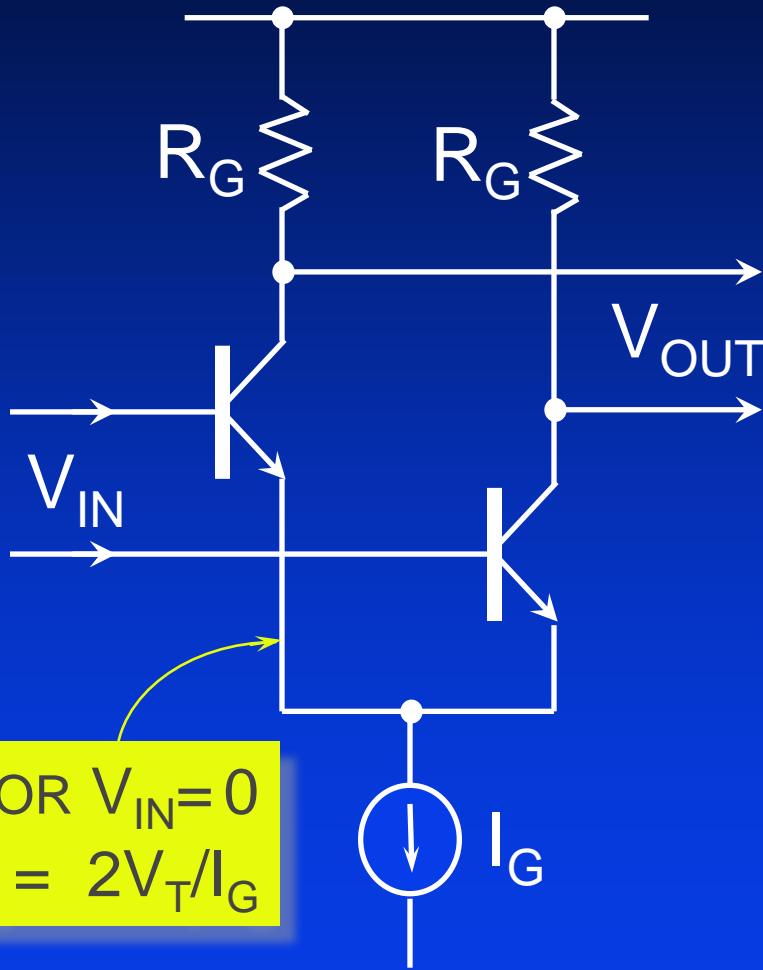
Today, one can use deep sub-micron CMOS to build log amps for operation up to several GHz.

In this presentation, we'll illustrate some design techniques using monolithic bipolar processes.

# tanh as LIMITER



# An A/O AMPLIFIER USING tanh



LARGE-SIGNAL FUNCTION:

$$V_{OUT} = I_G R_G \tanh (V_{IN}/2V_T)$$

SMALL-SIGNAL GAIN ( $V_{IN}=0$ )

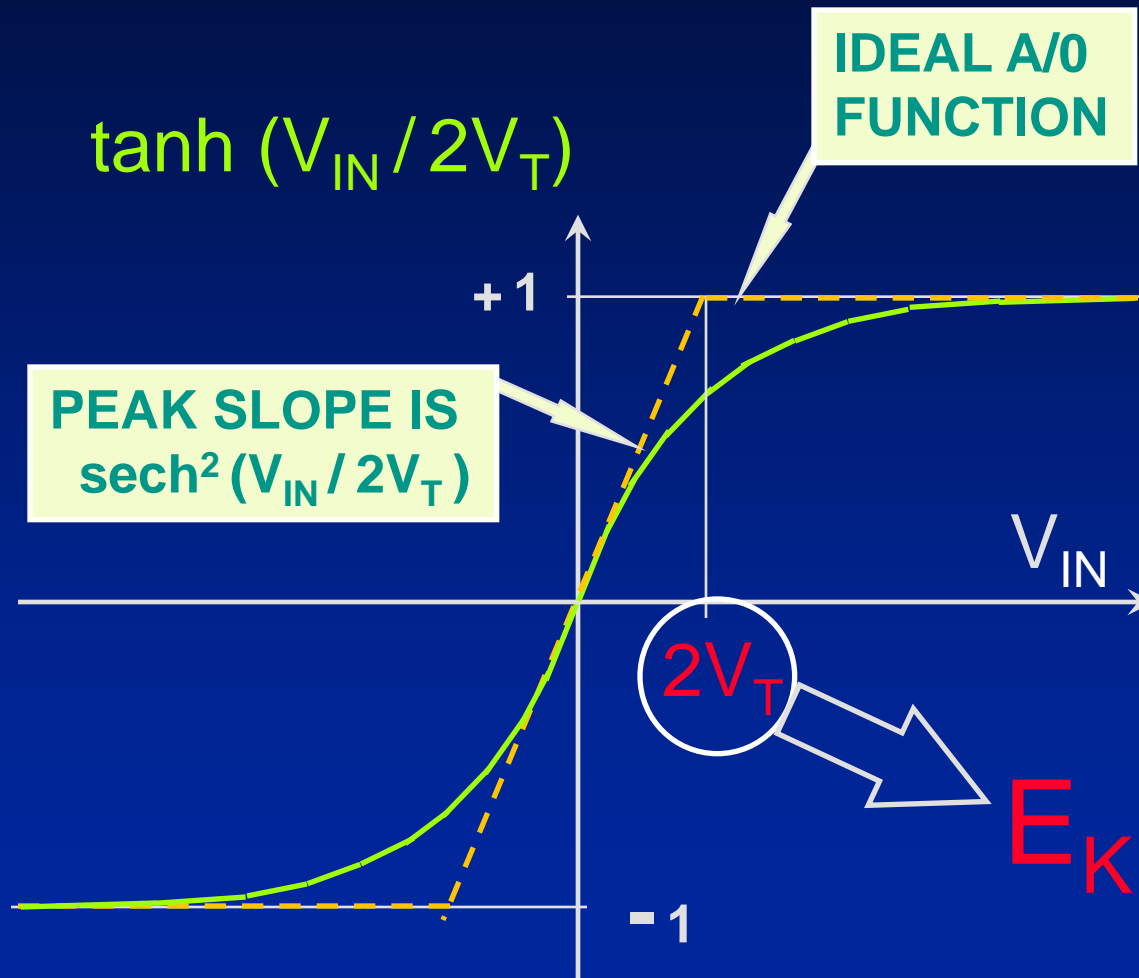
$$A_0 = R_L/r_e = I_G R_G/2V_T$$

INCREMENTAL GAIN

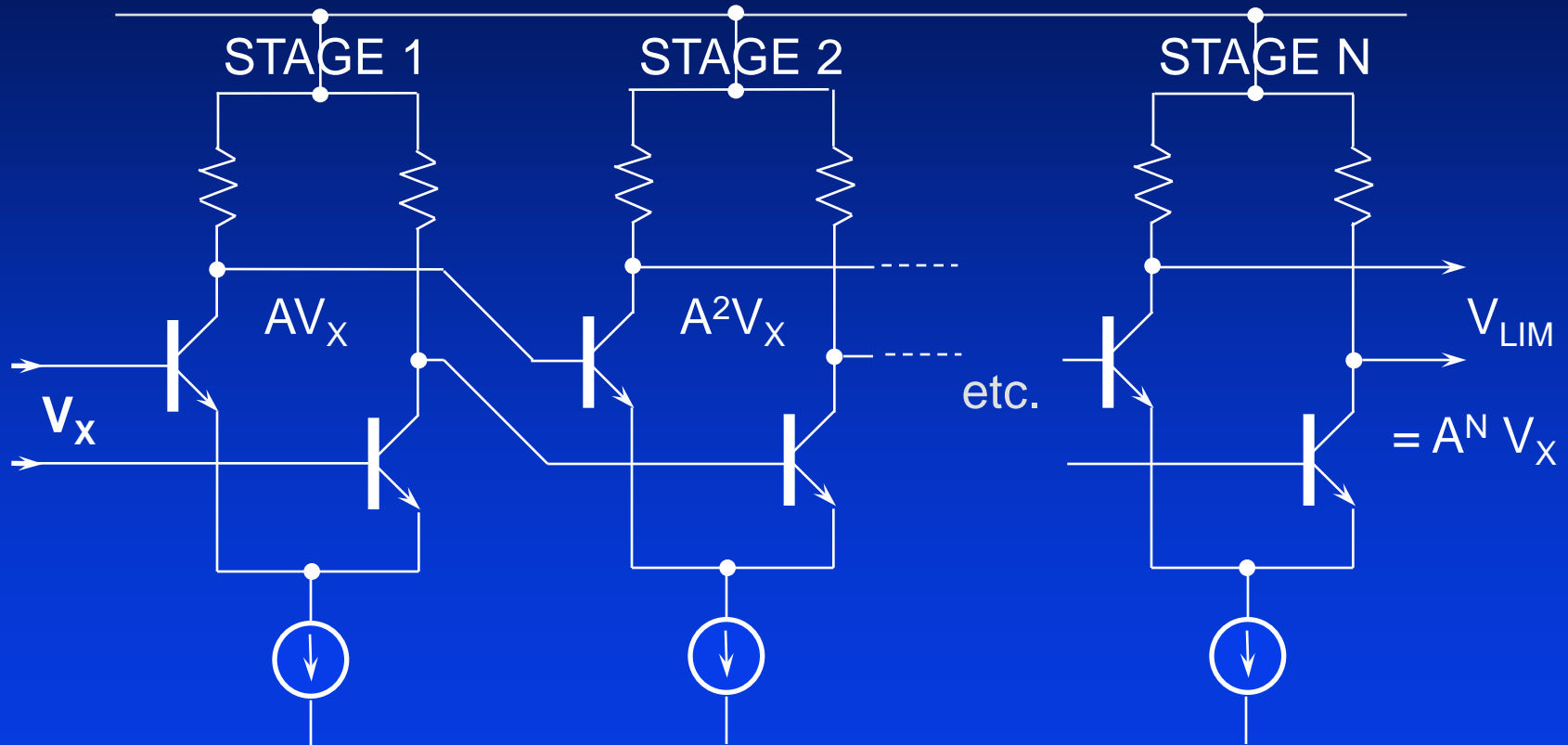
$$\partial V_{OUT} / \partial V_{IN} = A_0 \operatorname{sech}^2 (V_{IN}/2V_T)$$

where  $V_T = kT/q$

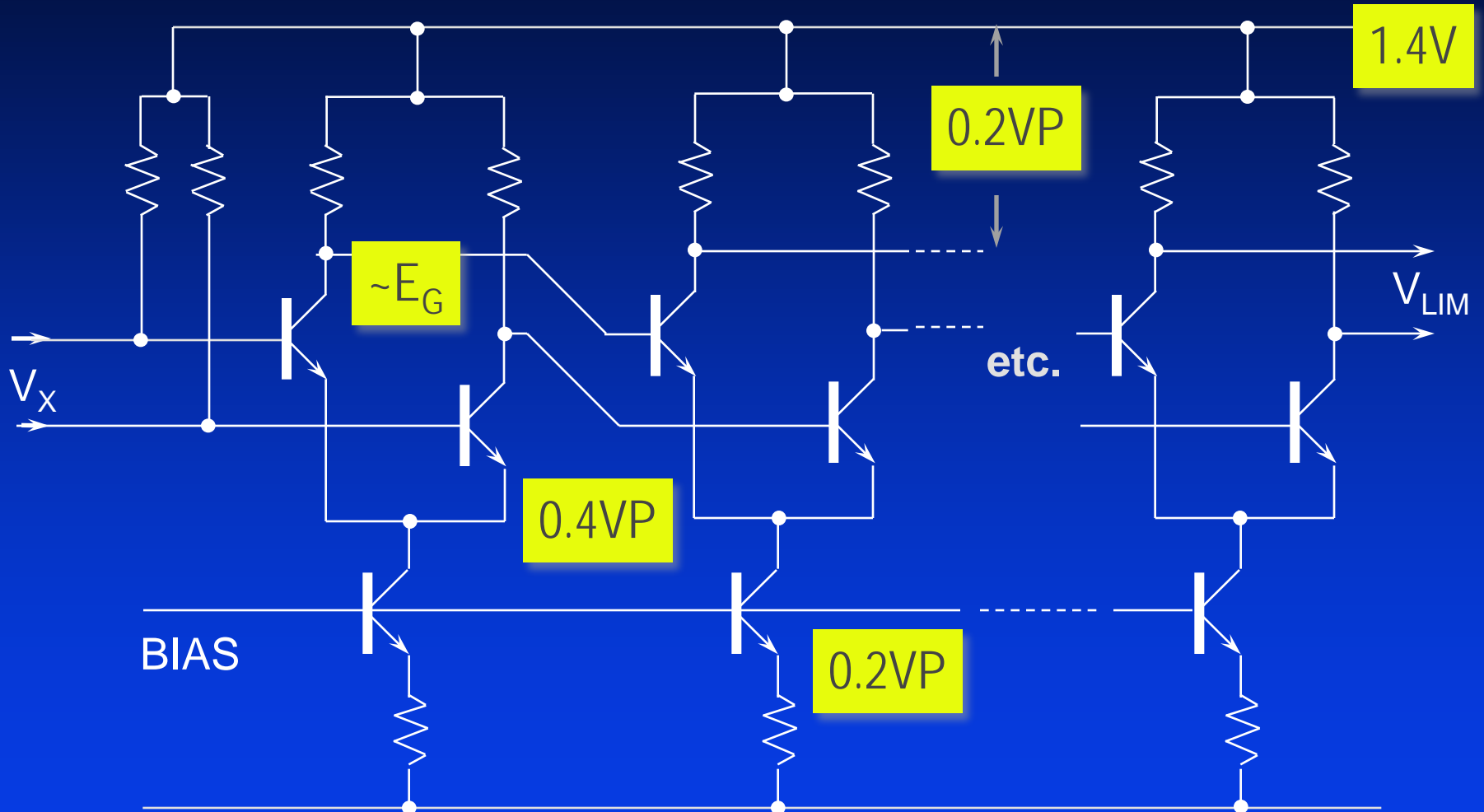
# tanh as LIMITER



# CASCADE OF tanh AMPLIFIER CELLS



# LOW SUPPLY VOLTAGE



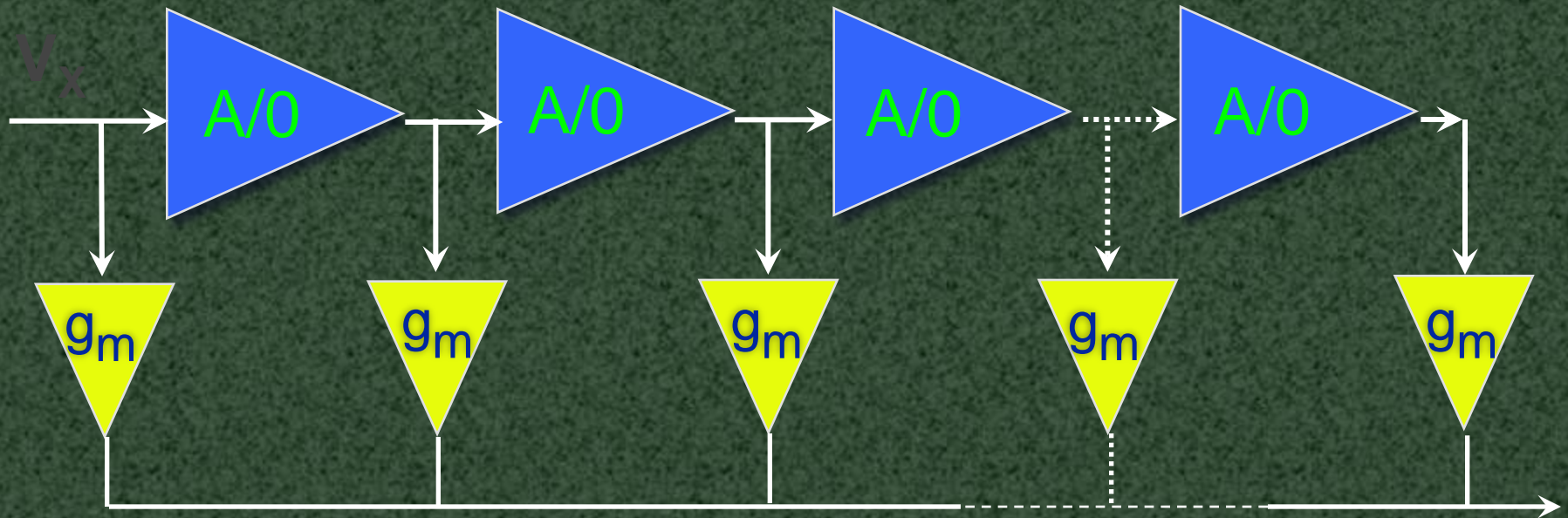
# BUT AS ALWAYS IN PRACTICE, THE DEVIL'S IN THE DETAILS....

.... SCORES OF THEM ....  
SUCH AS THE EFFECTS OF  
FINITE DC AND AC BETA;  
CHOICE OF "T-SHAPES";  
OHMIC RESISTANCES;  
FINITE INERTIA; etc.



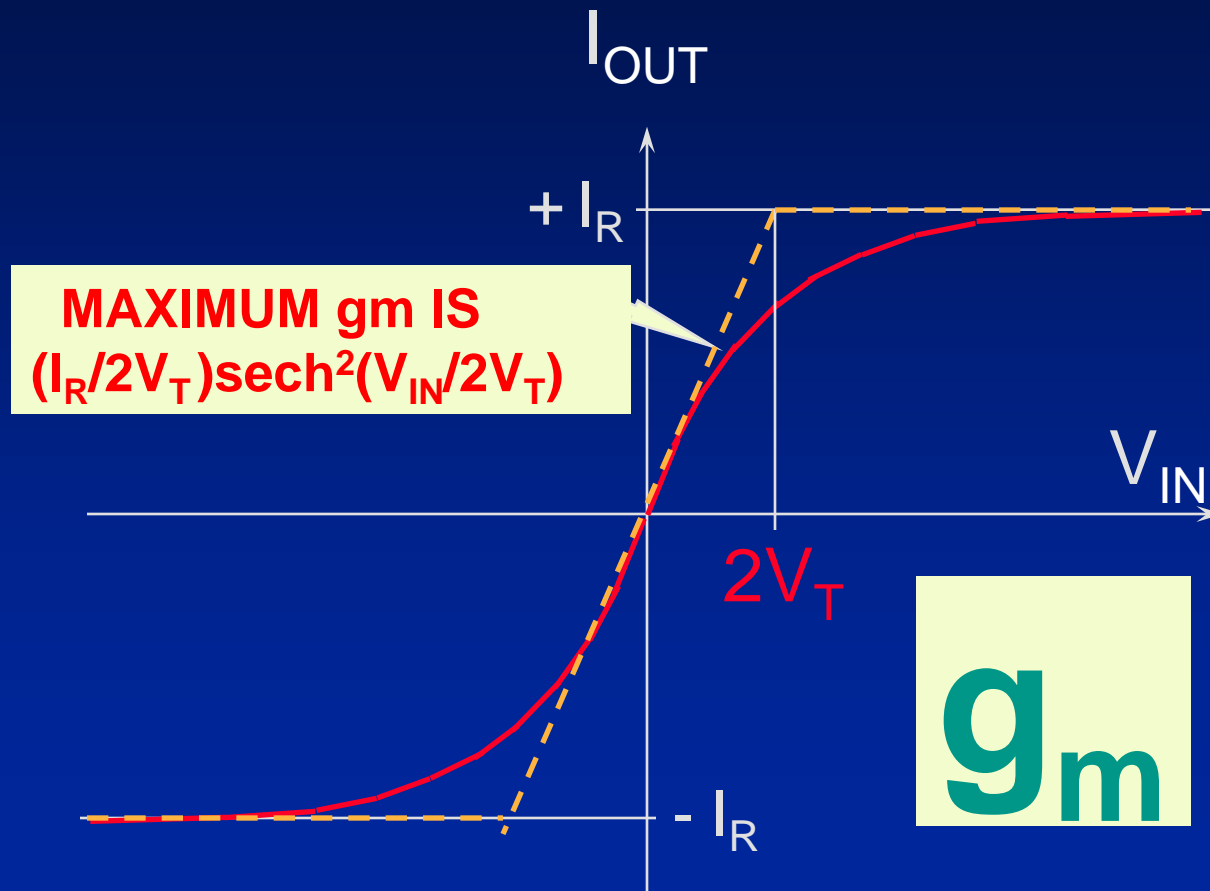
# AUXILIARY $g_m$ CELLS SUM IN CURRENT-MODE

STAGE 1      STAGE 2      STAGE 3 . . . . . STAGE N

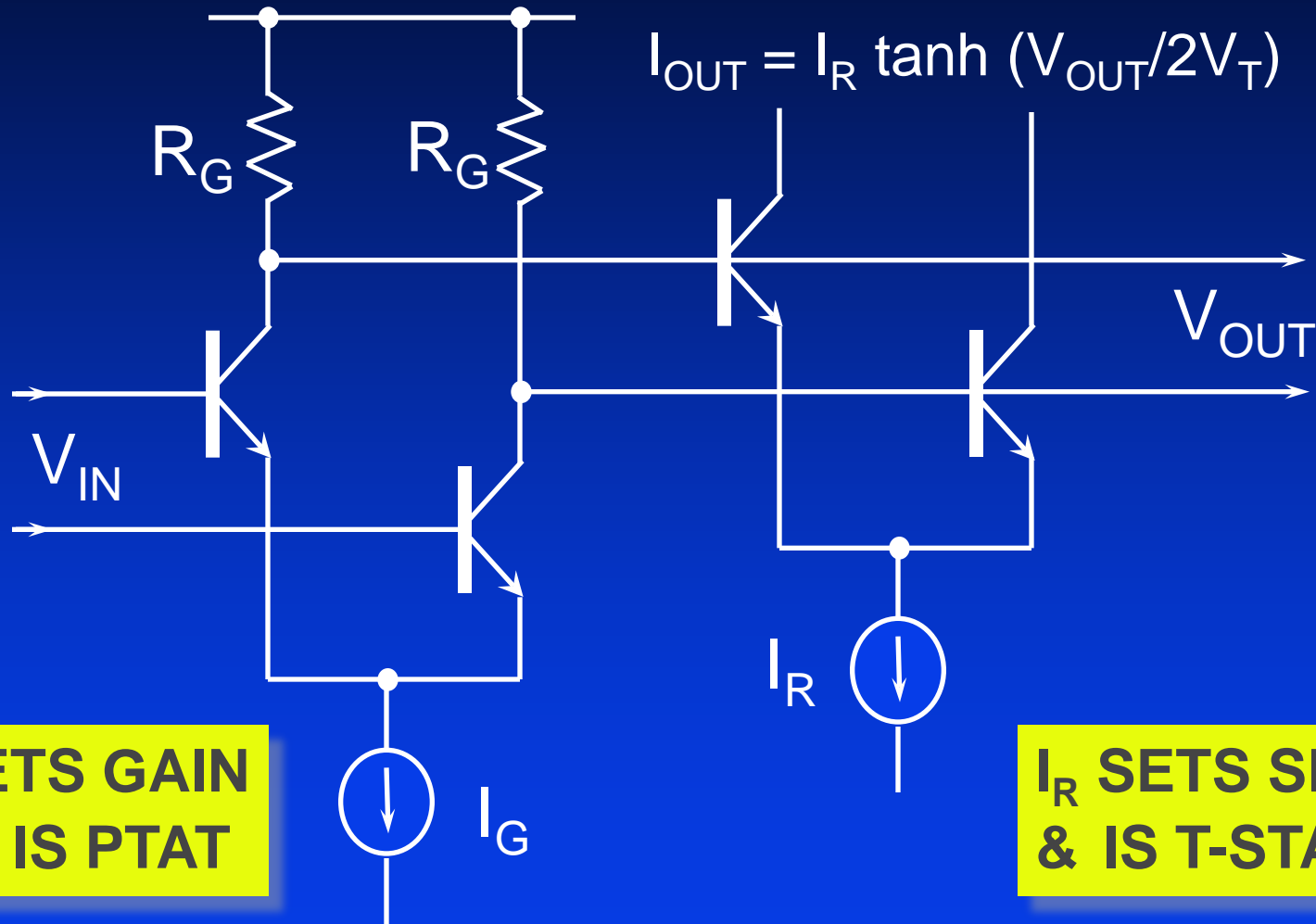


USEFUL SECONDARY FUNCTION:  $g_m$  CELLS ISOLATE GAIN STAGES

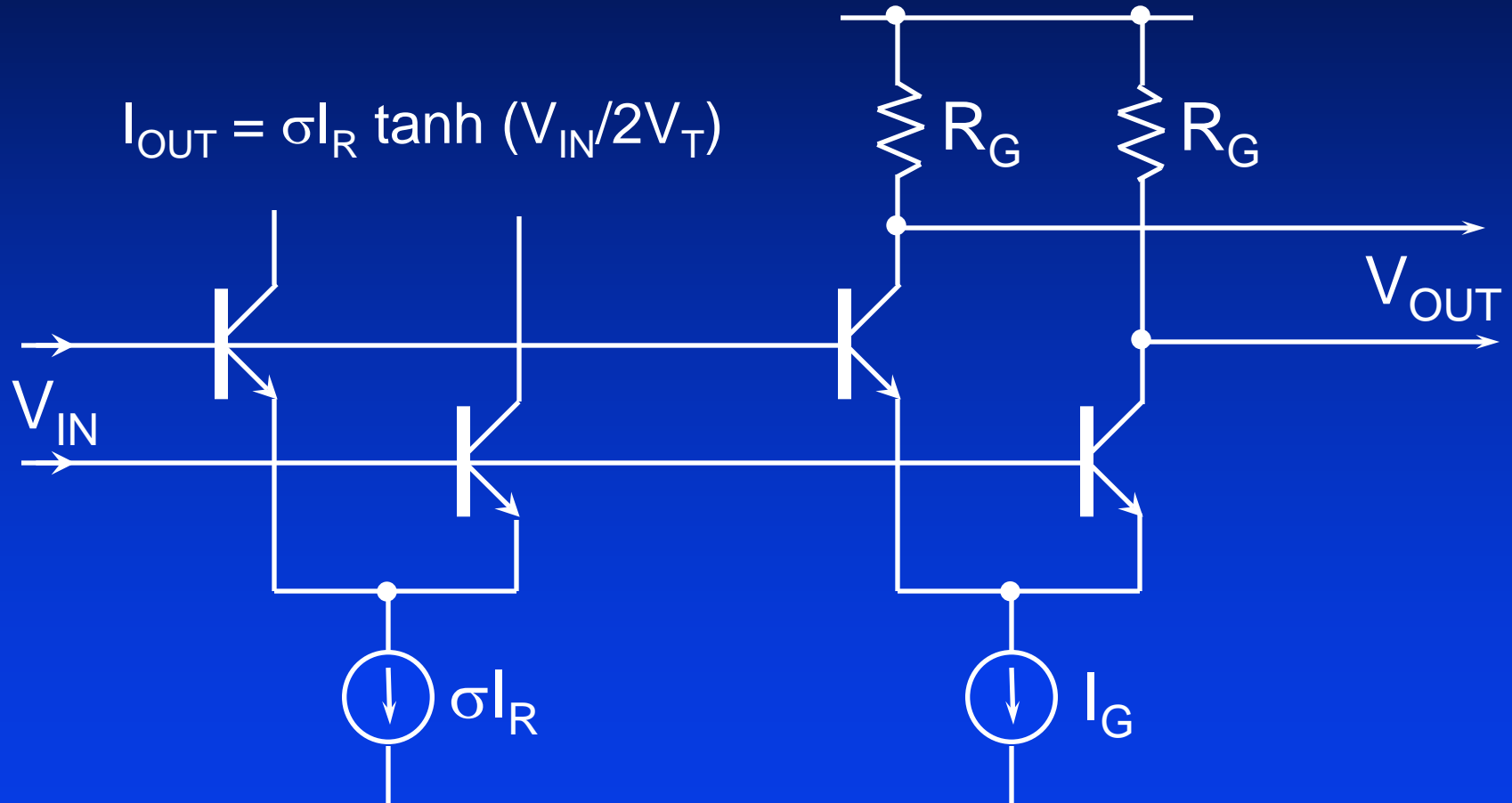
# AUXILIARY gm CELLS: STILL USE THE A/O IDEA



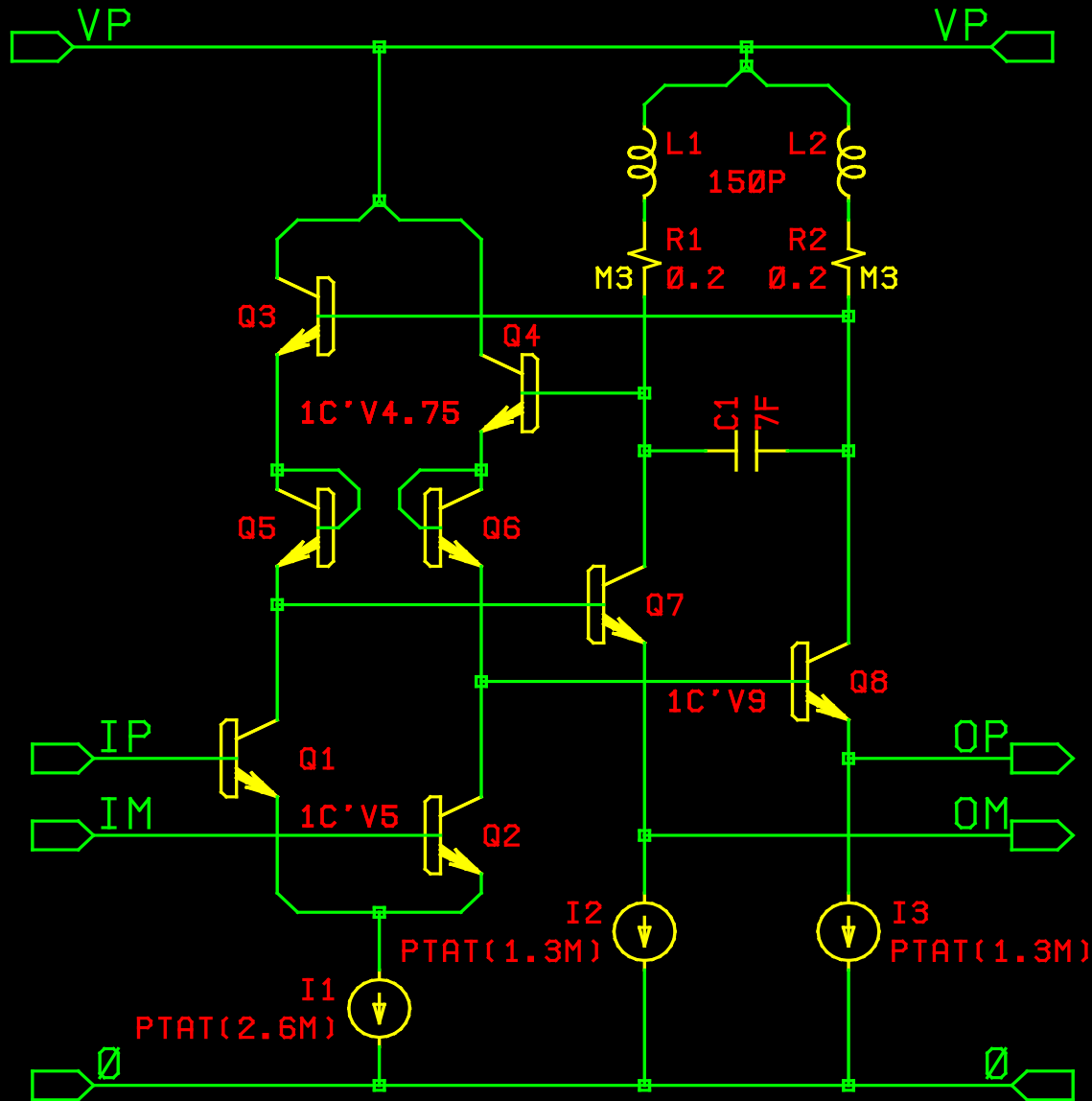
# AUXILIARY $g_m$ CELLS



# DIFFERENT FIRST STAGE

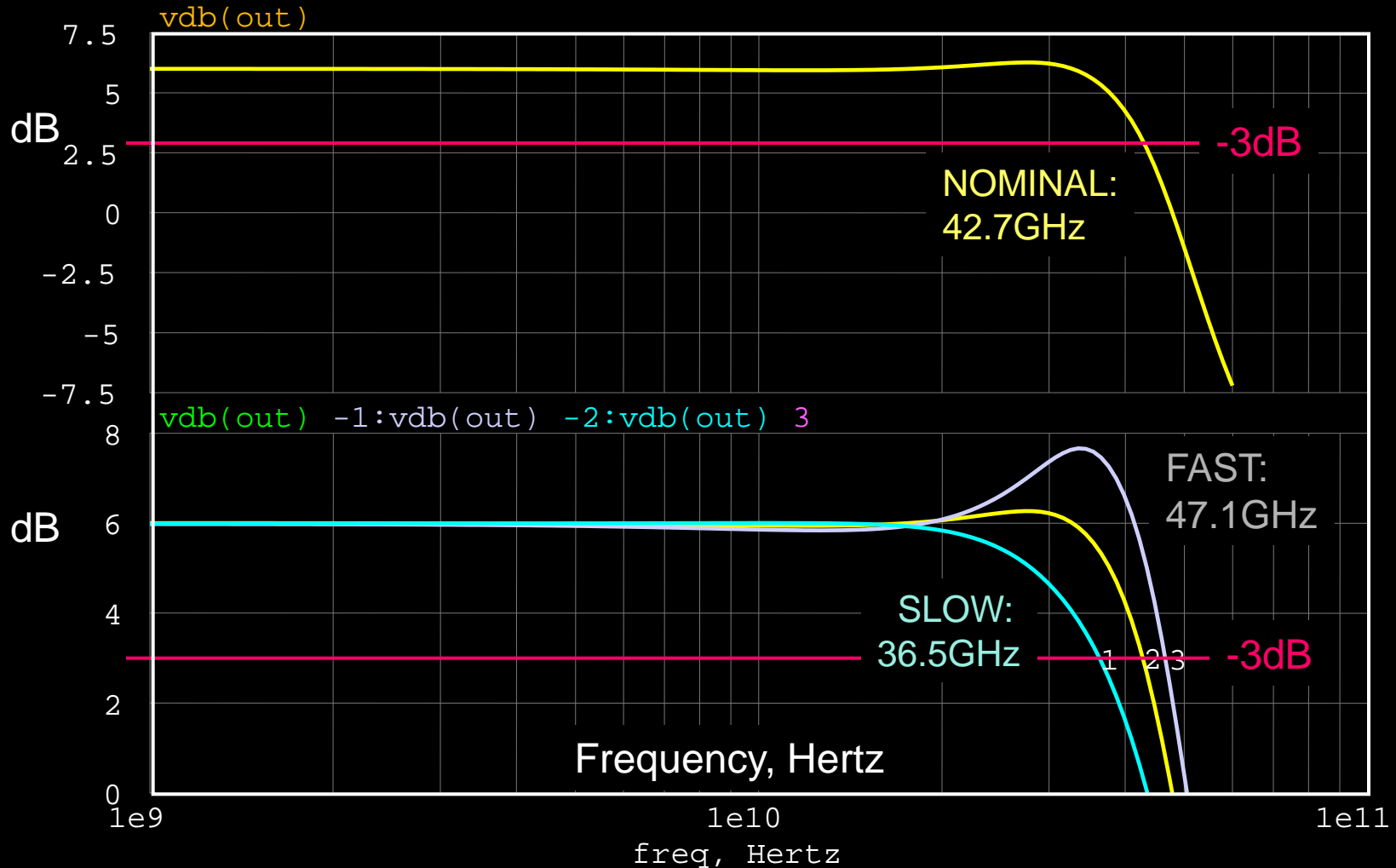


# A TRANSLINEAR AMPLIFIER

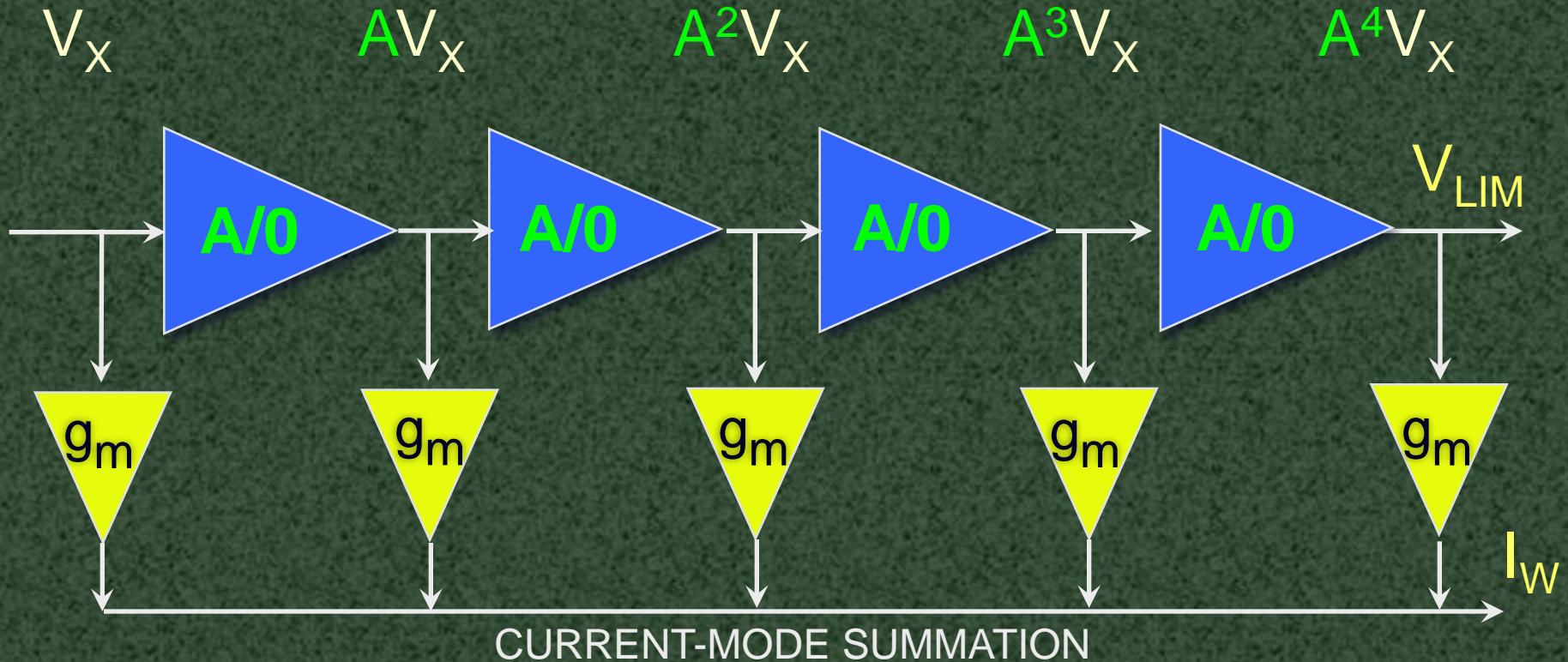


# TEN CASCADED STAGES, 150pH INDUCTORS

CASCADES: VCS=2.28V, VCM=3.3V, VPS = 4.45V; EF's are 5um; Uppers = 4.75um  
-cascade of ten stages with inductors included. f<sub>3dB</sub> for the single stage  
is 42.7GHz, nominal, 47.1GHz FAST and 36.5GHz SLOW, sigma=3

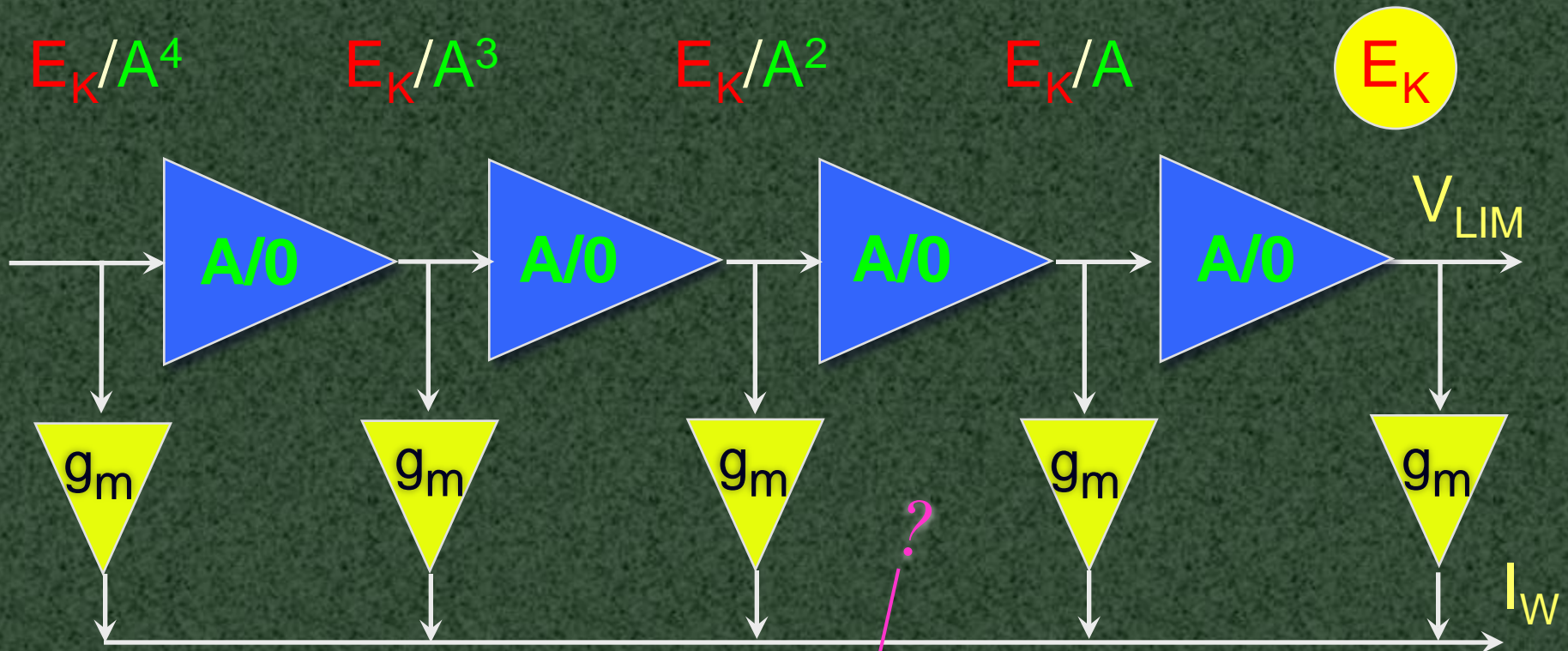


# A FOUR-STAGE EXAMPLE



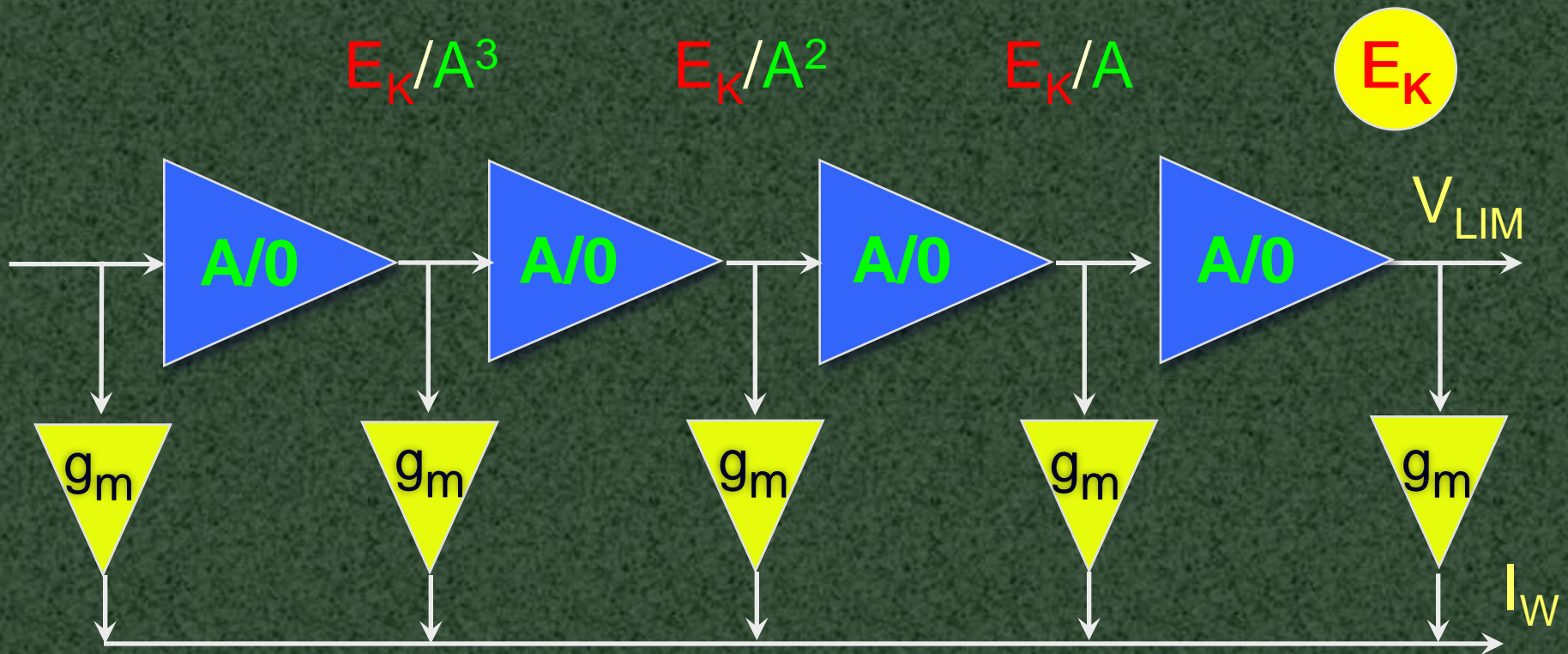
FOR SMALL INPUTS, SYSTEM IS A LINEAR AMPLIFIER

# A FOUR-STAGE EXAMPLE



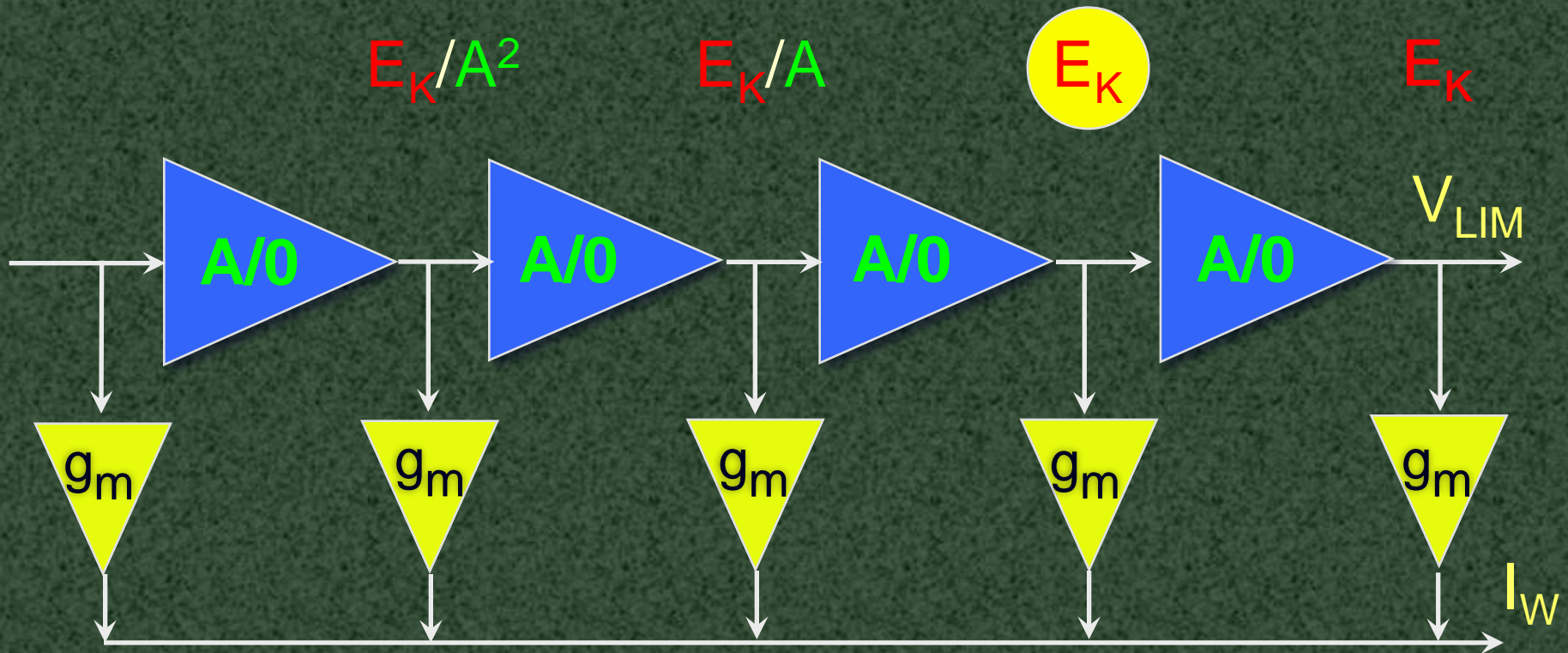
THE FIRST TRANSITION NOW OCCURS WHEN THE INPUT TO THE FINAL  $g_m$  CELL REACHES  $E_K$   
 AT THIS POINT THE INPUT IS  $E_K/A^4$  AND THE  
 OUTPUT CURRENT  $I_W = \lambda I_R (1 + A^{-1} + A^{-2} + A^{-3} + A^{-4})$

# A FOUR-STAGE EXAMPLE



WHEN THE INPUT IS A TIMES HIGHER, AT  $E_K/A^3$ ,  
THE OUTPUT IS  $I_W = I_R(2 + A^{-1} + A^{-2} + A^{-3})$

# A FOUR-STAGE EXAMPLE



WHEN THE INPUT IS A TIMES HIGHER, AT  $E_K/A^2$ ,  
 THE OUTPUT IS  $I_W = I_R(3 + A^{-1} + A^{-2} + A^{-3})$  etc.  
 etc.

# DETERMINATION OF SLOPE

## THE OUTPUT CHANGES

from  $I_W = I_R (1 + A^{-1} + A^{-2} + \dots A^{-N})$

for an input of  $V_X = E_K/A^{-N}$ ,

to  $I_W = I_R (2 + A^{-1} + A^{-2} + \dots A^{-(N-1)})$

for an input of  $V_X = E_K/A^{-(N-1)}$ .

The output changes by  $I_R(1 - A^{-N}) \cong I_R$

as the INPUT changes by the RATIO A

# DETERMINATION OF SLOPE

WE NEED NOT PROCEED FURTHER: THE LOG FUNCTION DEVELOPS IN A SIMILAR FASHION TO THAT DETERMINED EARLIER FOR THE MORE IDEALIZED CASE.

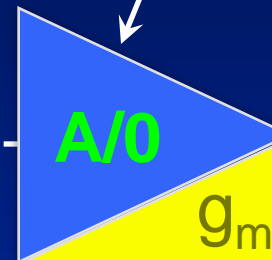
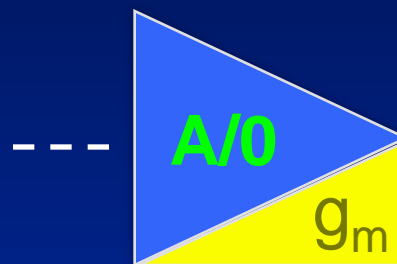
THE LOG SLOPE – NOW IN CURRENT FORM – IS SIMPLY

$$I_W = \frac{I_R}{\lg t (A)}$$

WHICH IS FULLY DECOUPLED FROM  $E_K$

# ELIMINATION OF PTAT $E_K$

SPECIALIZED PTAT CELL  
DETERMINES THE GAIN  $A$



0 to  $\pm E_K$

$I_R$

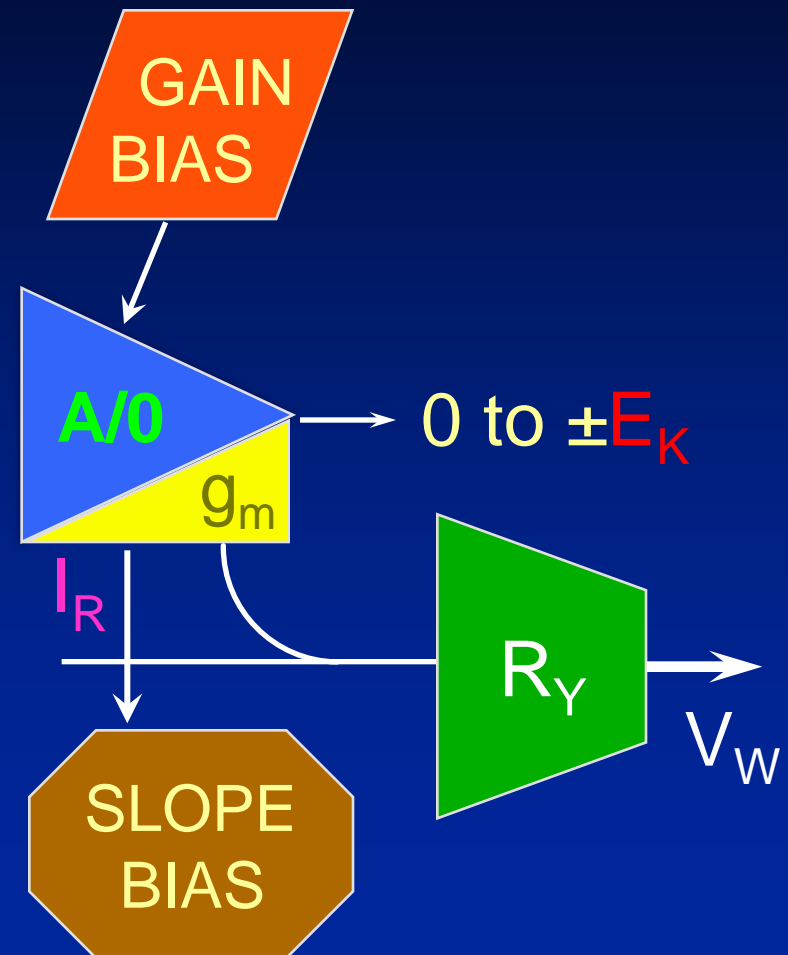
0 to  $\pm I_R$

VOLTAGE REFERENCE  
SETS ACCURATE BIAS  $I_R$



# ELIMINATION OF PTAT $E_K$

- OUTPUT CURRENT OF THE  $g_m$  CELL CHANGES BY  $I_R$  FOR A CHANGE IN RATIO OF  $A$  AT THE AMPLIFIER'S INPUT
- CONVERTED TO A VOLTAGE USING A TRANSRESISTANCE OUTPUT STAGE, OF VALUE  $R_Y$
- THUS, THE CHANGE IN  $V_W$  IS  $I_R R_Y$  FOR EVERY FRACTION  $\ln(A)$  AT THE MAIN INPUT

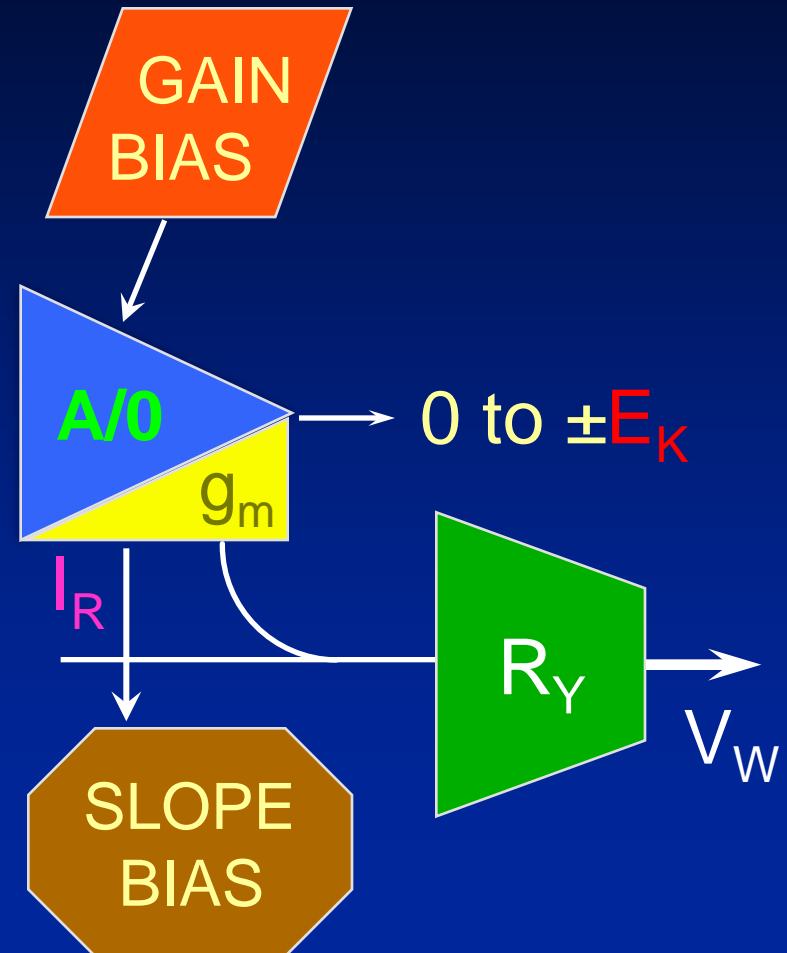


# ELIMINATION OF PTAT $E_K$

- THE NEW SLOPE IS

$$V_Y = \frac{I_R R_Y}{\lg t(A)}$$

- THE DEPENDENCE ON A PTAT  $E_K$  HAS THUS BEEN ELIMINATED

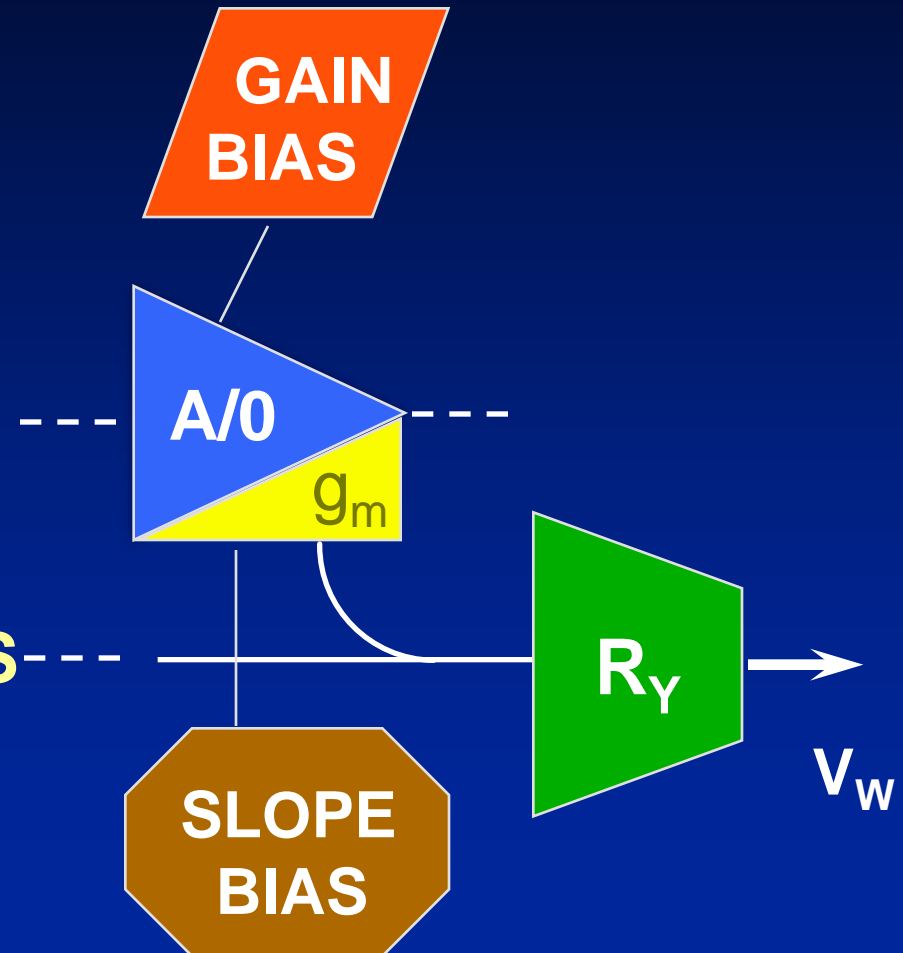


# ELIMINATION OF PTAT $E_K$

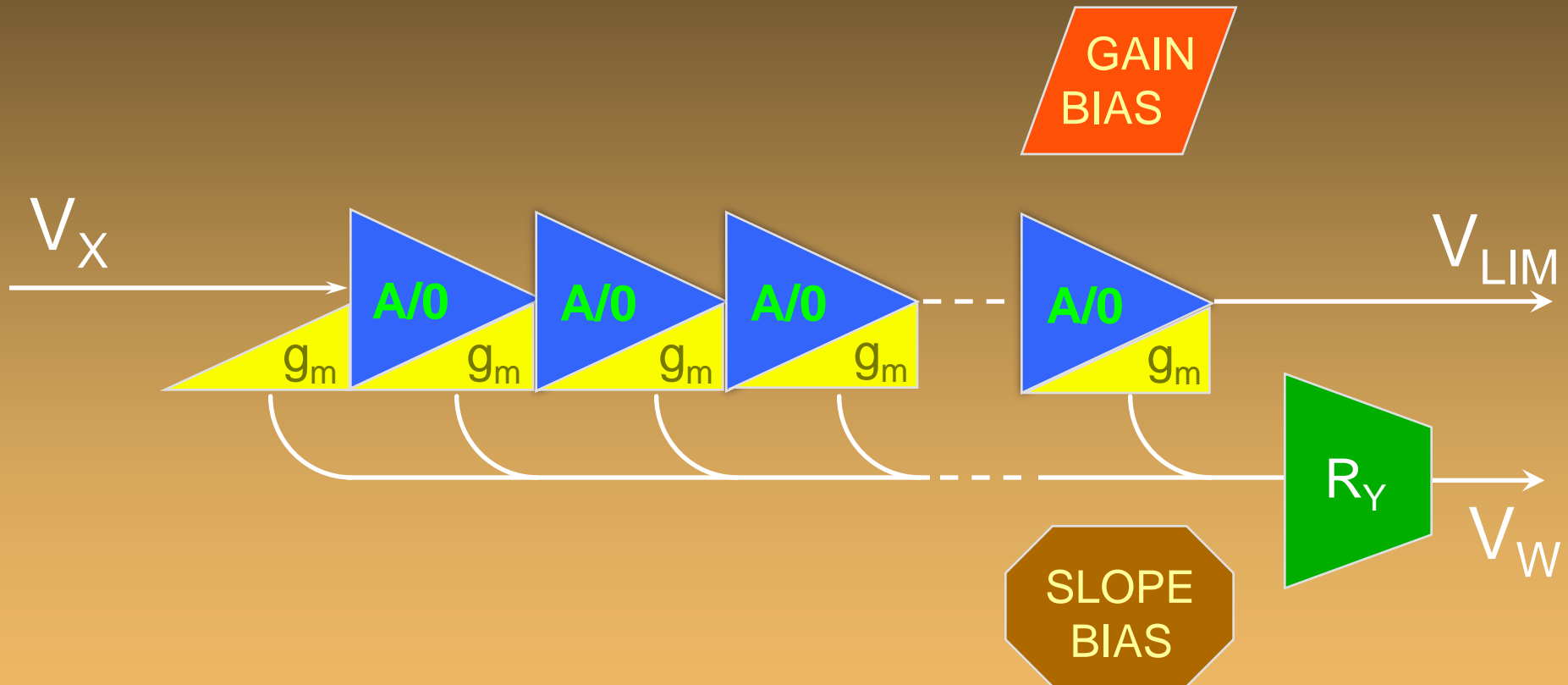
- THE NEW SLOPE IS

$$V_Y = \frac{I_R R_Y}{\lg t(A)}$$

- THE DEPENDENCE ON THE PTAT  $E_K$  HAS THUS BEEN ELIMINATED



# COMPLETE A/O LOG-AMP



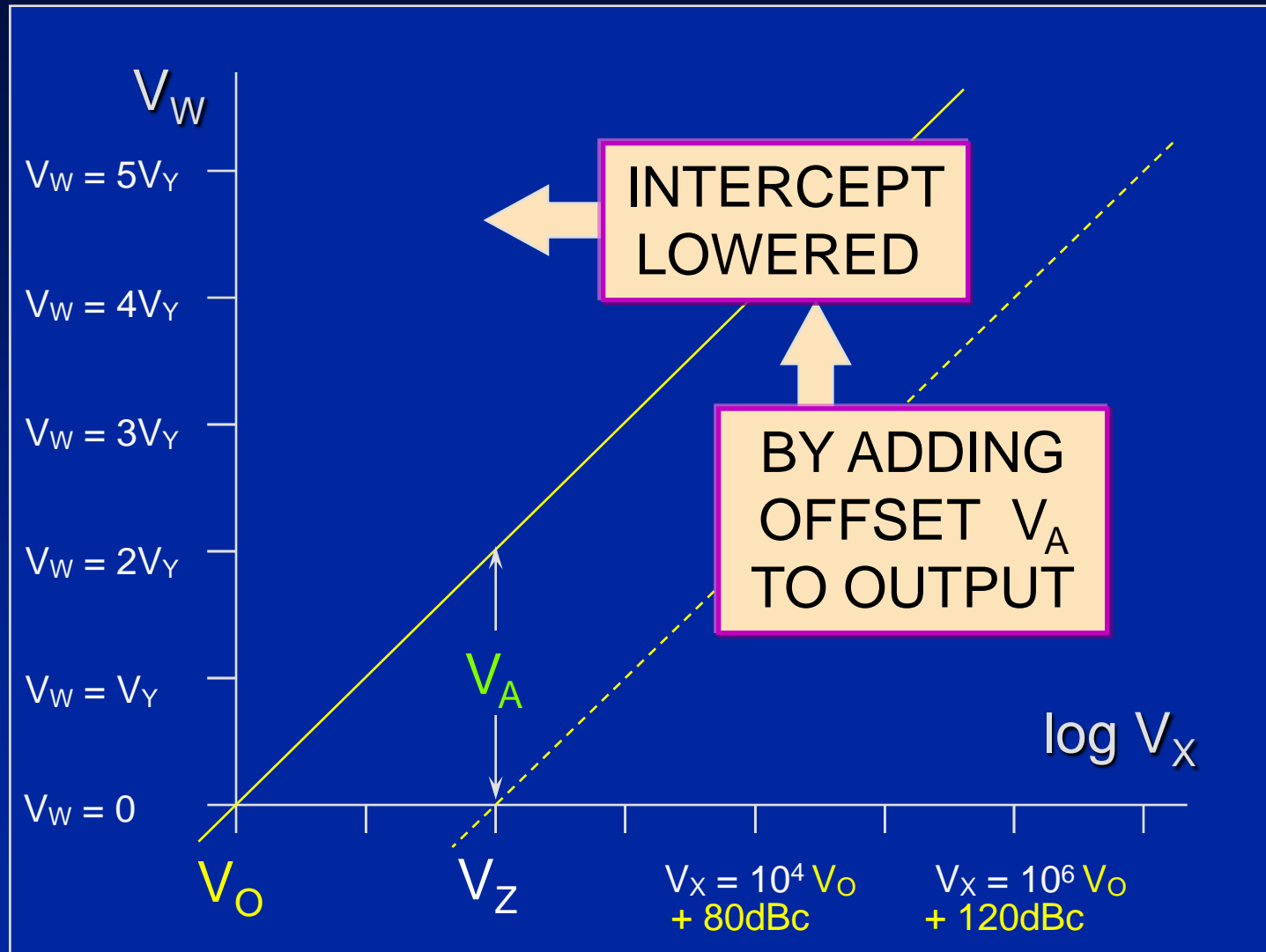
# HOWEVER, THE INTERCEPT IS STILL DEPENDENT ON $E_K$

$$V_Z = \frac{E_K}{A^N + 1/(A-1)}$$

... and  $E_K$  in the 'natural' system comprising the log amp backbone is inherently PTAT.

The next few steps demonstrate how this can readily be compensated, to provide a stable intercept versus temperature.

# INTERCEPT MANIPULATION

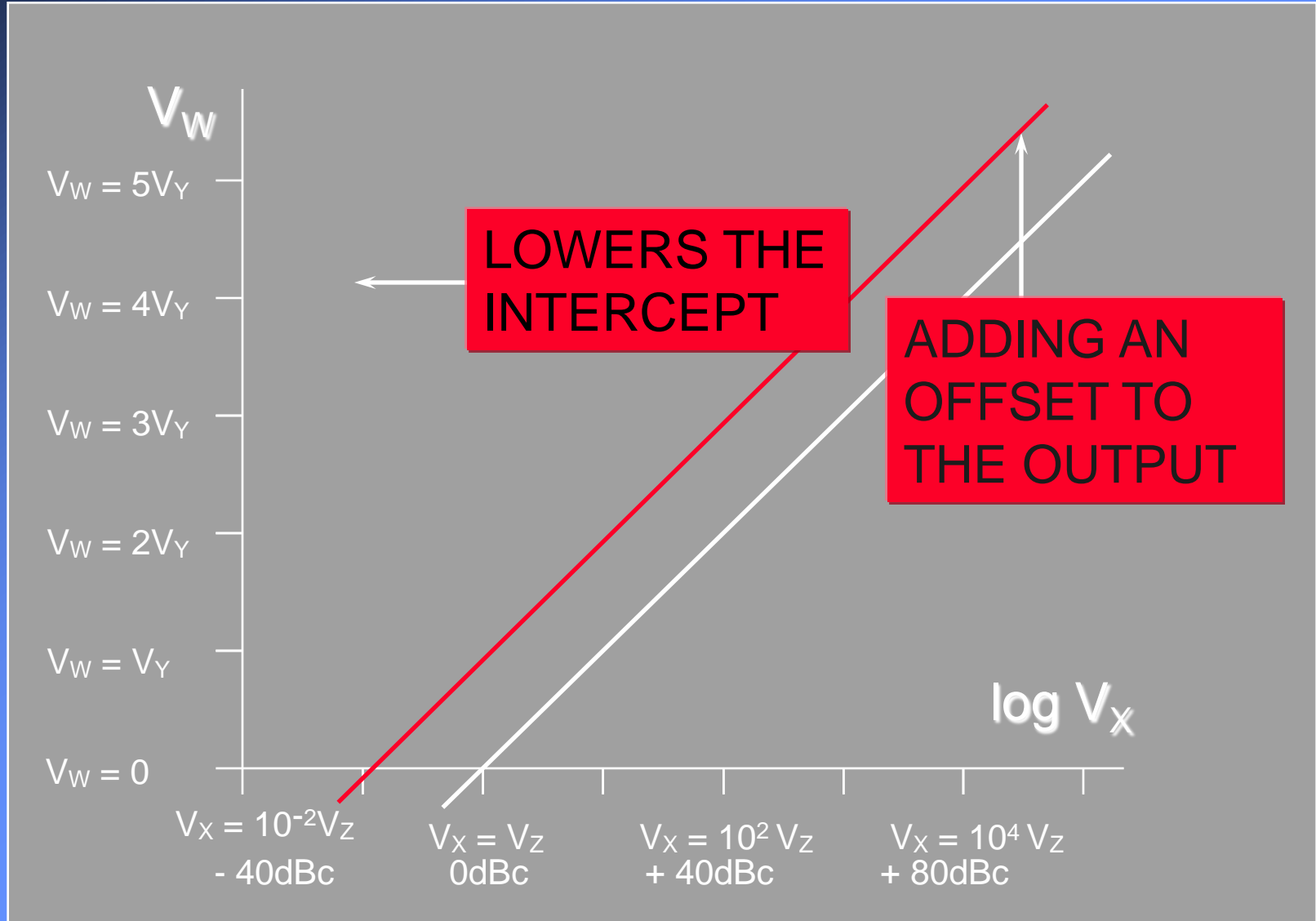


# INTERCEPT MANIPULATION

$$\begin{aligned}V_W &= V_Y \log (V_X/V_Z) \\&= V_Y \log (V_X V_O / V_Z V_O) \\&= V_Y \log (V_X/V_O) + V_A\end{aligned}$$

where  $V_A = V_Y \log (V_O / V_Z)$

# INTERCEPT POSITION IS EASILY ALTERED

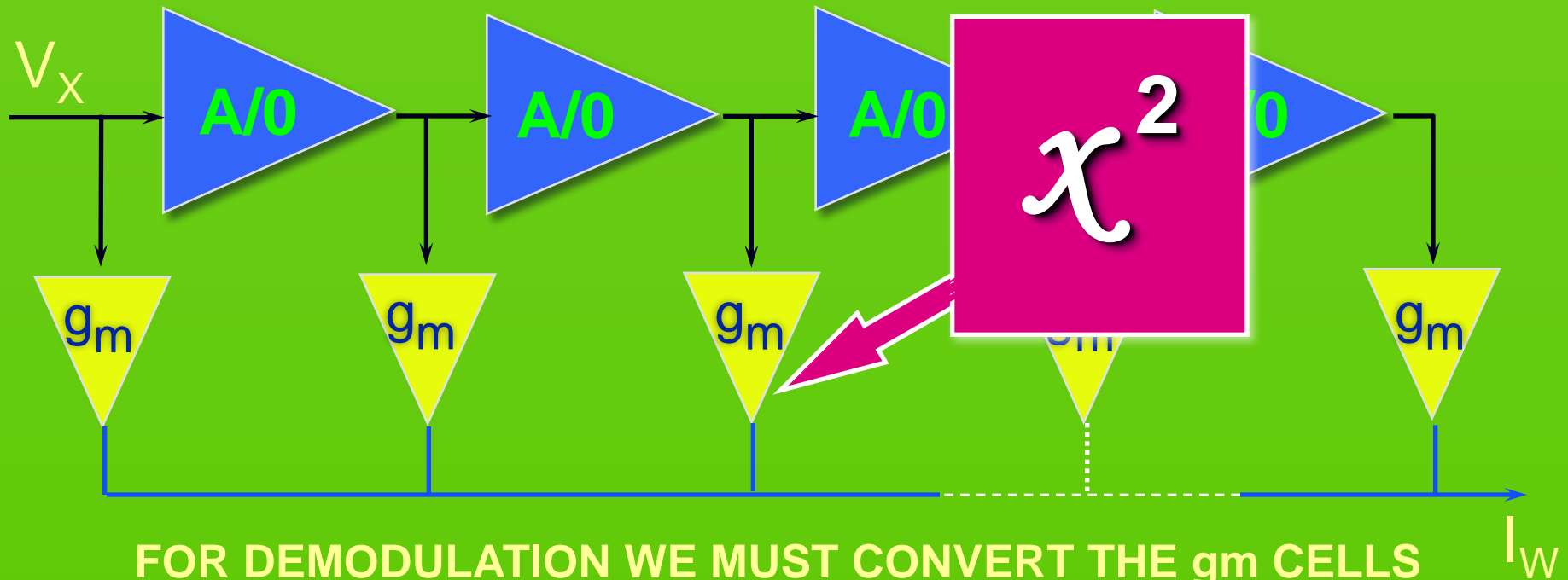


# AUXILIARY $g_m$ CELLS SUM IN CURRENT-MODE

STAGE 1

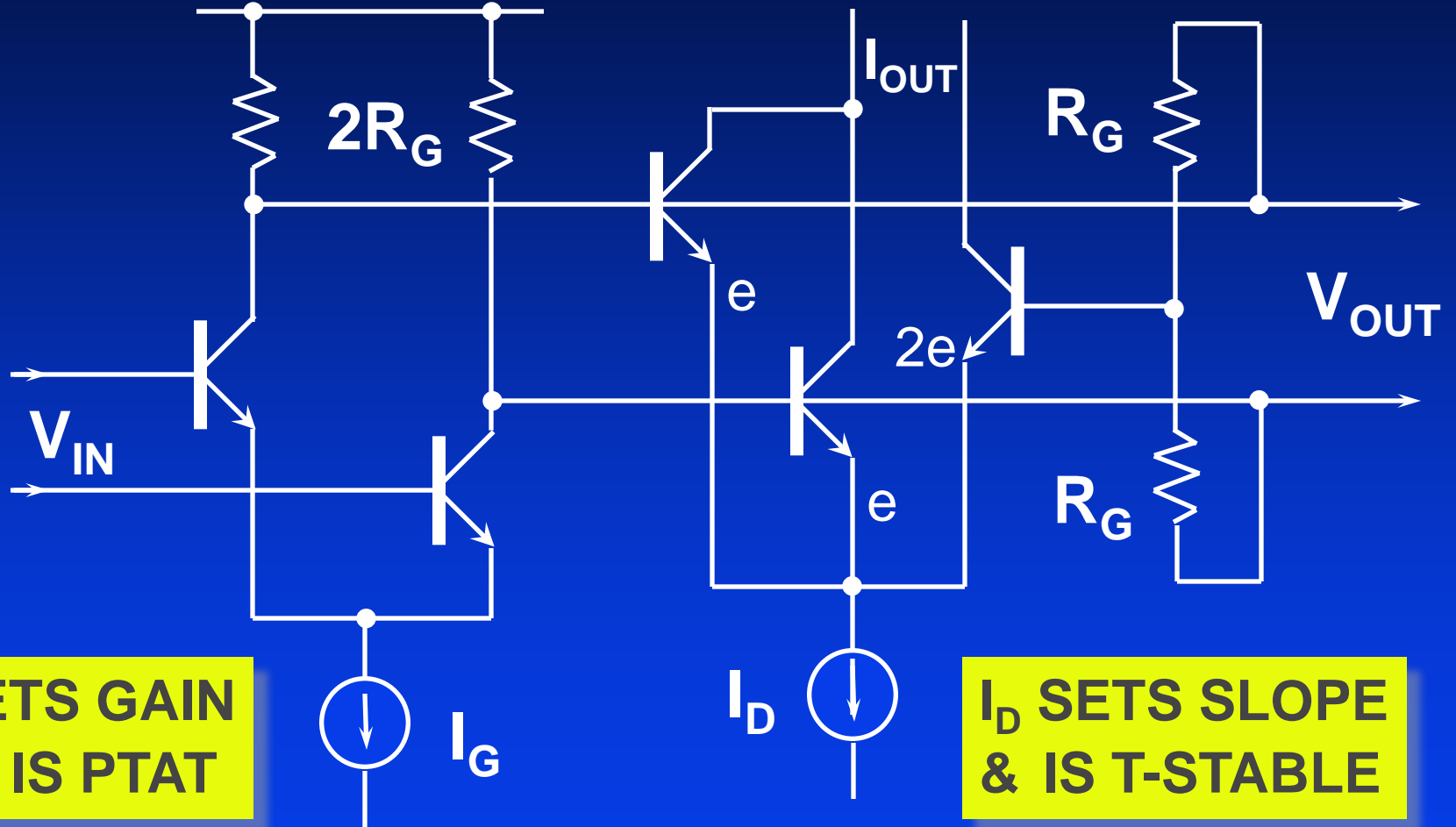
STAGE 2

STAGE 3 . . . . . STAGE N

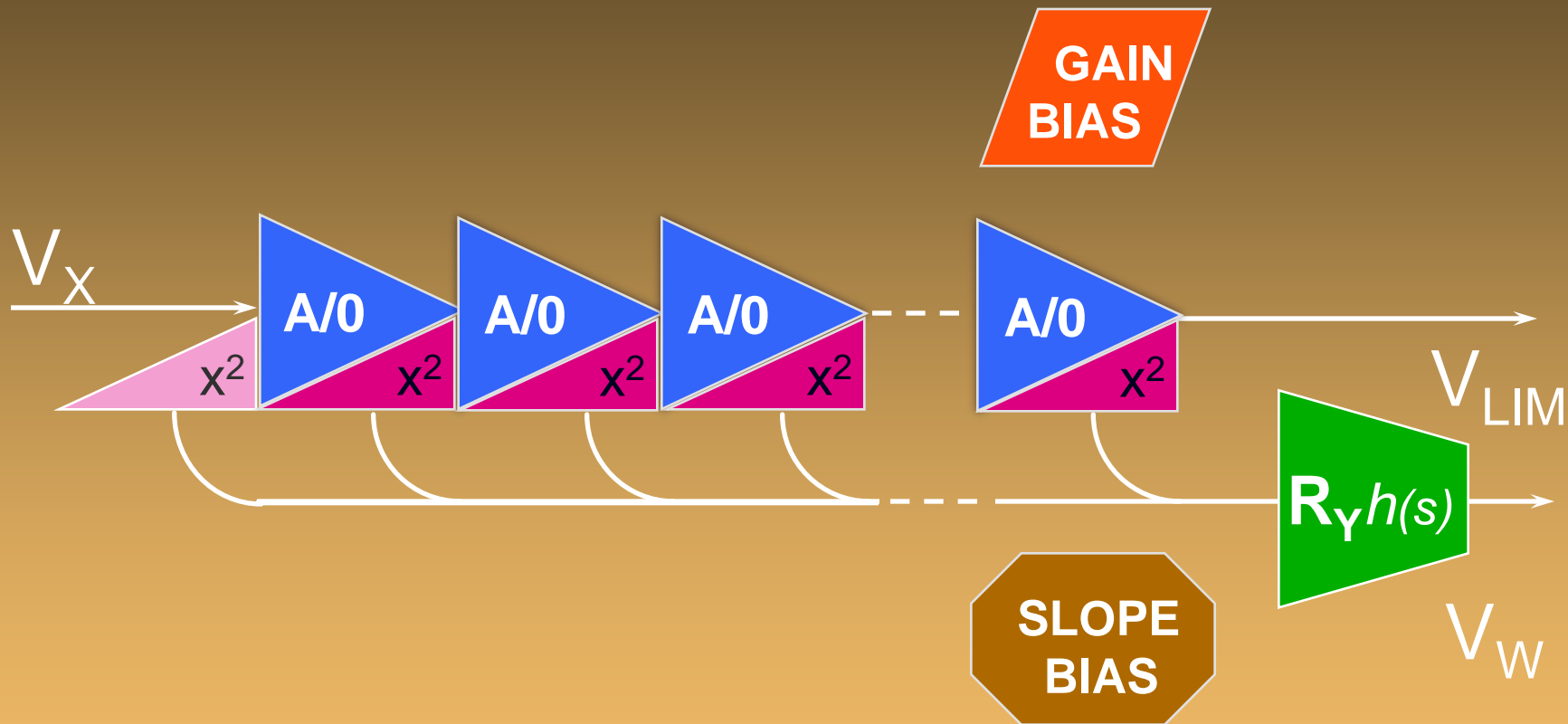


FOR DEMODULATION WE MUST CONVERT THE  $g_m$  CELLS  
INTO RECTIFIERS (DETECTORS) PREFERABLY FULL-WAVE

# SQUARE-LAW DETECTOR



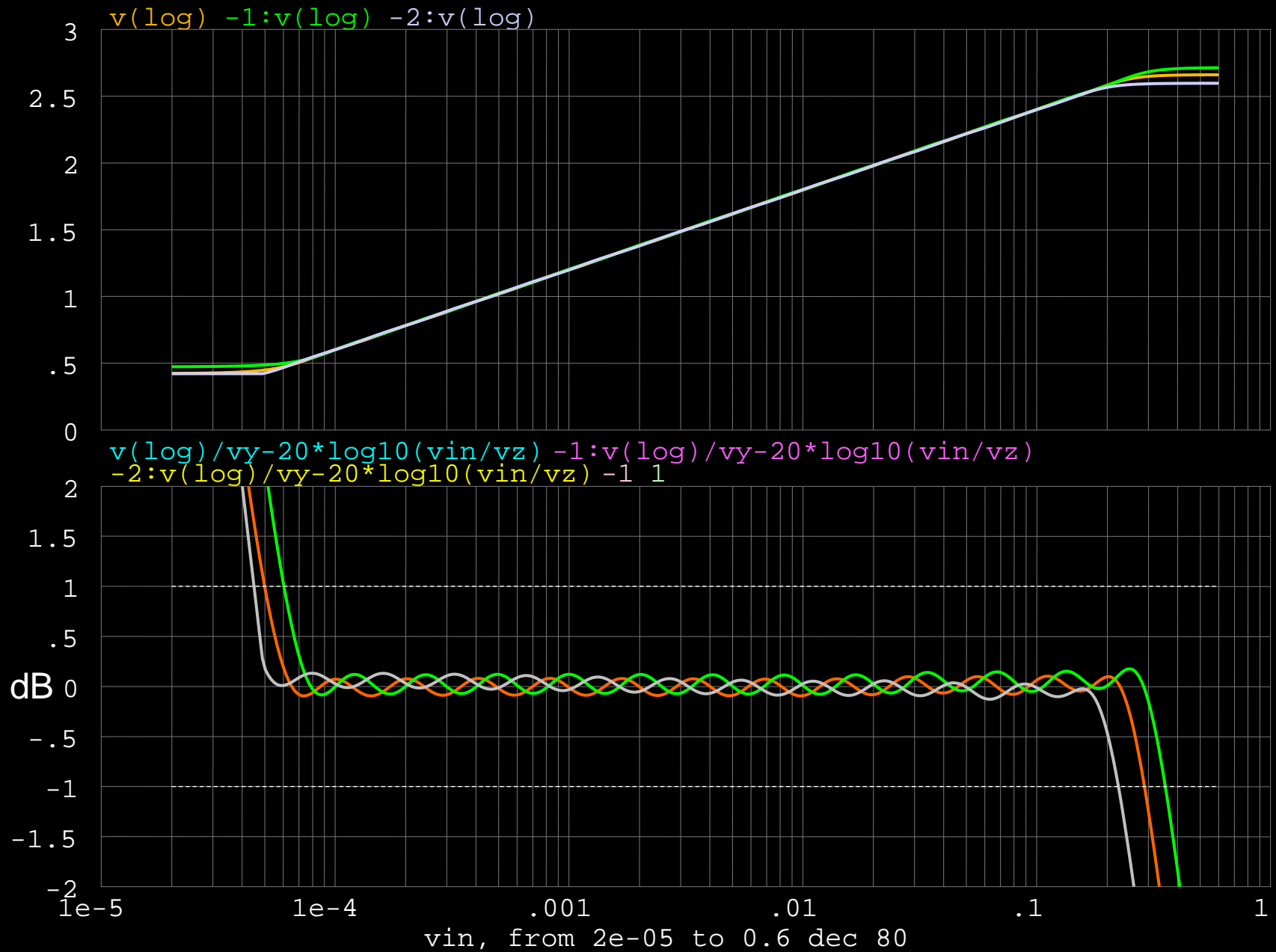
# DEMODULATING LOG-AMP



THE OUTPUT I-V CONVERTER IS NOW ALSO A FILTER

# APPROXIMATION RIPPLE

- THE LOG CONFORMANCE ERROR WHEN USING  $\tanh$  GAIN CELLS NOW TAKES ON AN ESSENTIALLY SINUSOIDAL FORM. FOR  $A = 4$  THE RIPPLE IS CLOSE TO  $\pm 0.2\text{dB}$ .
- SMALL ADJUSTMENTS ARE MADE TO THE WEIGHTING OF THE  $g_m$  CELLS TO IMPROVE ACCURACY OF THE LOGARITHMIC FIT



# DYNAMIC RANGE ISSUES

RECALL THAT A BASIC REQUIREMENT OF A LOG-AMP IS THAT IT MUST HAVE VERY ZERO-SIGNAL GAIN, TO ACCURATELY RESPOND TO VERY SMALL INPUTS.

CHAINED A/O AMPLIFIERS HAVE A GAIN OF  $A^N$ . FOR EXAMPLE, FOR  $A = 4$  (12dB) &  $N=8$ , THE GAIN IS 65,500 (~96dB).

# DYNAMIC RANGE ISSUES

FOR A MODEST OVERALL BANDWIDTH  
OF 3.2 GHz, AND THIS GAIN OF 65,000,  
THE *GAIN-BANDWIDTH PRODUCT* IS  
IS AN ASTRONOMICAL 208,000 GHz !

# DYNAMIC RANGE ISSUES

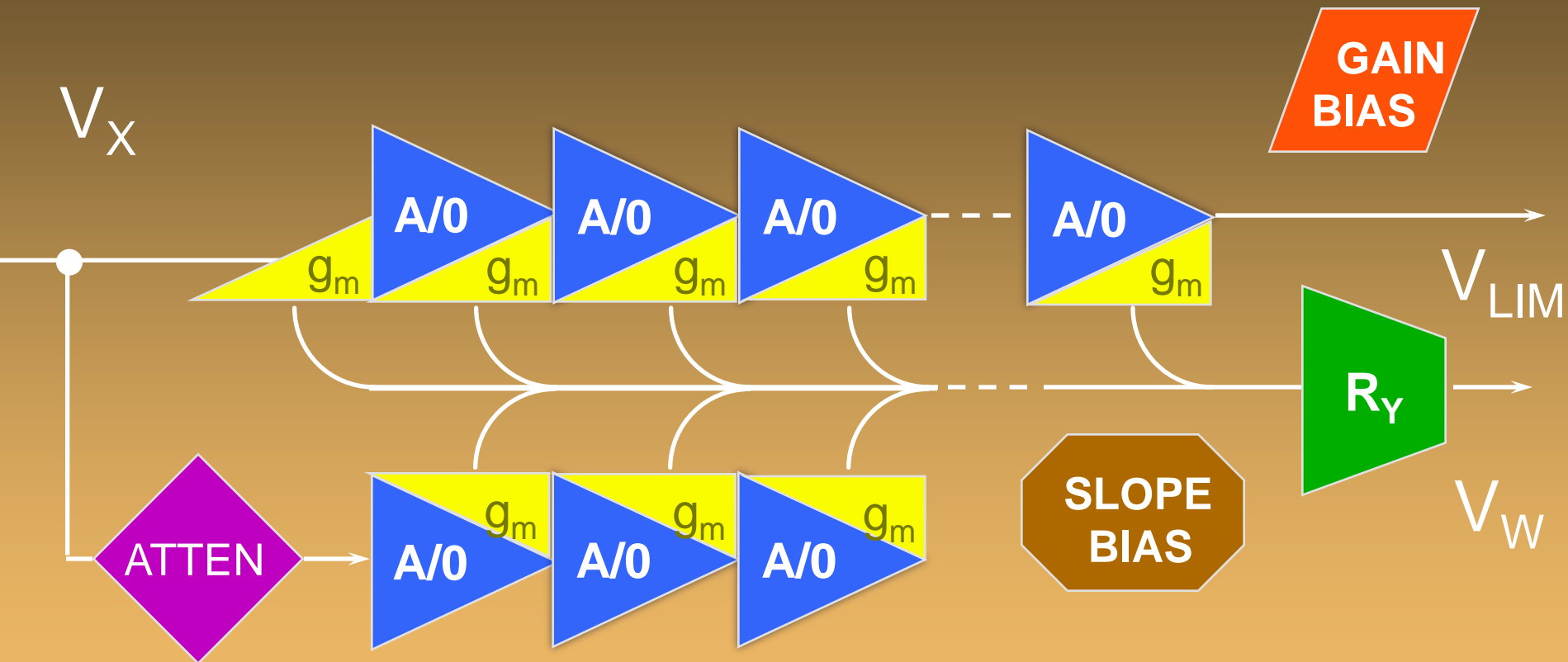
THE DYNAMIC RANGE OF A LOG-AMP IS LIMITED BY ITS FIRST-STAGE NOISE (say,  $1\text{nV}/\sqrt{\text{Hz}}$ ) & BY THE PEAK INPUT AT WHICH IT REMAINS LOGARITHMIC, ROUGHLY  $\pm 2E_K$ . THIS IS ABOUT 74dB.

# SOLUTIONS

ONE SOLUTION IS TO LOWER THE BANDWIDTH, BUT THIS MAY NOT BE PERMISSIBLE. ANOTHER IS TO LOWER THE NOISE; THAT'S HARD.

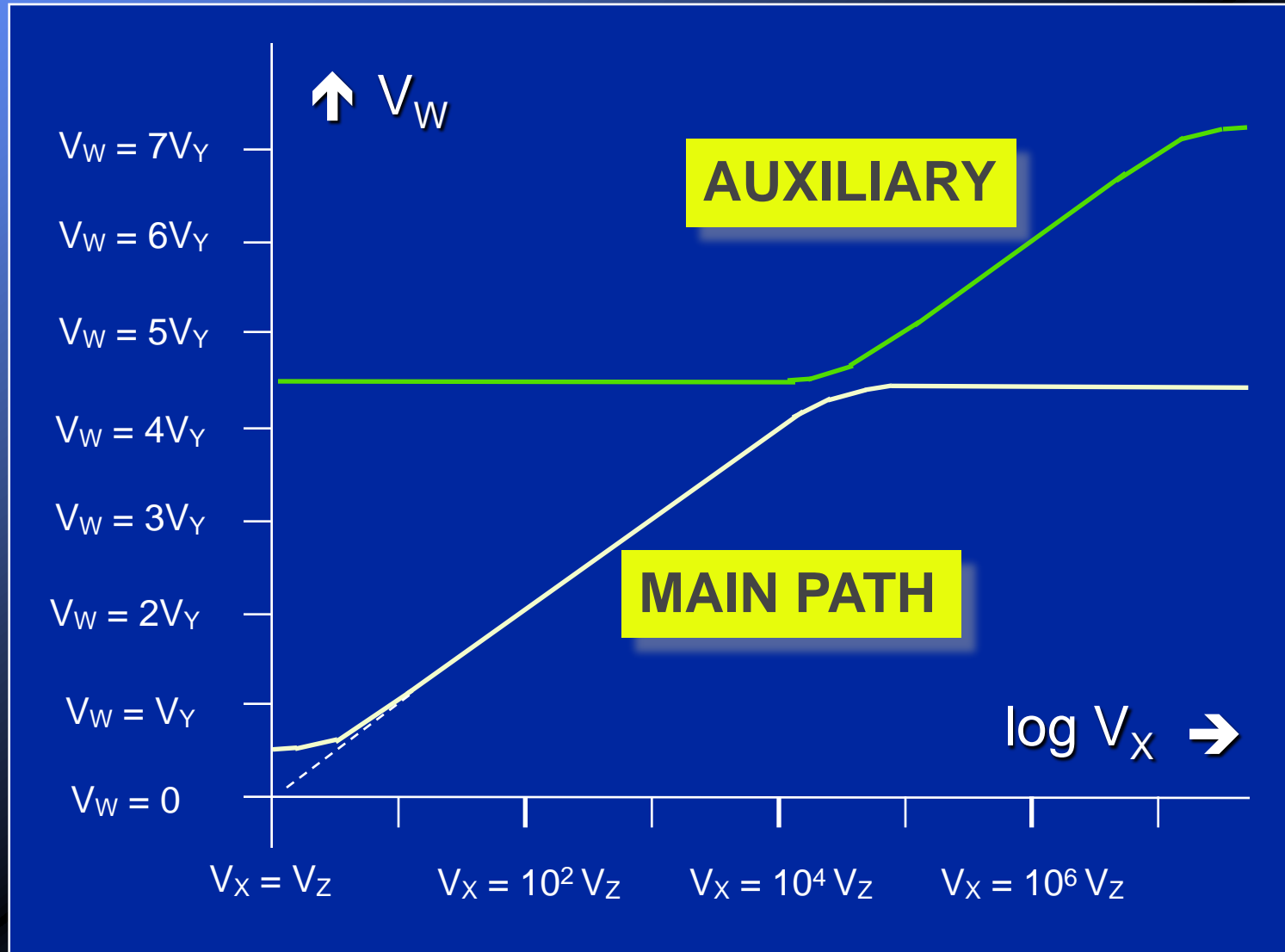
A BETTER SOLUTION IS TO RAISE TOP END OF THE DYNAMIC RANGE BY USING AN AUXILIARY LOG-AMP

# WIDE RANGE LOG-AMP



ATTENUATION RATIO MUST BE  $A^k$  WITH  $k$  INTEGER

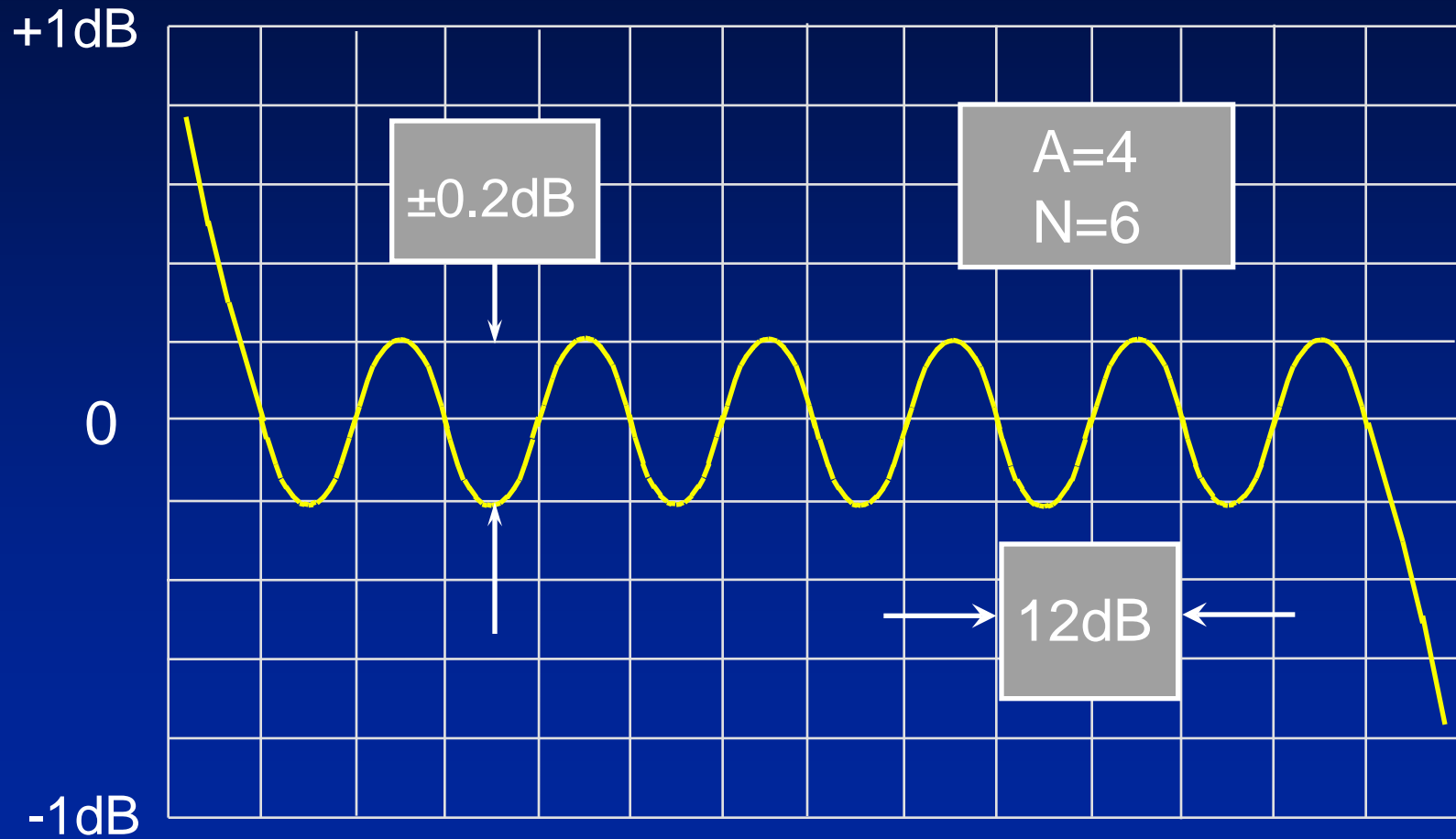
# WIDE RANGE LOG-AMP



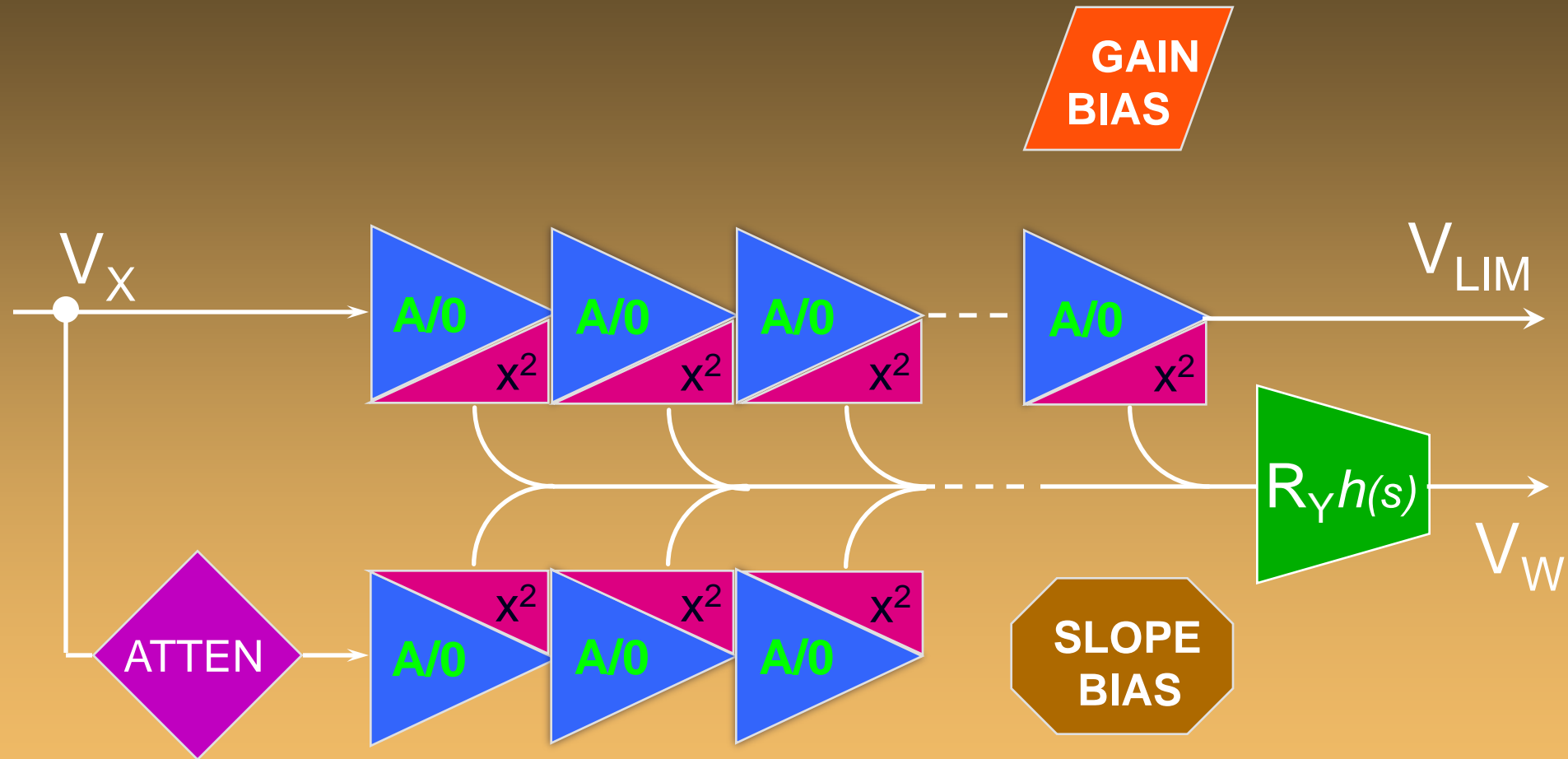
# WIDE RANGE LOG-AMP

USING THIS TECHNIQUE, A DYNAMIC RANGE OF  $>100\text{dB}$  CAN BE ACHIEVED, EVEN AT WIDE BANDWIDTHS. WITH  $1\text{nV}/\sqrt{\text{Hz}}$  AND  $\Delta f = 400\text{MHz}$ , THE NOISE VOLTAGE IS  $20\mu\text{V}$  RMS; THE LARGEST INPUT MAY BE LIMITED ONLY BY  $BV_{\text{CBO}}$ , SAY,  $4\text{V}$ , ABOUT  $2.8\text{V}$  RMS. THIS IS A DYNAMIC RANGE OF  $103\text{dB}$ .

# APPROXIMATION ERROR FOR NON-IDEAL (tanh) CELLS (DC)

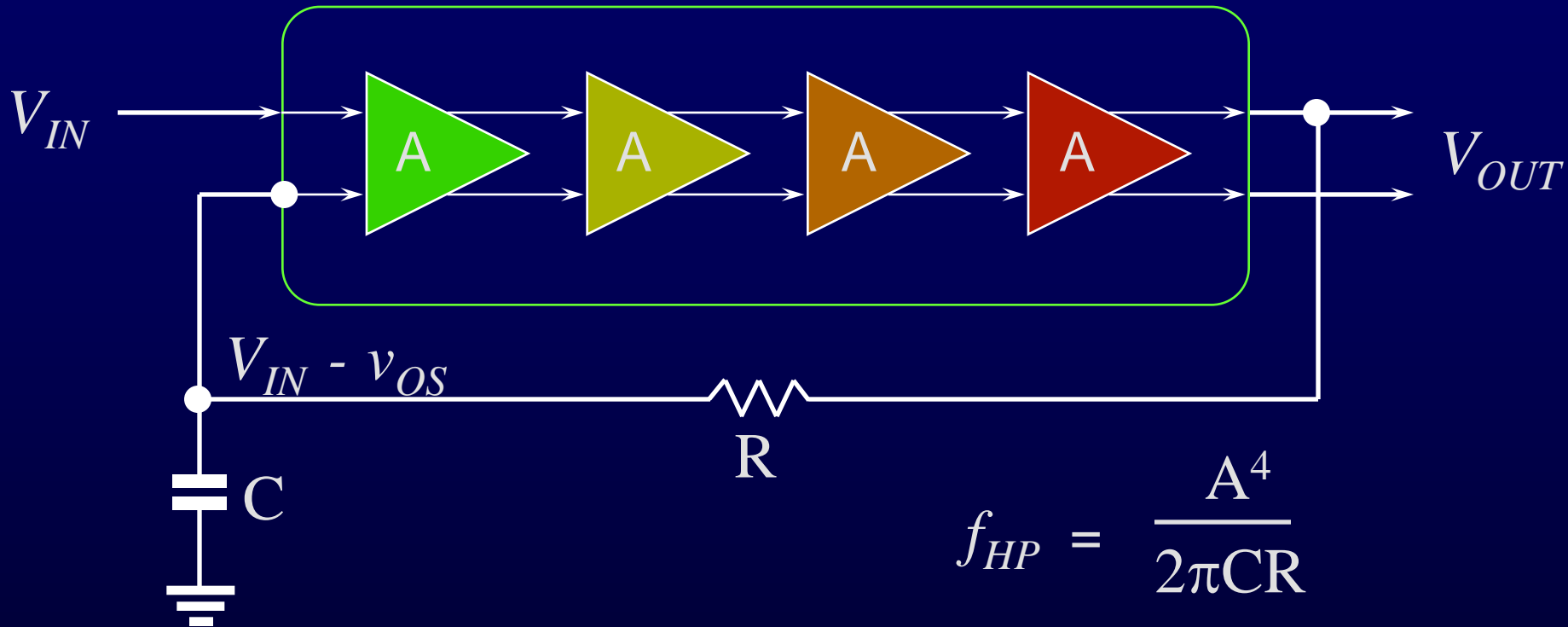


# WIDE RANGE LOG-AMP



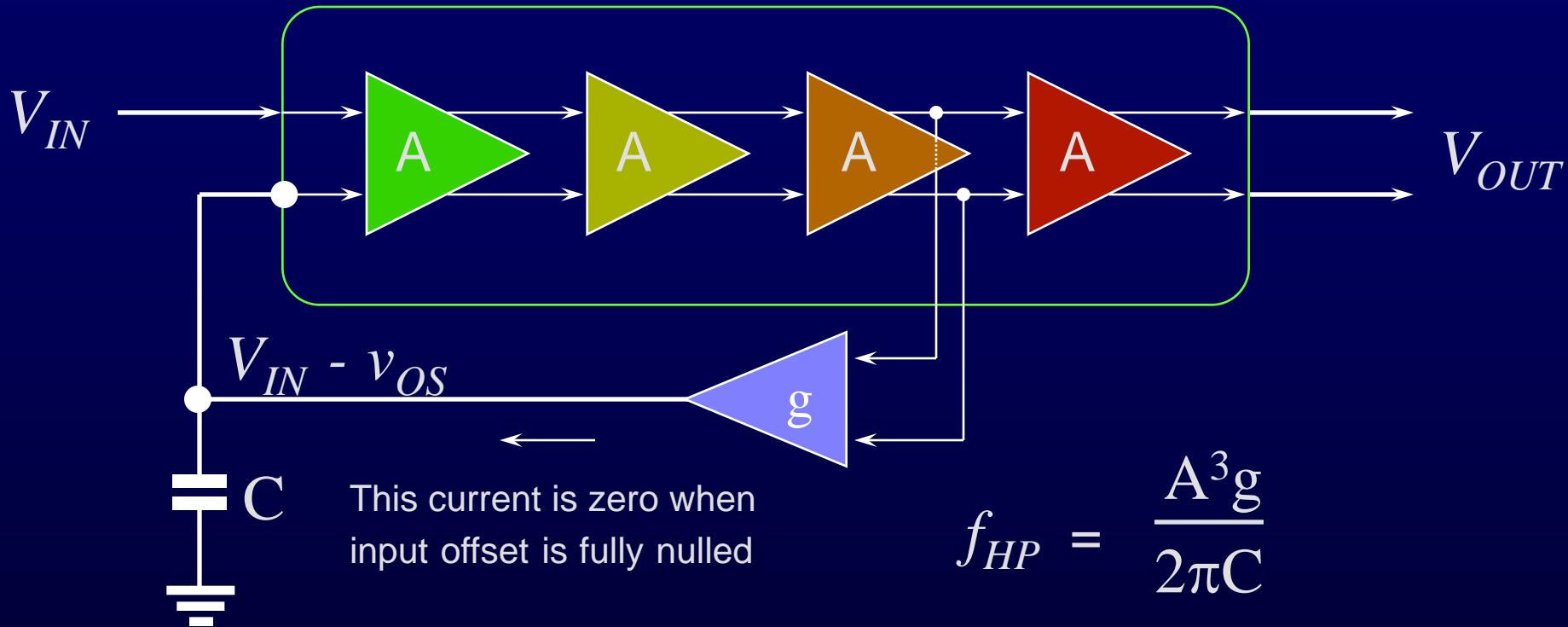
THE OUTPUT I-V CONVERTER IS NOW ALSO A FILTER

# REMOVING DC OFFSET



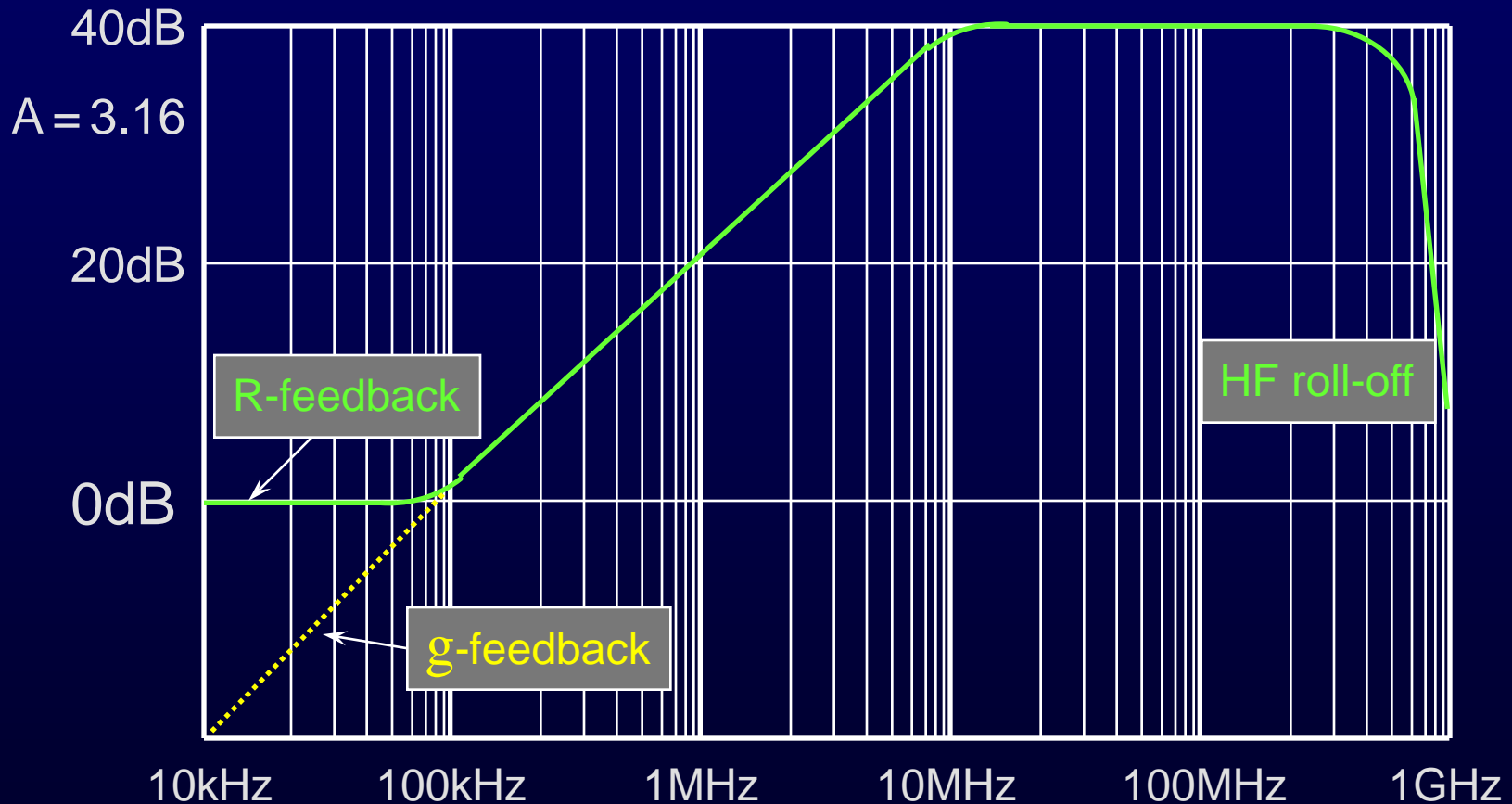
To avoid breaking the chain, while at the same time cope with DC offsets a global feedback path can be used to stabilize the operating point (null the offset)

# REMOVING DC OFFSET

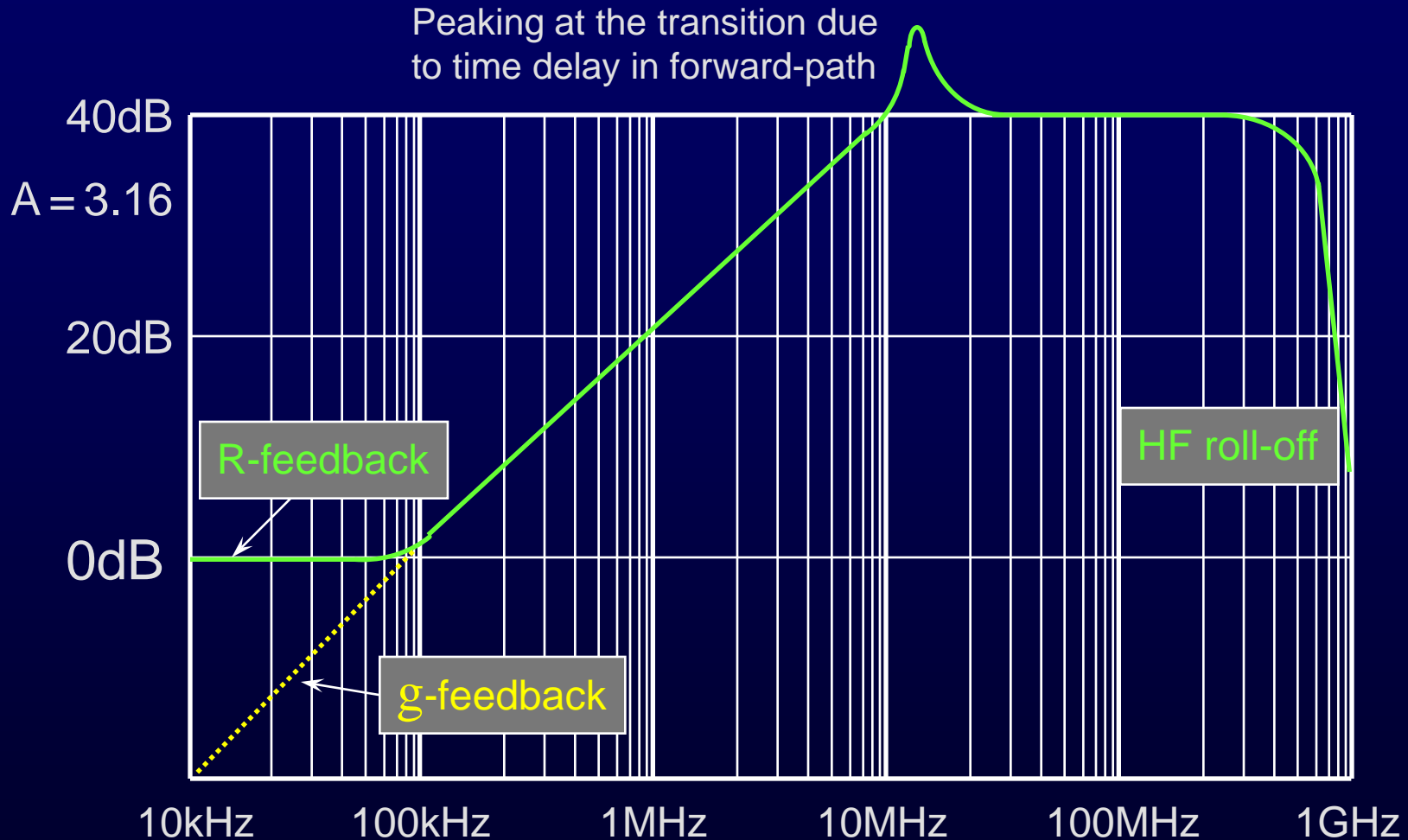


This method is often useful in coping with DC offset since it forms an integrator that fully nulls the error.

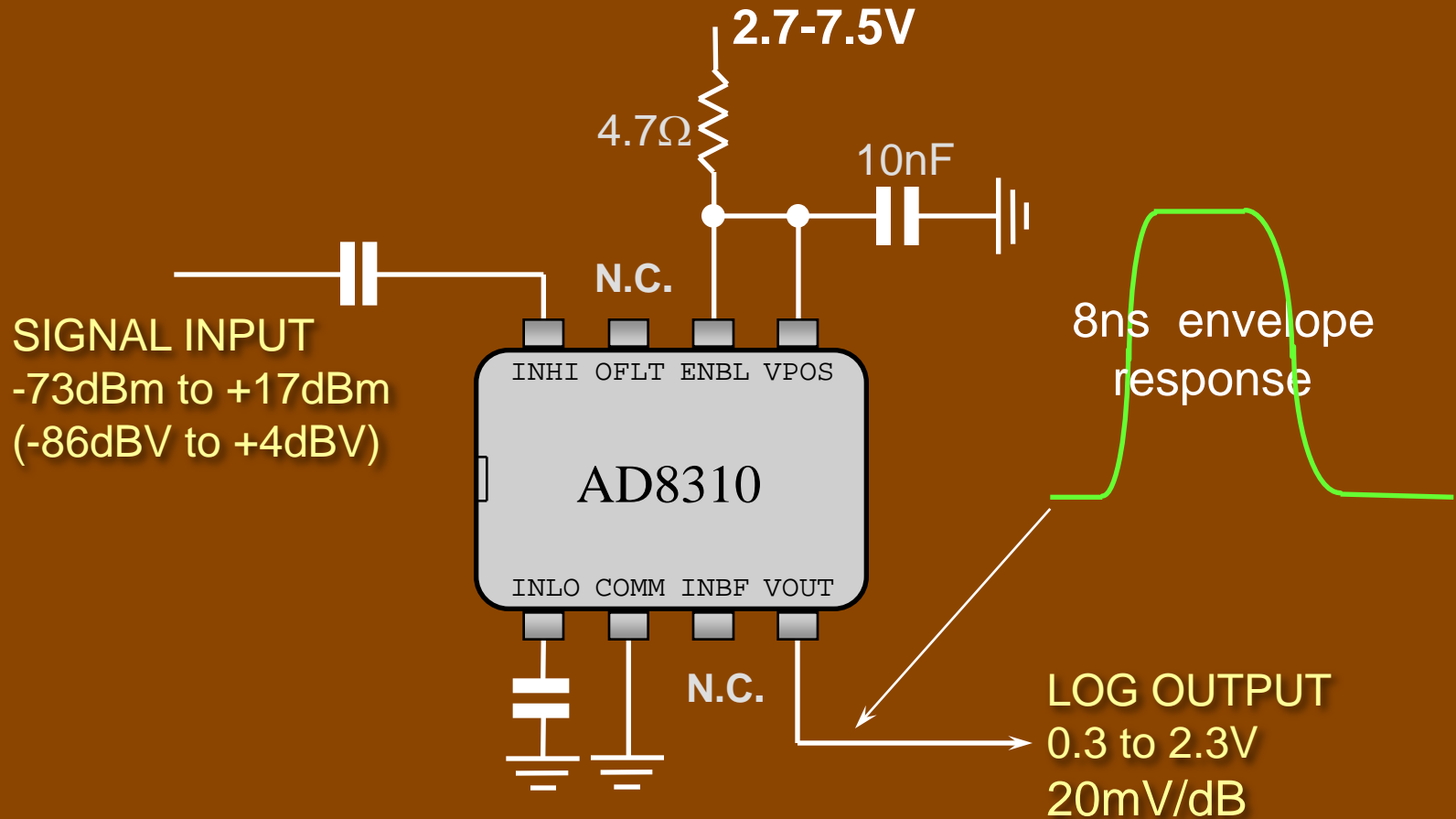
# REMOVING DC OFFSET



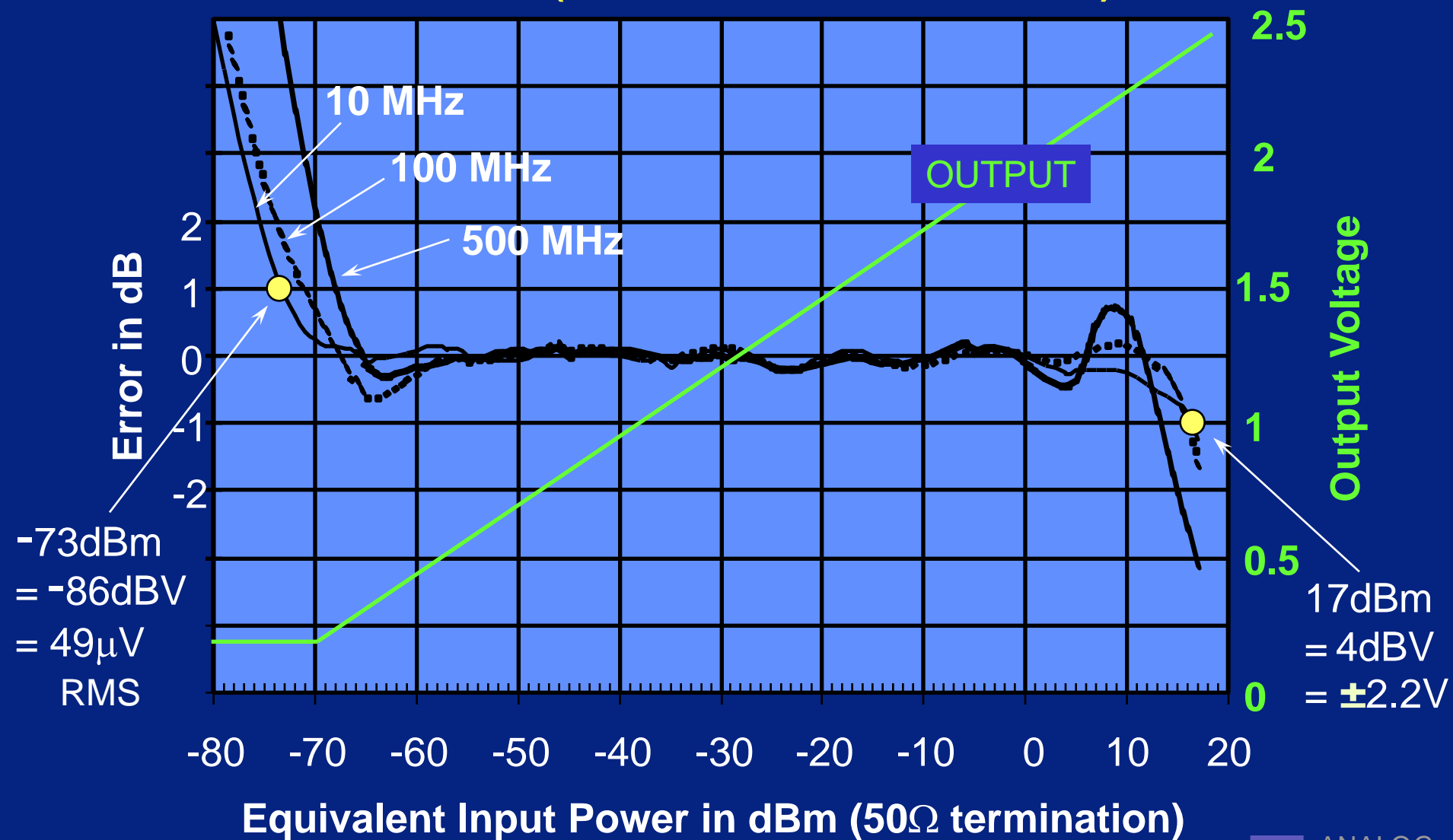
# REMOVING DC OFFSET



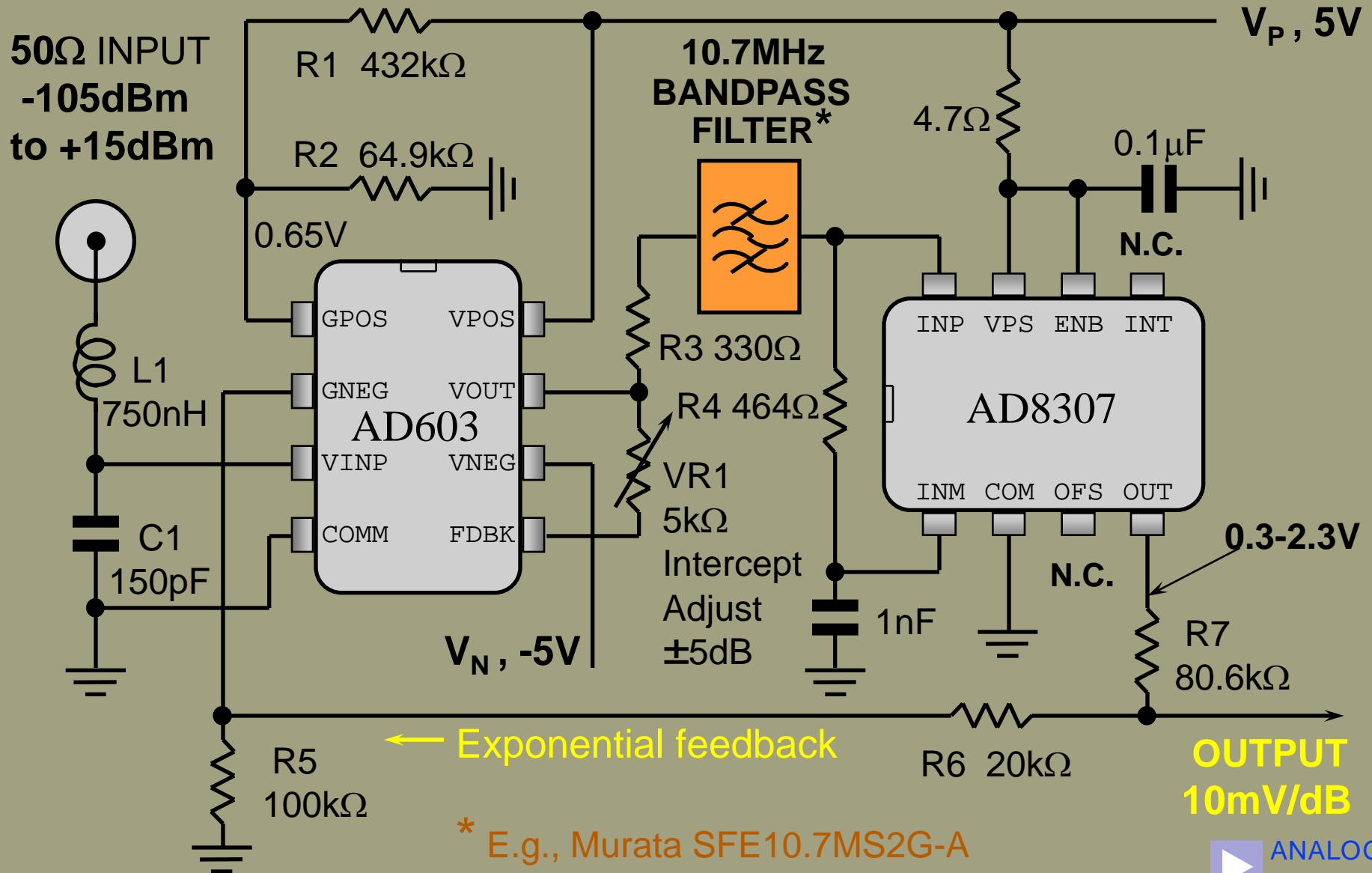
# AD8310: Fast, Lo-Z Voltage Out



# Logarithmic Conformance of AD8307 (from Data Sheet)



# 120+ dB MEASUREMENT SYSTEM



# EFFECT OF WAVEFORM

- THESE ANALYSES HAVE BEEN FOR DC INPUTS ONLY
- AN AMPLITUDE-SYMMETRIC PULSE OR SQUAREWAVE INPUT WOULD PRODUCE THE SAME RESULT
- FOR STANDARD WAVEFORMS, IT IS EASY TO CALCULATE THE EFFECT ON THE CALIBRATION

# EFFECT OF WAVEFORM

- WAVEFORM HAS NO EFFECT ON THE SLOPE CALIBRATION,  $V_Y$
- IT AFFECTS ONLY THE INTERCEPT

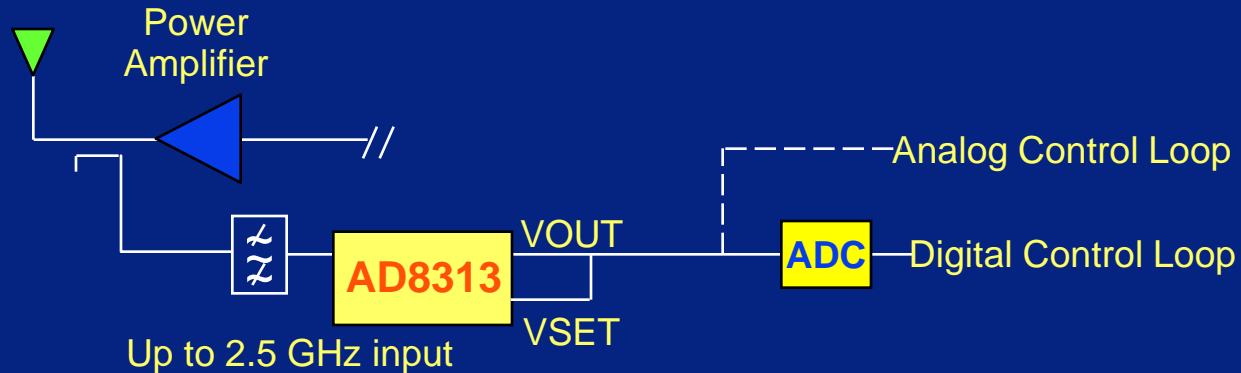
# EFFECT OF WAVEFORM

- FOR A SINUSOIDAL INPUT  $E_s \sin \omega t$   
THE OUTPUT WILL BE THE SAME AS  
THAT FOR A DC INPUT OF  $E_s/2$
- THAT IS, INTERCEPT VALUE FOR  
SINE IS EFFECTIVELY DOUBLED



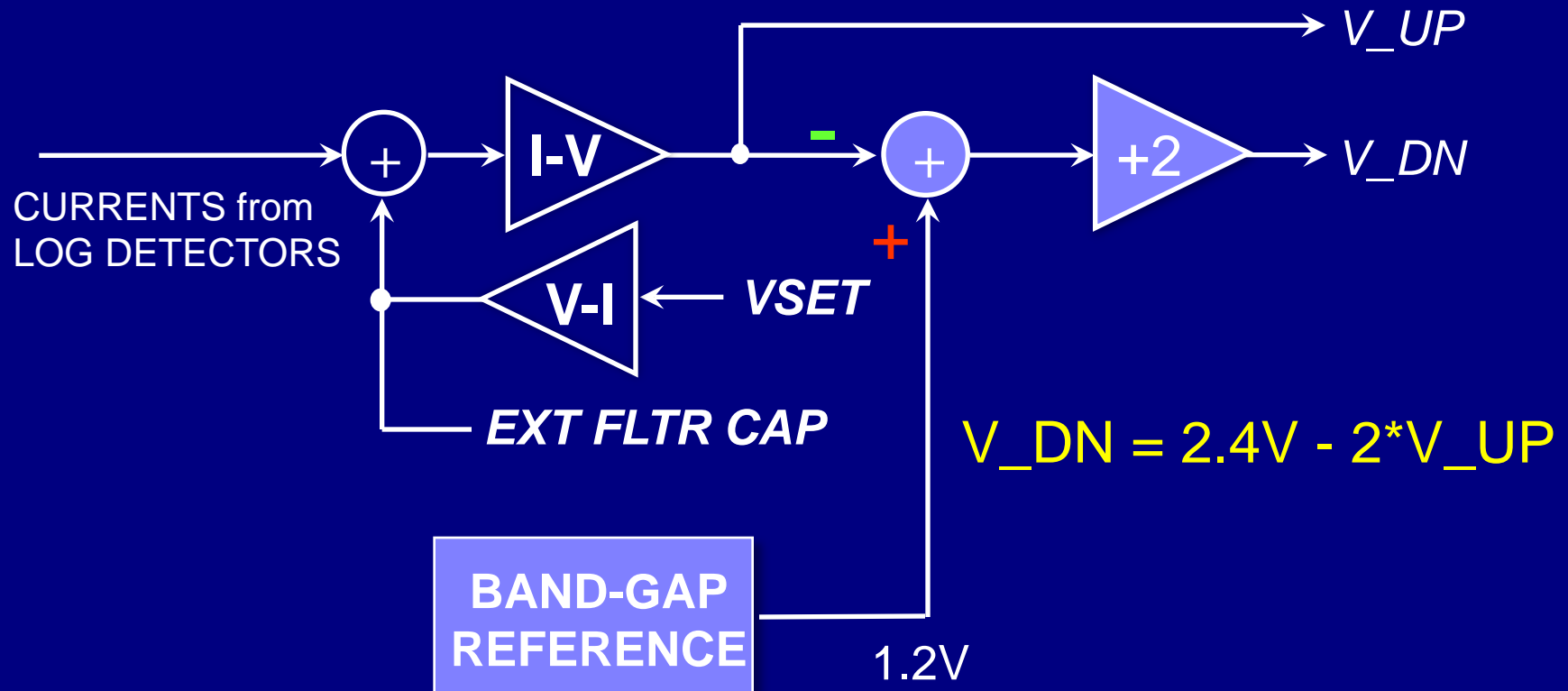
# **RF Power Detectors**

# Direct RF Detection using AD8313



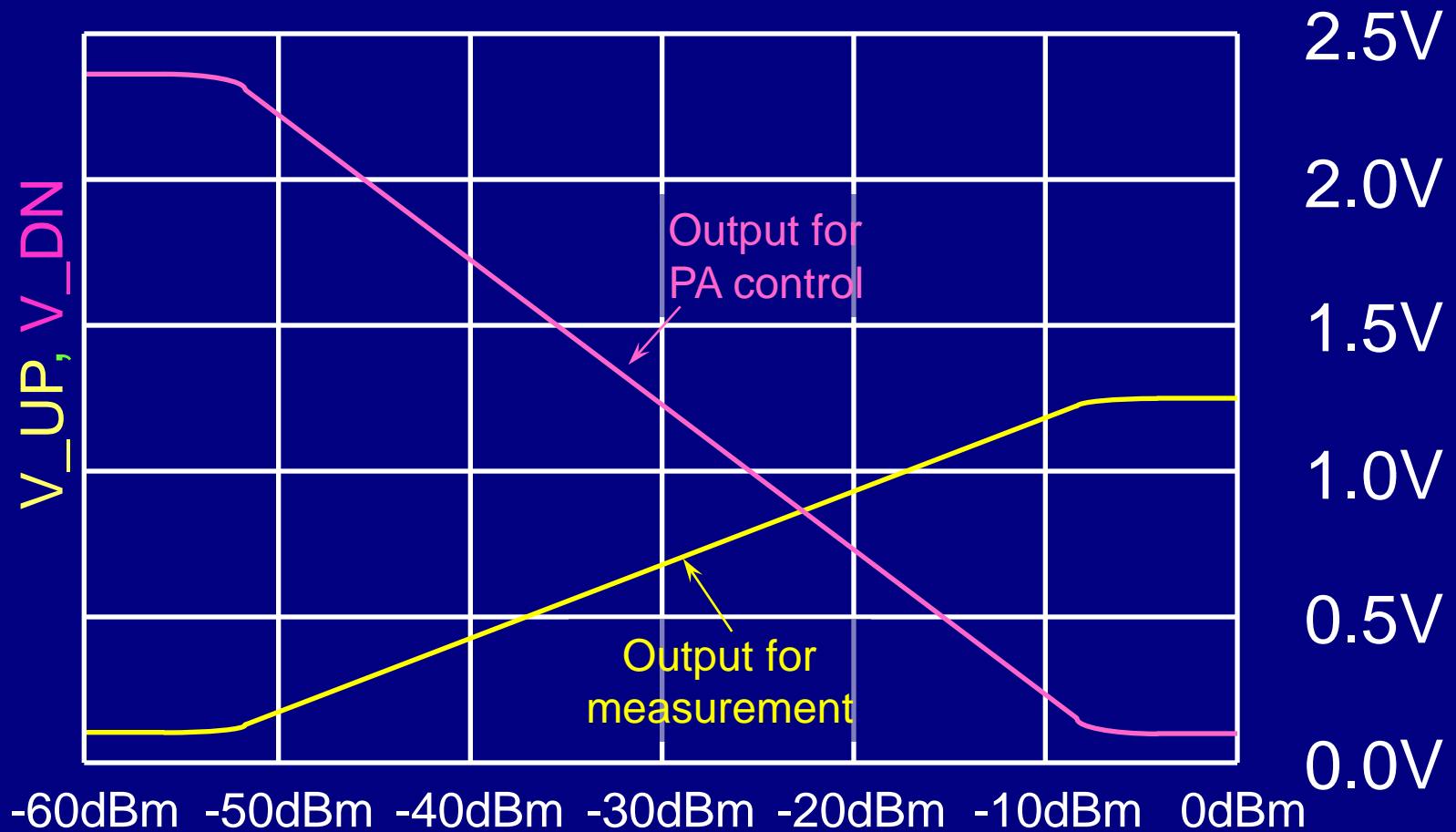
- Direct RF Detection to >2.5 GHz
- 70 dB total dynamic range
- $\pm 1$  dB accuracy over central 62 dB
- 8 pin Micro-SOIC
- Provides RSSI at antenna frequency
- Released August 1998

# AD8314 Low Cost 40dB Log Amp

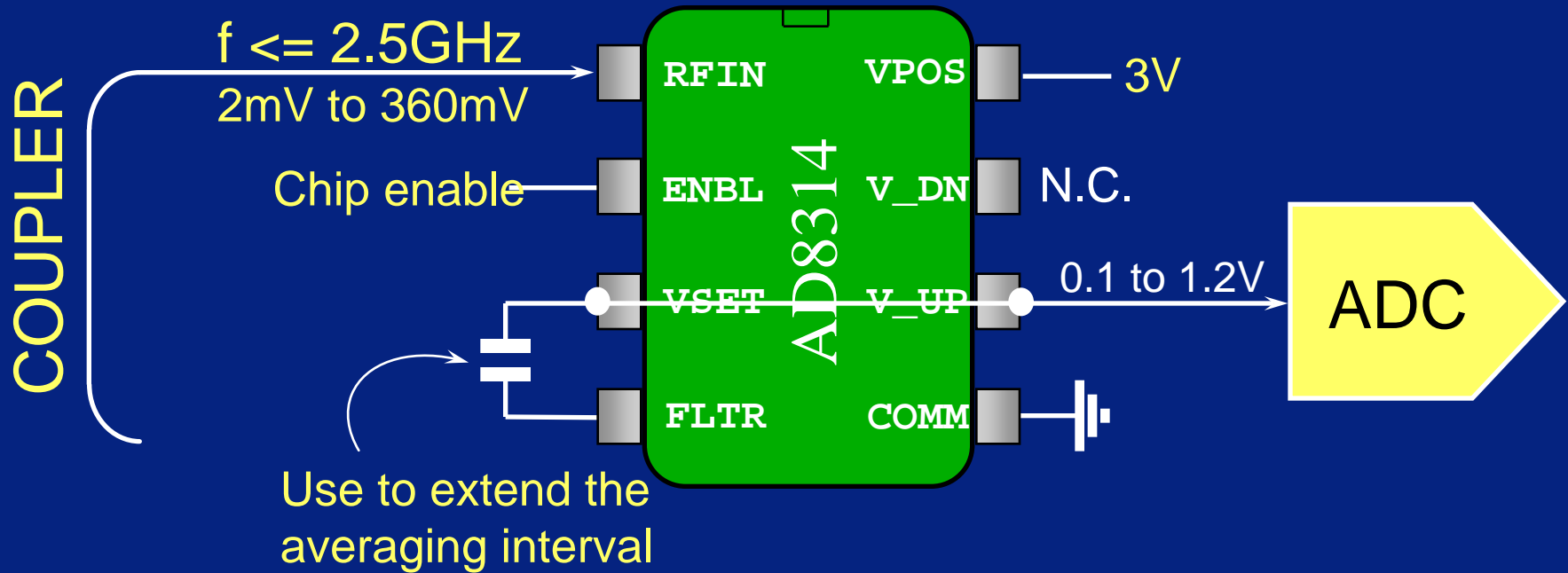


Output Interface of AD8314

# AD8314 40dB Log-Amp is a Detector & Controller



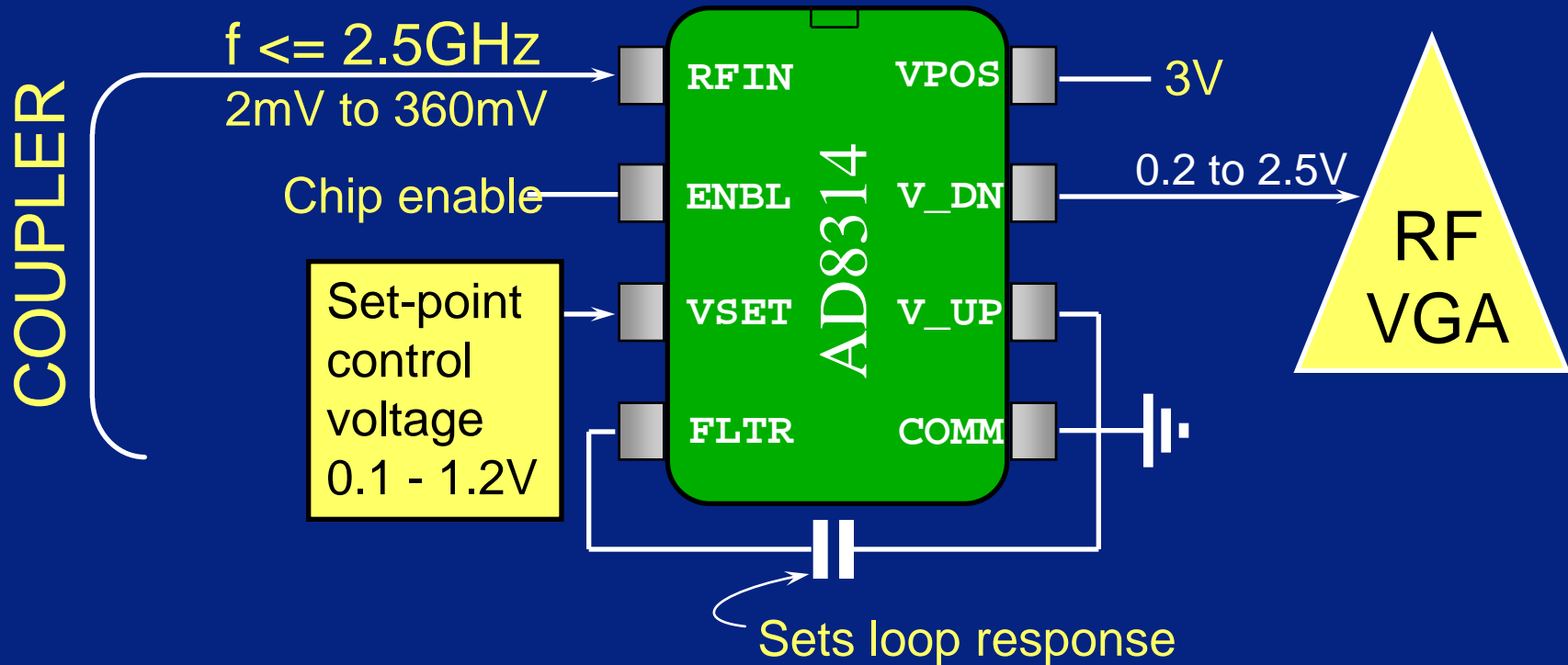
# AD8314 in Measurement Mode



Output is  $0.5 \log_{10}(V_{IN} \text{ in mV})$



# AD8314 in Controller Mode



V\_UP not used as an output in this mode  
but is needed to set the loop bandwidth



# One-chip Network Analyser

# AD8302: A GAIN/PHASE DETECTOR

- ♦ A Network Analyser on a Chip! - Almost!

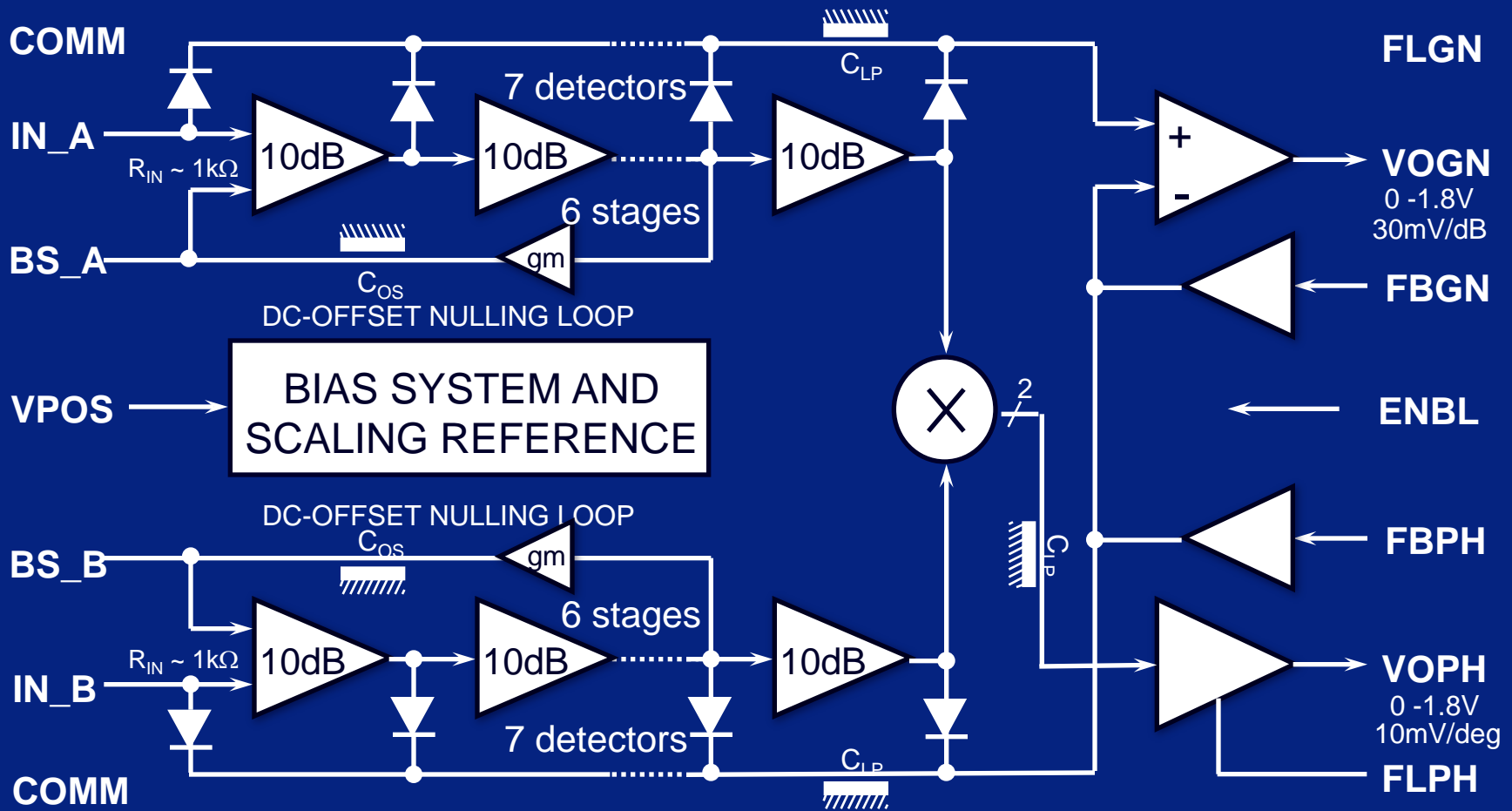
$$V_{\text{GAIN}} = V_G \log (V_A/V_B) \quad V_G = 30\text{mV/dB}$$

$$V_{\text{PHS}} = V_P ( \phi_1 - \phi_2 ) \quad V_P = 10\text{mV/deg}$$

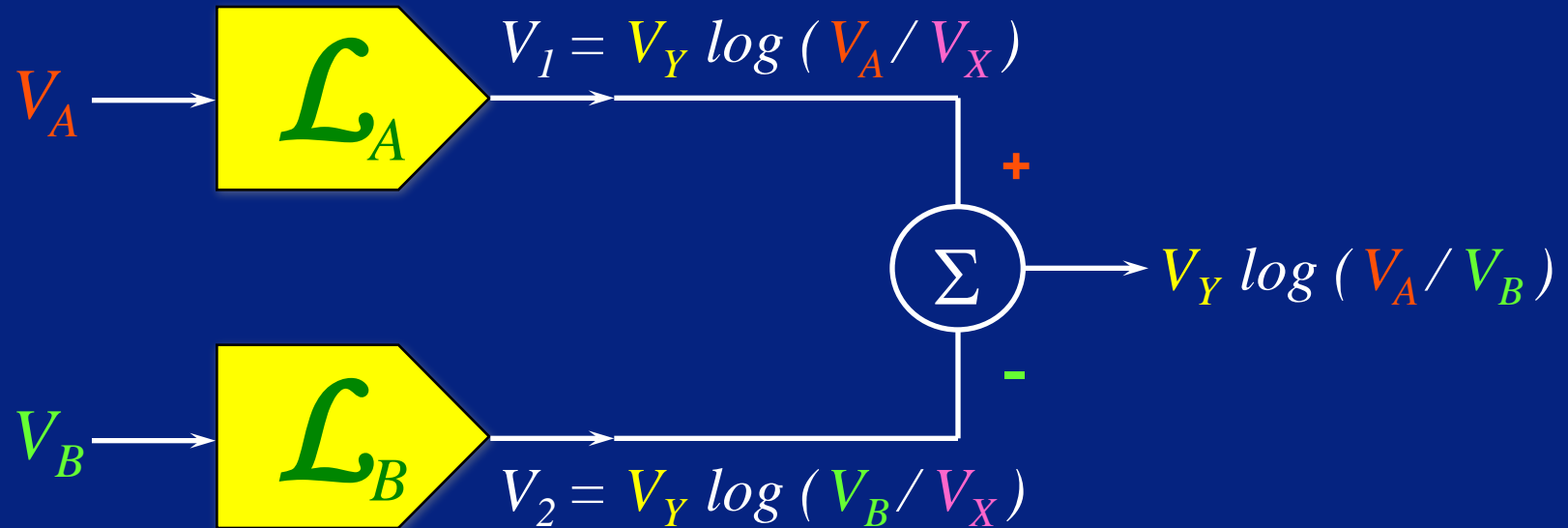
- ♦ Operates from LF to >3 GHz

## Applications

- Power Amplifier Phase/Gain Control  
.... *independent of actual power level*
- Monitoring of System Gain/Loss (e.g. Return Loss)
- System Diagnostics
- Linear Phase Demodulator

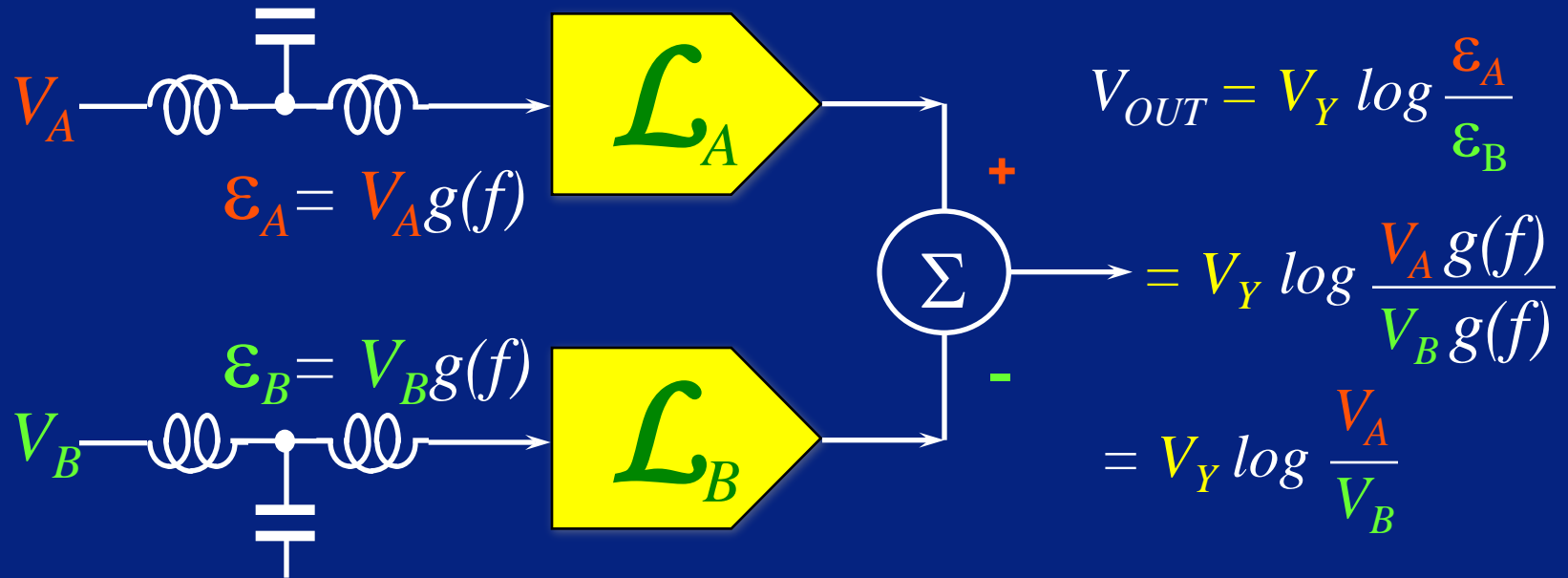


# TRUE GAIN MEASUREMENT



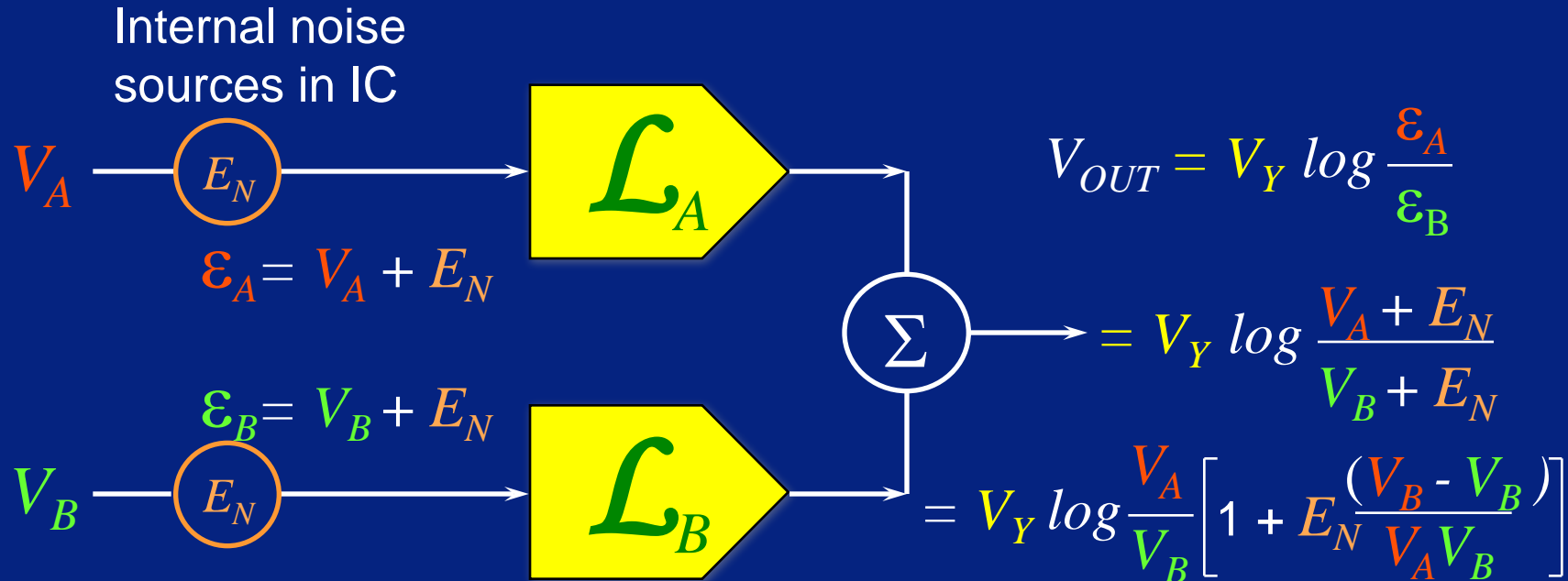
By subtracting the output of the B-channel log-amp from that of the A-channel log-amp, the intercept  $V_X$  is eliminated and the resulting difference is a measure of the RATIO of  $V_A / V_B$

# CANCEL PACKAGE RESONANCES



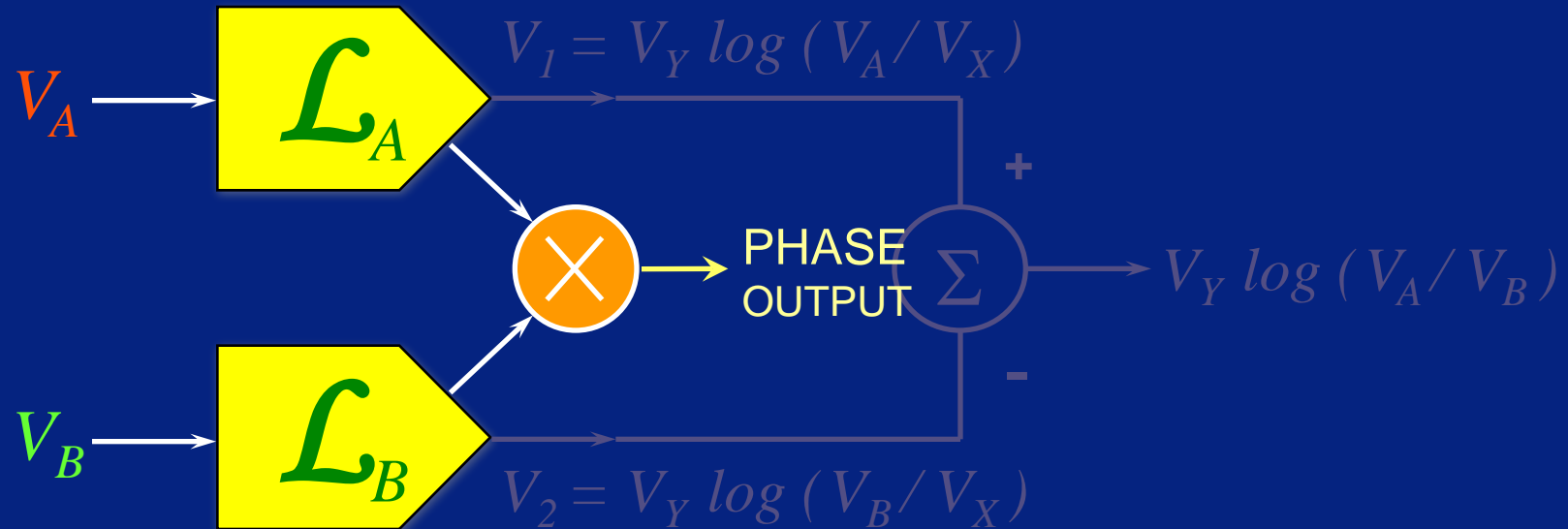
Both channels have the same HF resonances and other HF transmission effects  $g(f)$ , but these are canceled in taking the difference which remains a measure of the RATIO of  $V_A/V_B$

# LOW NOISE-INDUCED ERRORS



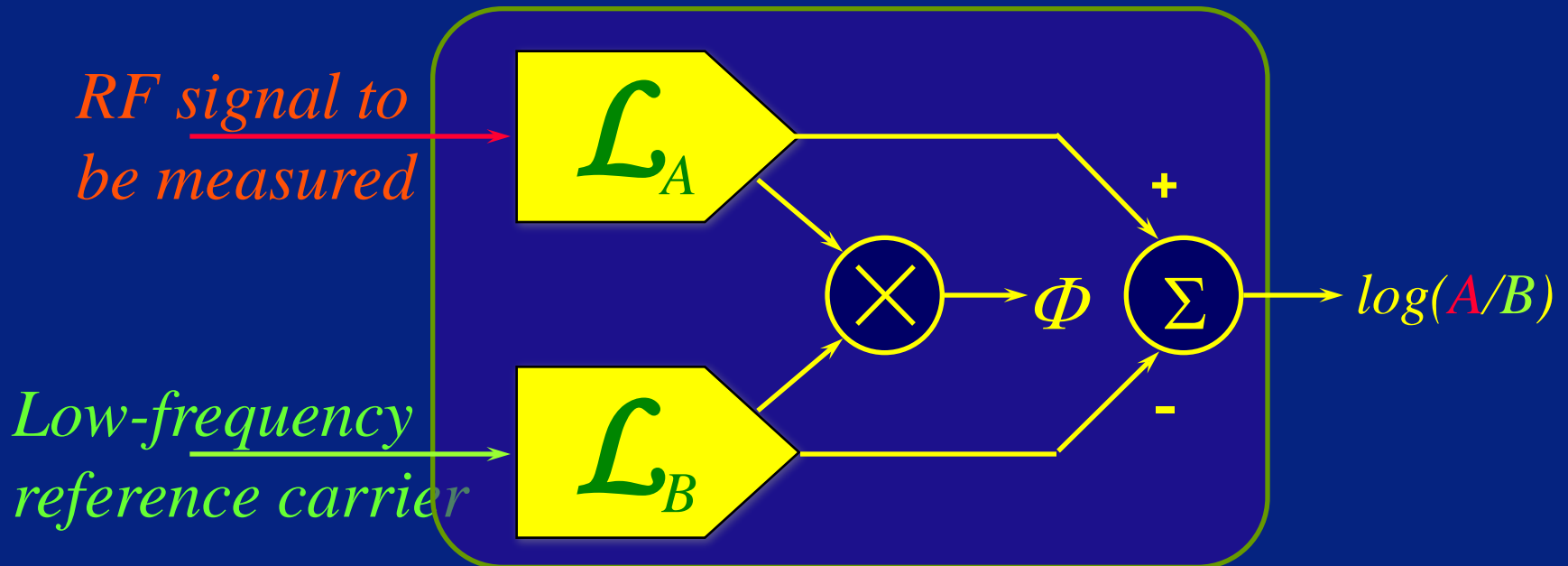
Both channels have the same additive noise at input but its effect is reduced in the output. For example, if  $V_A = 3 E_N$  and  $V_B = 2 E_N$  the measurement error of ratio is only 1.3dB

# PHASE MEASUREMENT LF-3GHz



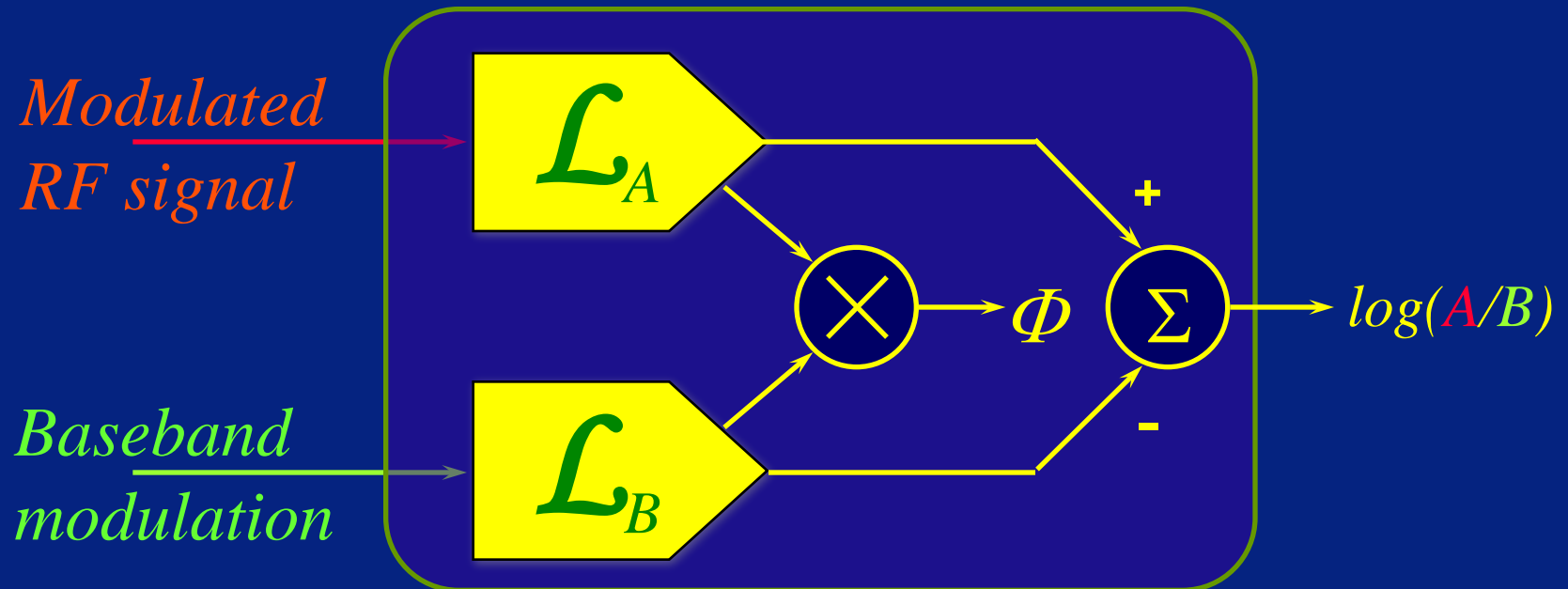
Logarithmic amplifiers also provide very high gain and limiting action: using a special type of analog multiplier between the limiter outputs, phase measurements can be made up to 3GHz

# APPLICATIONS



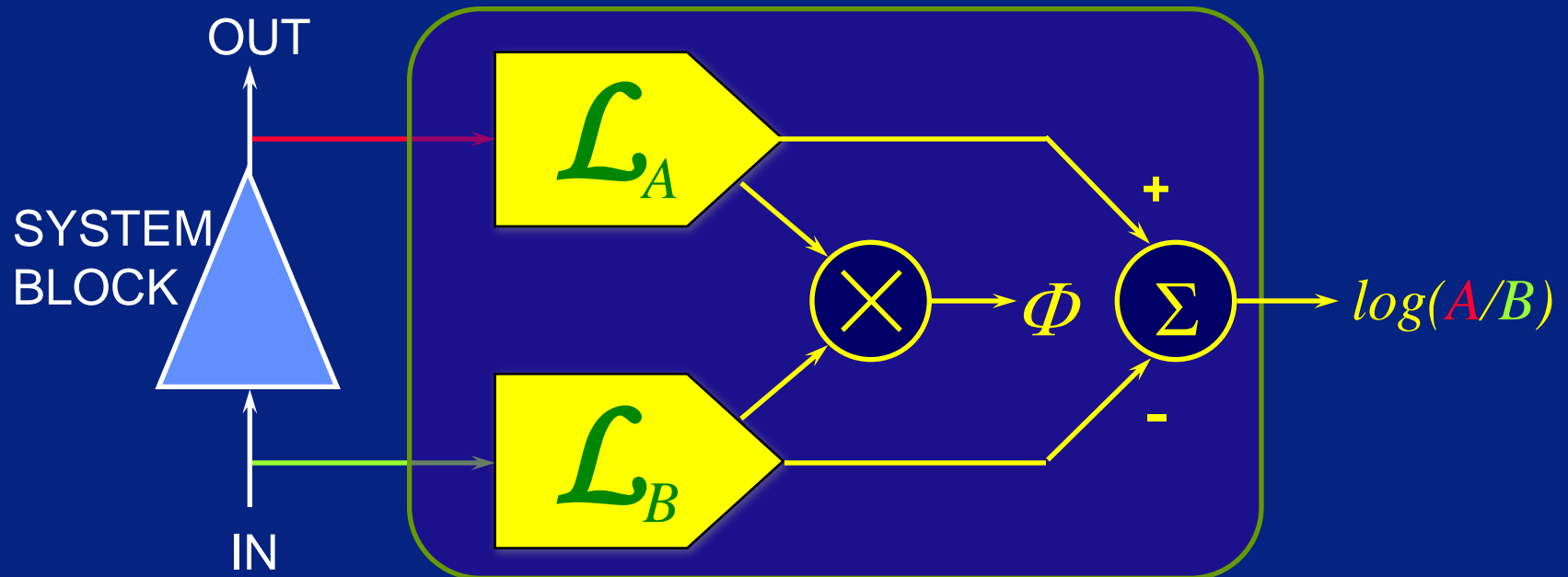
In this case, a low-frequency carrier provides a very high calibration reference for the intercept

# APPLICATIONS



Here, the reference is provided by the baseband modulation & system measures conversion gain

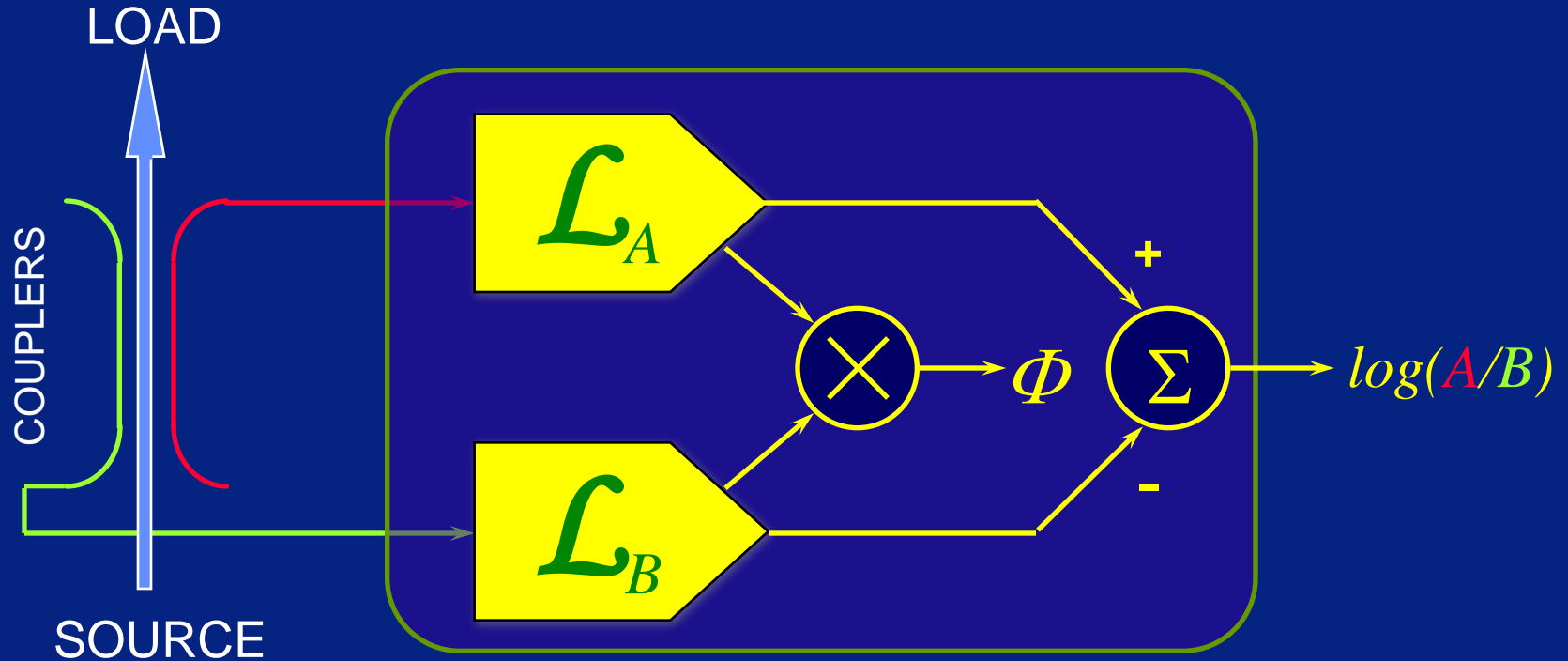
# APPLICATIONS



True gain of system block is measured independent of the actual power levels

# APPLICATIONS

▶ ANALOG  
DEVICES



Measurement of return loss  
independent of power level

▶ ANALOG  
DEVICES

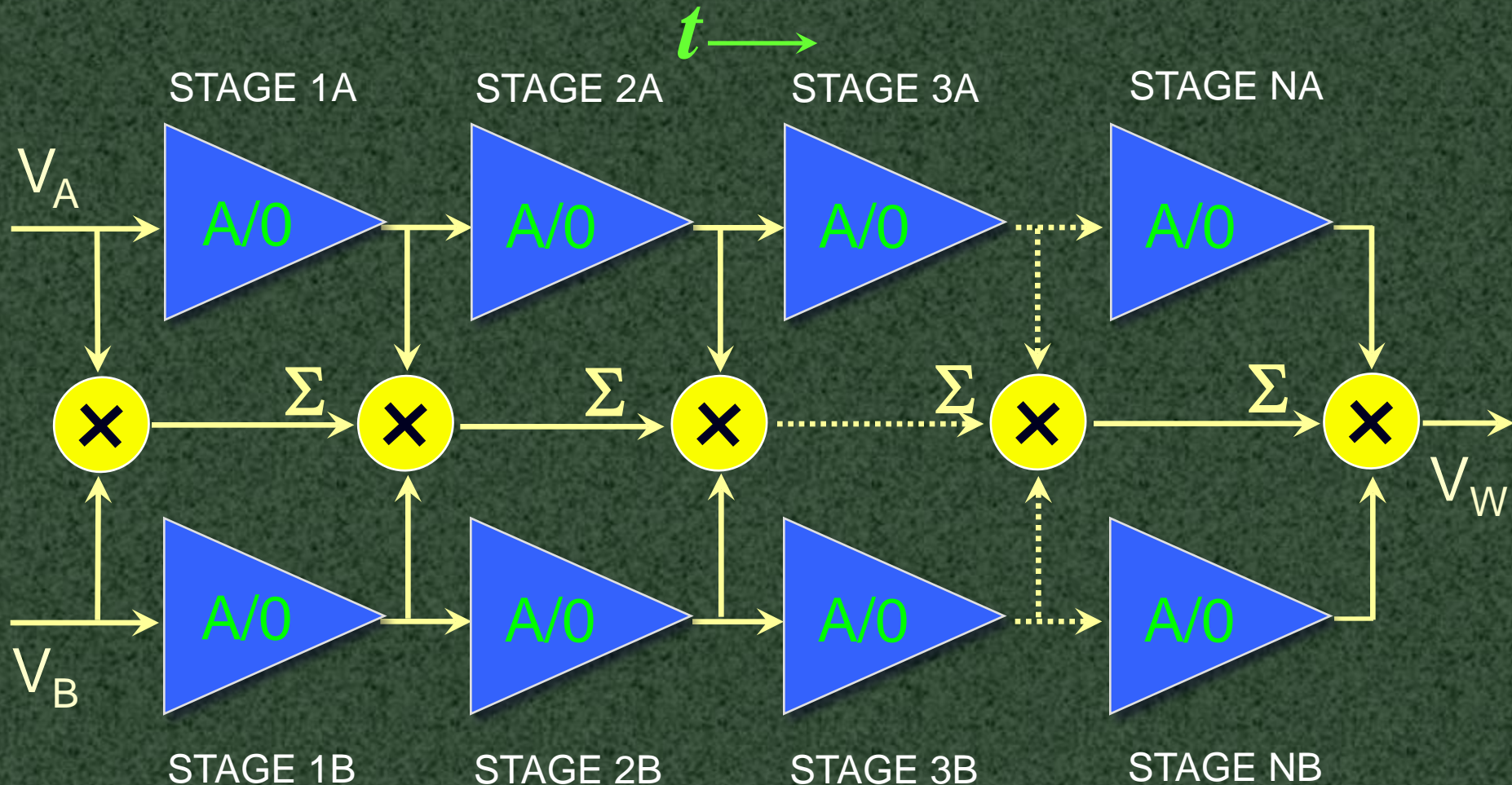


# SYNCHRONOUS LOG AMP

# SYNCHRONOUS LOG AMP

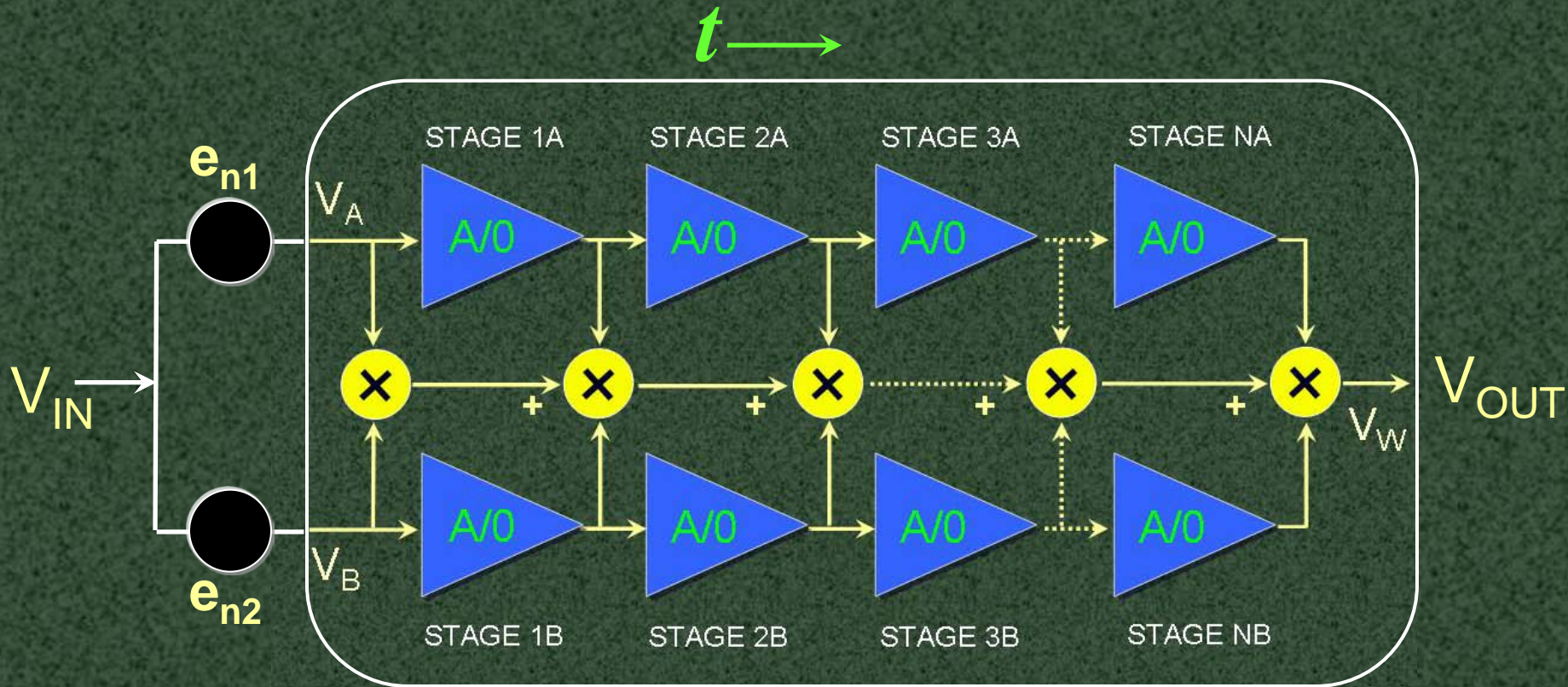
- AN 'SLA' IS DUAL LOG-AMPS ACTING IN PARALLEL
- INSTEAD OF 'SQUARE-LAW' DETECTORS IT USES ANALOG MULTIPLIERS BETWEEN CORRESPONDING NODES AS SIGNALS PROGRESS DOWN THE CASCADE
- CURRENT-MODE OUTPUTS OF ALL MULTIPLIERS IS SUMMED, AND THIS VARIABLE IS CONVERTED BACK TO THE VOLTAGE DOMAIN
- NUMEROUS APPLICATIONS: LOWER EFFECTIVE INPUT NOISE; TUNABLE TO SINGLE FREQUENCY;  $\sinh^{-1}$ ; etc

# SYNCHRONOUS LOG AMP



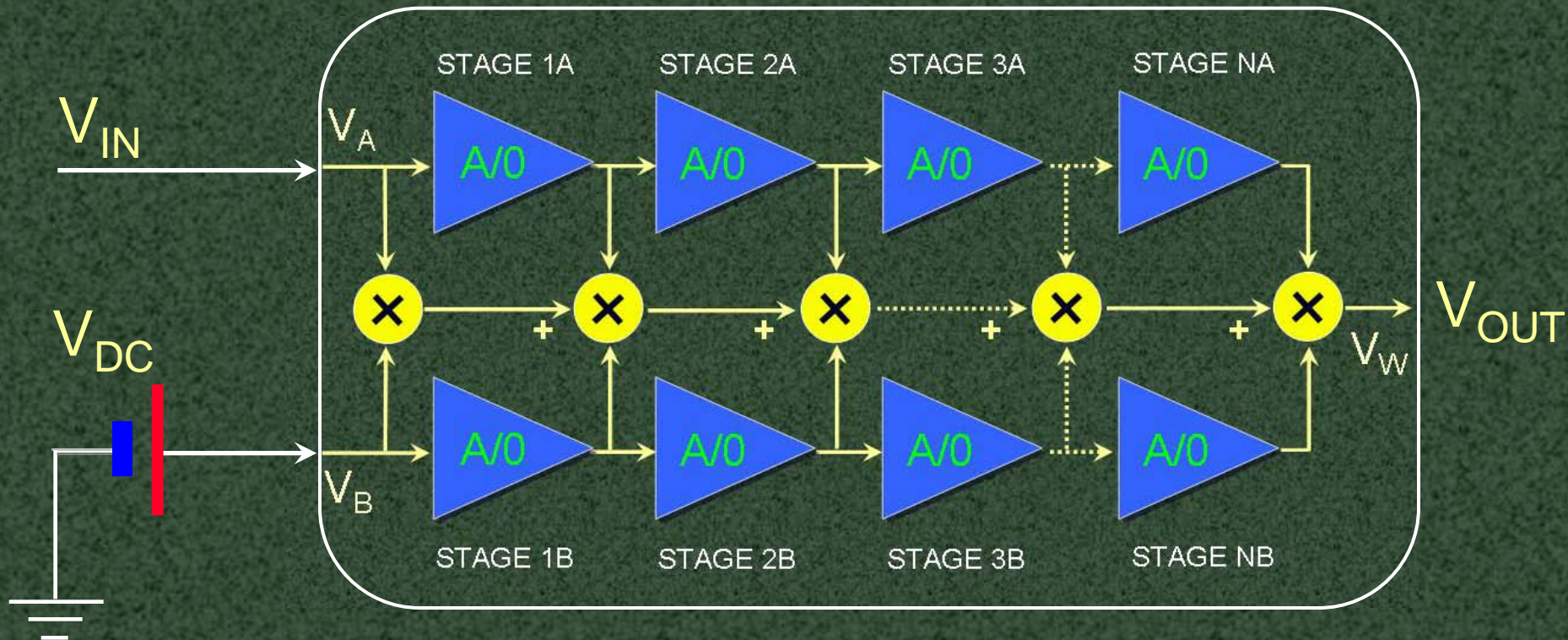
$2 \times N$  STAGES,  $N$  IS TYPICALLY 12

# FIRST APPLICATION

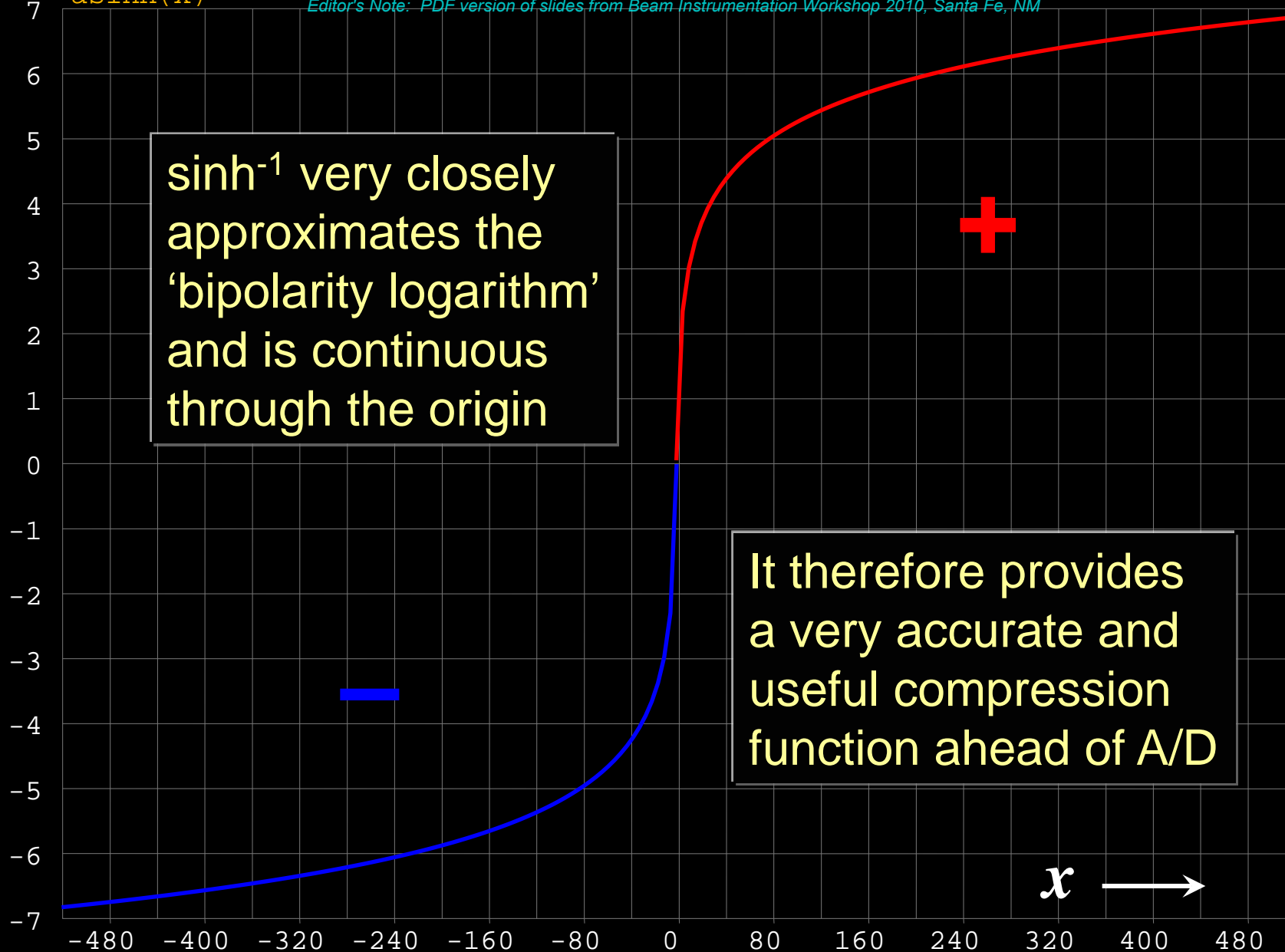


NOISE SOURCES  $e_{N1}$  and  $e_{N2}$  ARE UNCORRELATED  
SO THEIR CROSS-PRODUCT AVERAGES TO ZERO

# SECOND APPLICATION



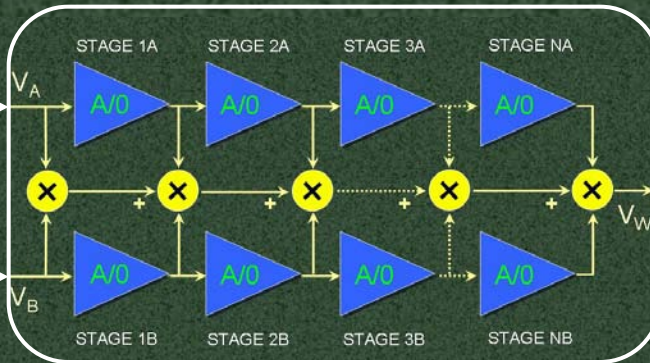
THE FIXED VOLTAGE  $V_{DC}$  TURNS THIS LOG AMP INTO A NON-DEMODULATING “ $\sinh^{-1}$ ” MACHINE



# PRE-A/D COMPRESSION

SIGNAL/  
SENSOR  
SOURCE

SCALING  
VOLTAGE  
↑  
Adapt as  
needed to  
application



LOGARITHMICALLY  
COMPRESSED AND  
SCALED SIGNAL

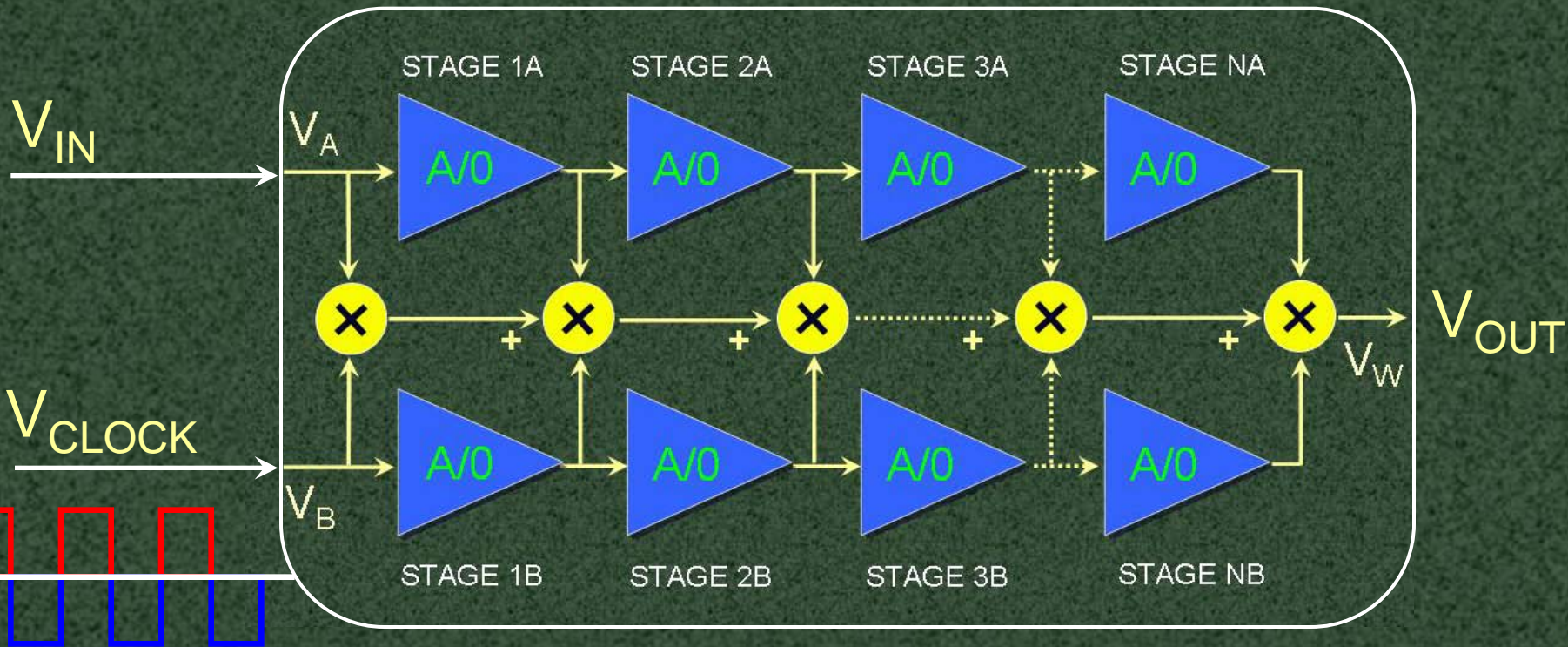
ADC

DATA

NOW, EACH LSB  
REPRESENTS A  
*FIXED NUMBER  
OF DECIBELS*  
(e.g., 0.1dB/LSB)

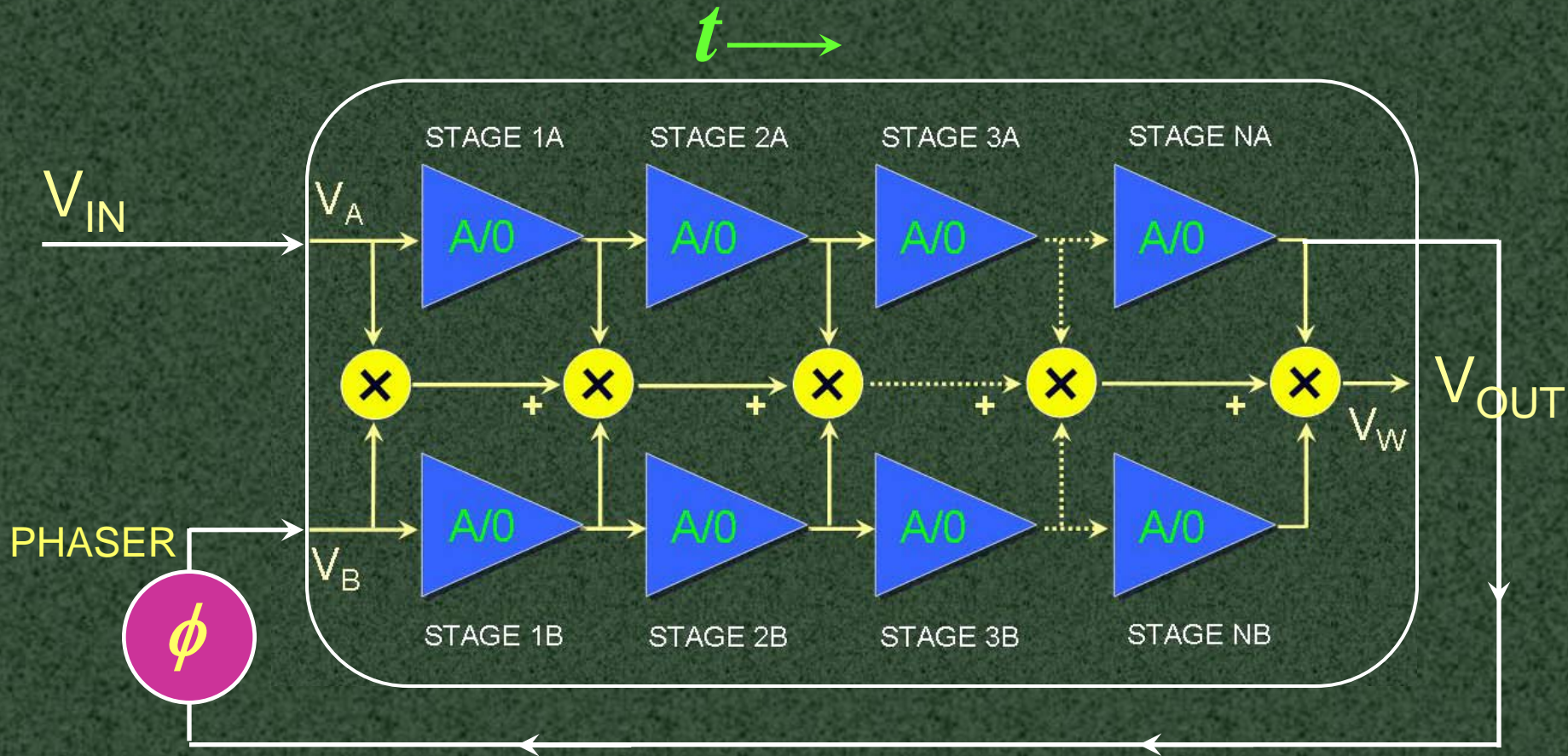
# THIRD APPLICATION

$t \rightarrow$




THE CLOCK VOLTAGE TURNS THIS LOG AMP  
INTO A SYNCHRONOUS DEMODULATOR

# FOURTH APPLICATION

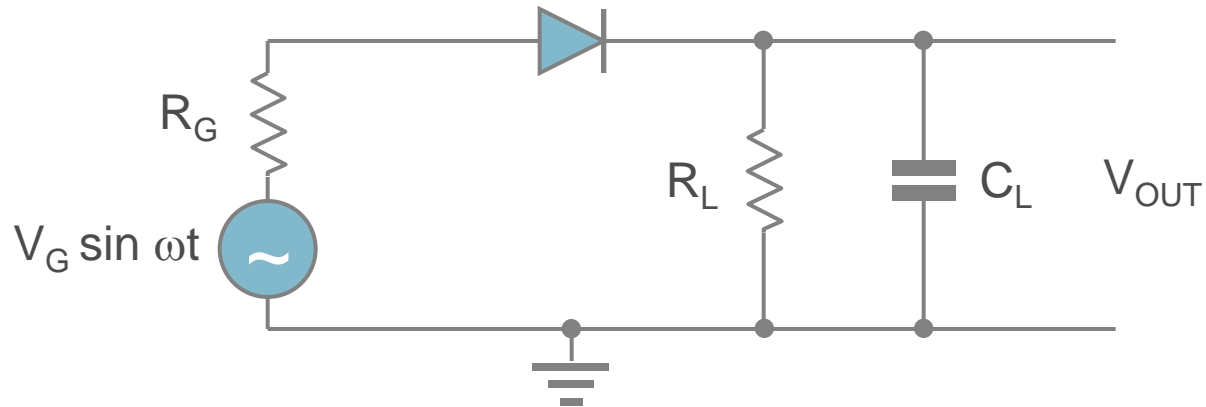


THE CLOCK VOLTAGE IS RECOVERED FROM THE LAST LIMITER OUTPUT: SYSTEM BECOMES A SELF-CLOCKED 'HOMODYNE'

The logo consists of a large, horizontally-oriented oval with a blue border. The interior of the oval is filled with a fine, repeating pattern of small blue and white squares, creating a textured effect. Centered within this oval is the text "DLVA" in a large, white, serif typeface.

**DLVA**

# A PRIMITIVE SCHOTTKY DETECTOR

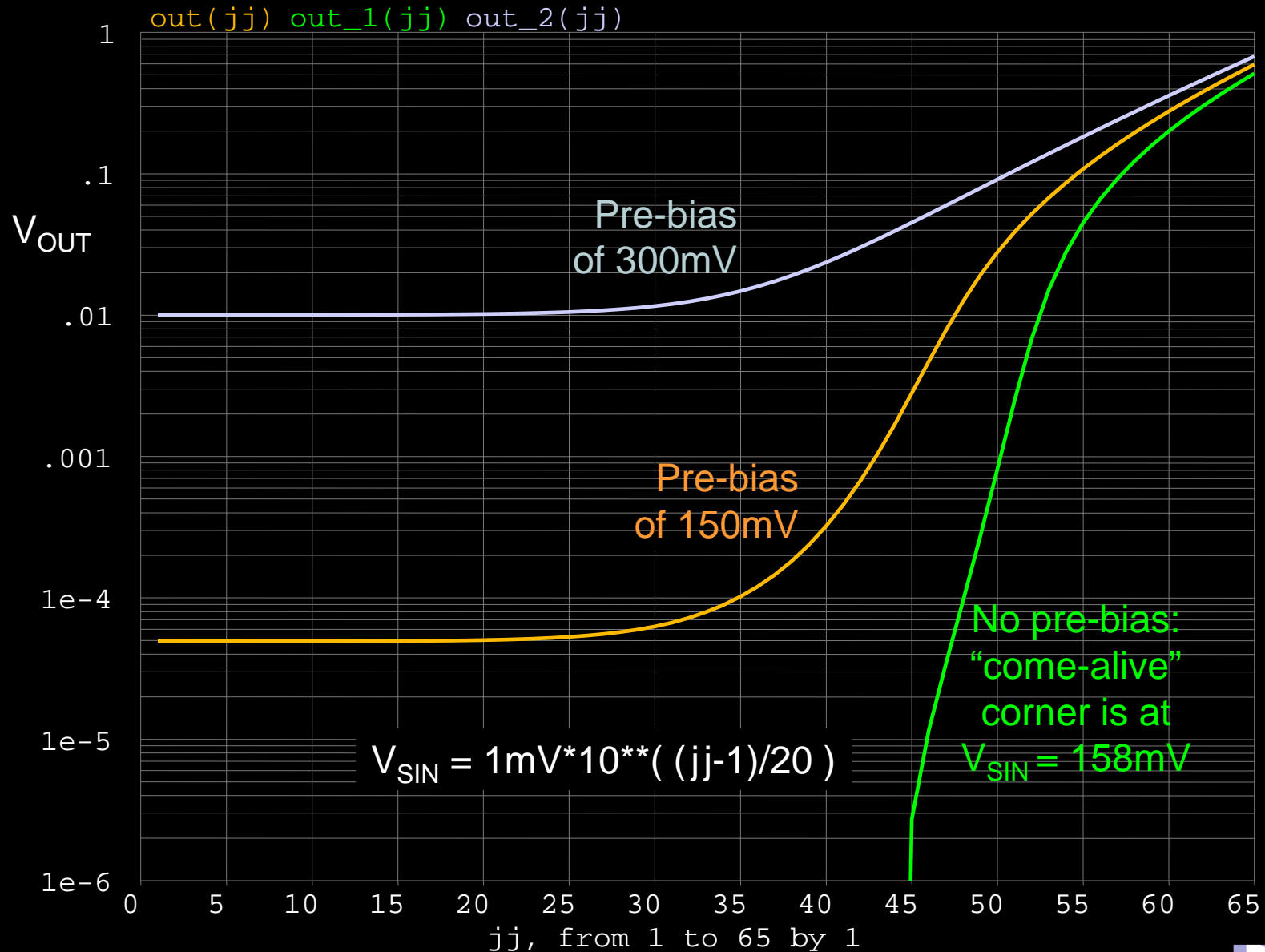


The great appeal of this Schottky-diode detector is its inherently wide bandwidth, passing effortlessly through the SHF band (3-30GHz) and still working very well into the EHF band (30-300GHz).

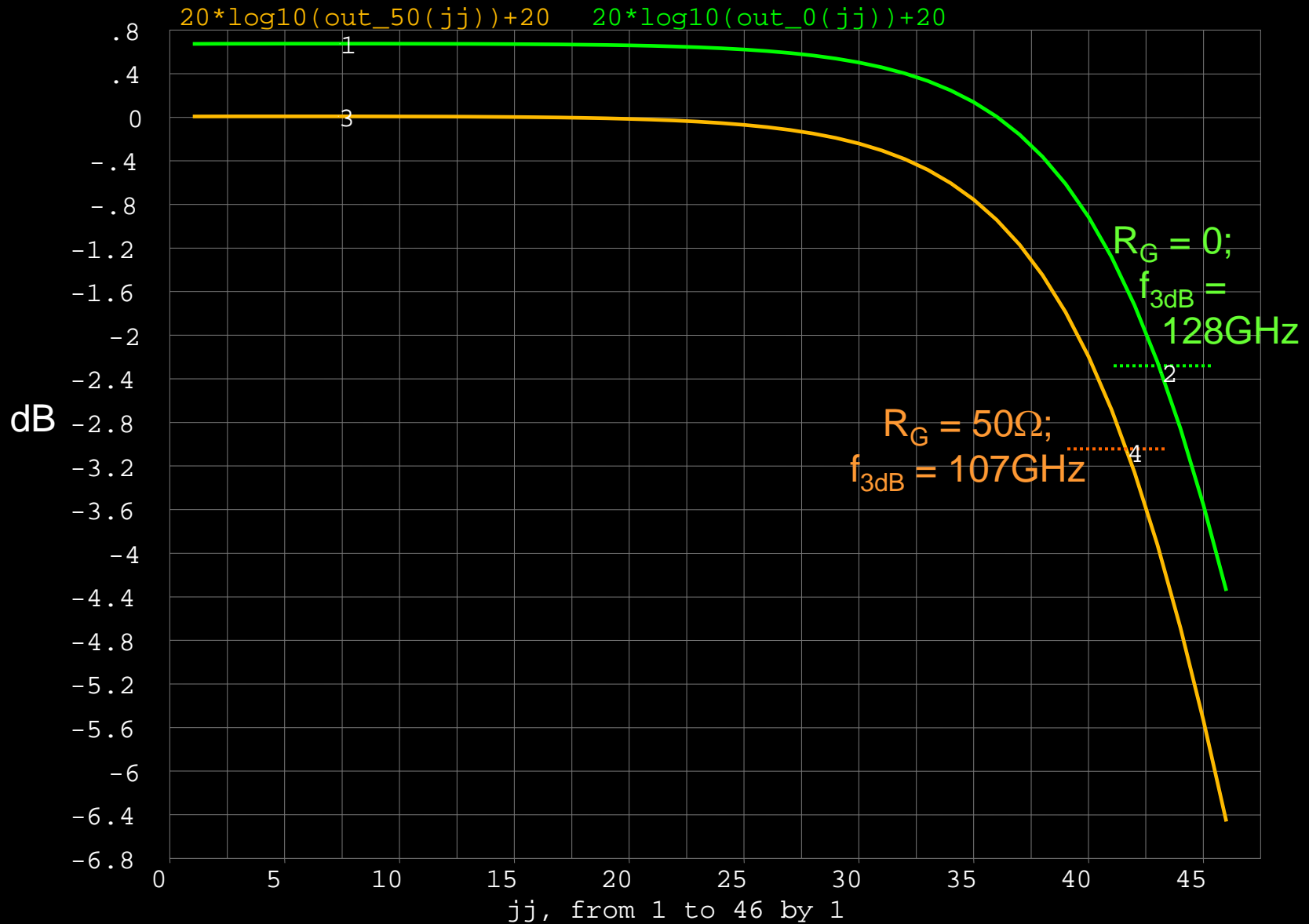
However, it suffers from extremely poor sensitivity, severe nonlinearity, and temperature dependence.

Thus, the challenge is to find ways to persuade some form diode detector to be free of these limitations, with the aim of providing dynamic range of at least 50dB.

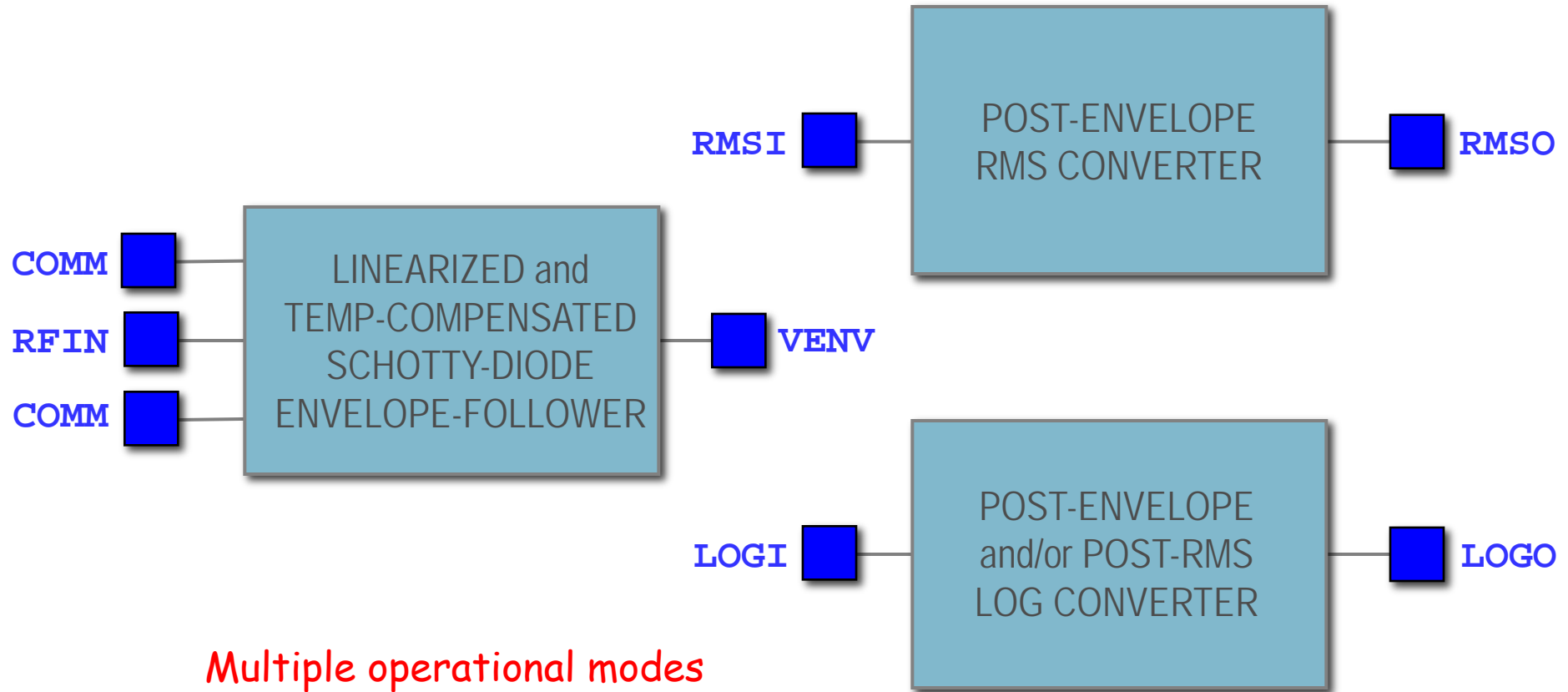
# AMPLITUDE RESPONSE, $f_{\text{sin}} = 1\text{GHz}$ , $1\text{mV}$ to $1.58\text{V}$ in $1\text{-dB}$ steps



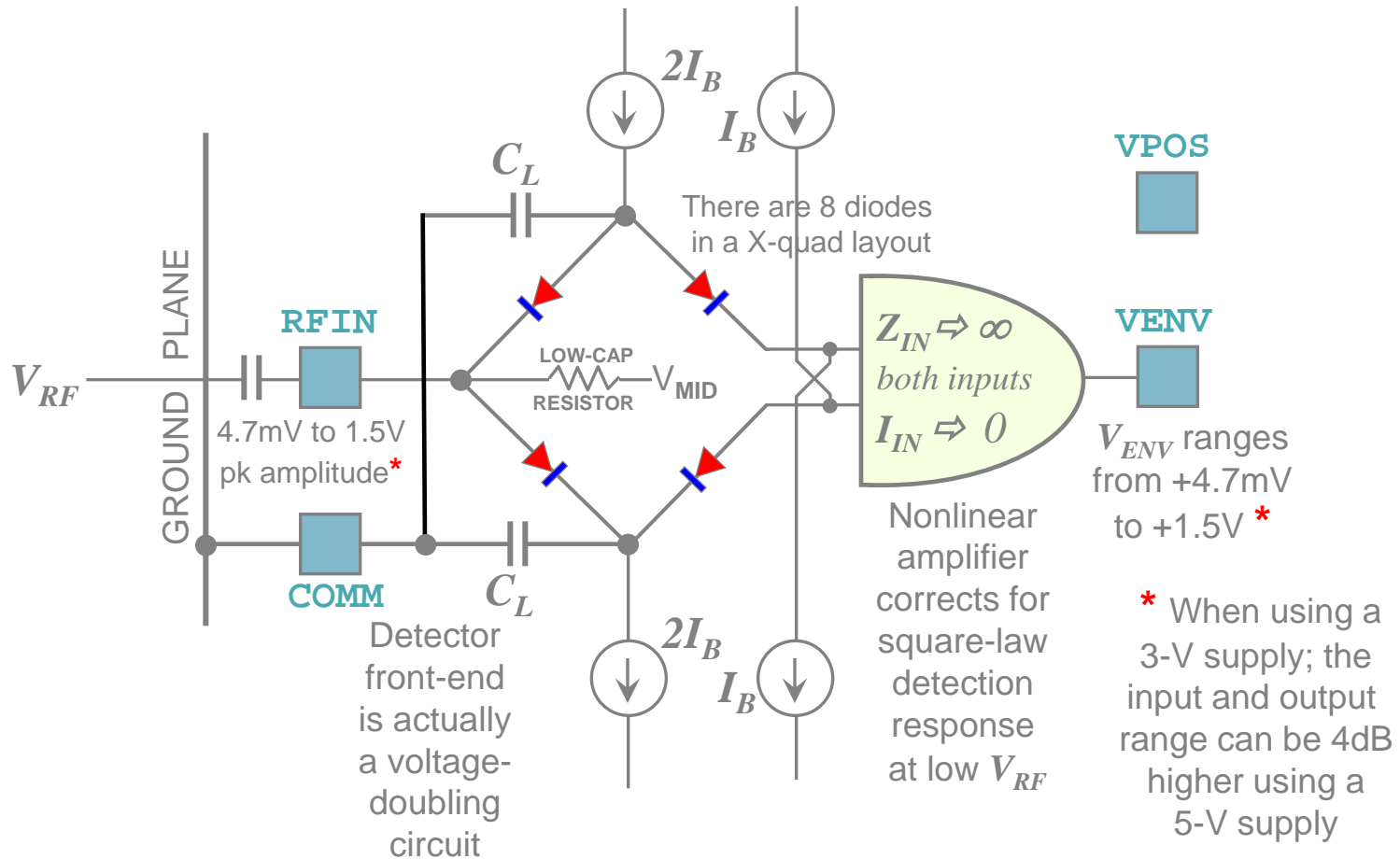
# FREQUENCY RESPONSE, $V_{\sin} = 300\text{mV}$ , 1GHz to 316GHz



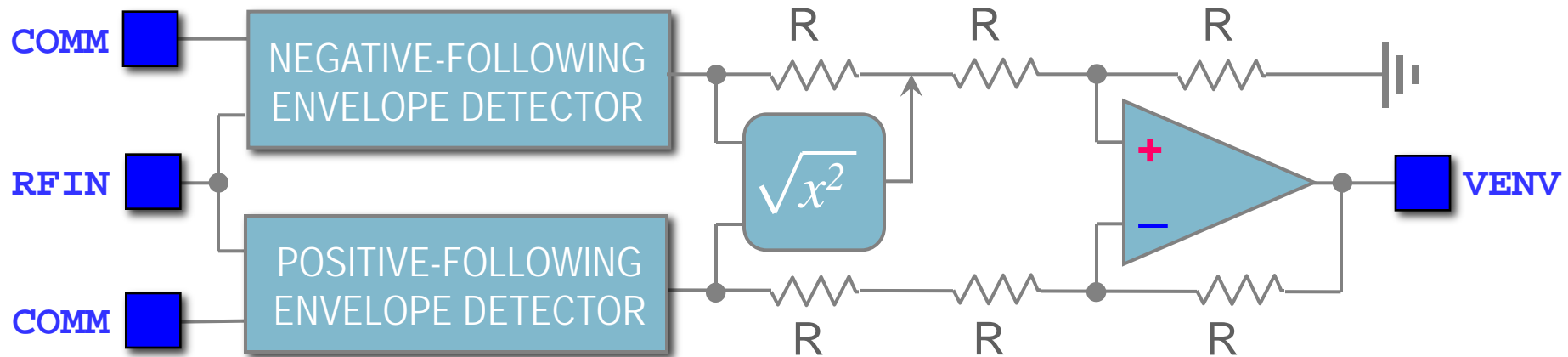
# DLVA: BASIC BLOCK DIAGRAM



# THE FRONT-END

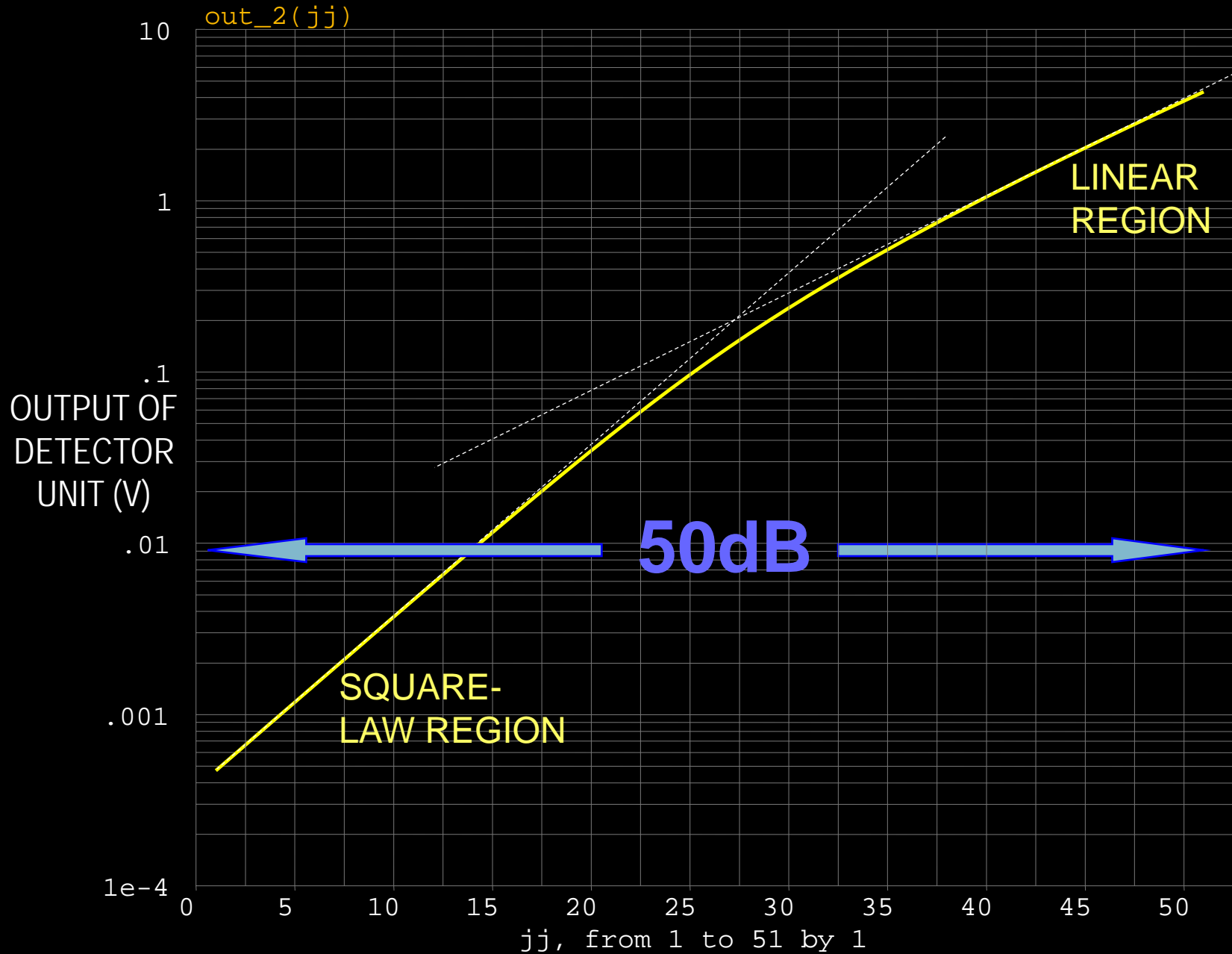


# FRONT-END PROCESSING

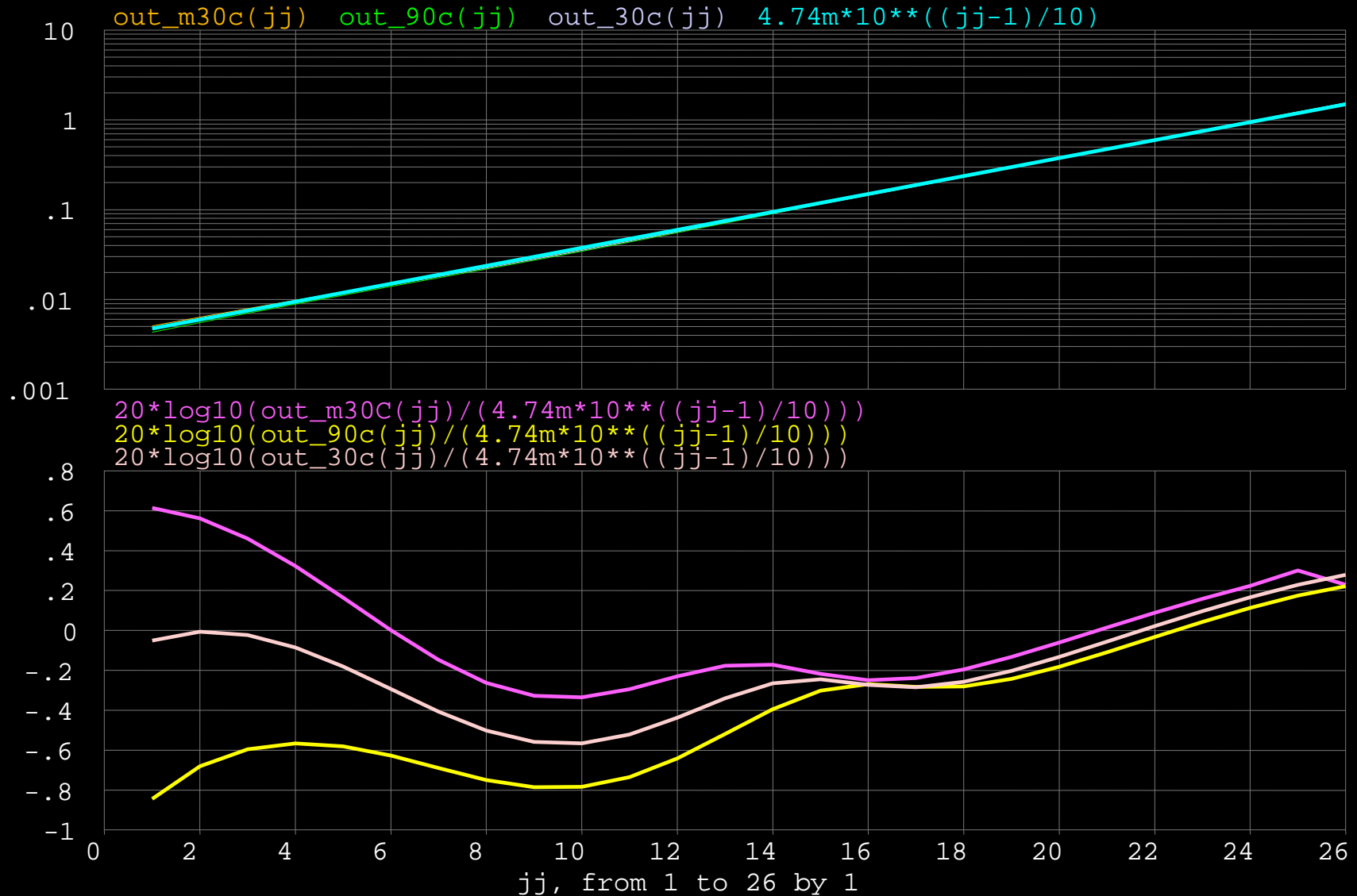


Because the input detectors provide voltage-doubling, the gain of the output section is 0.5.

The square-rooting cell is a particular sort of translinear circuit, having the double-amplitude differential voltage applied to its ultra-low-offset transconductance stage and the resulting current is used to generate the needed correction current to the output amplifier. To maintain fast response time, this current is kept fairly high and then attenuated in the resistor string.



An overnight run of March 20-21, 2010, checking temperature error at  
fsin = 500MHz, after small adjustment to SQRT scaling. Using inputs  
from 4.74mV to 1.500V (=50.006dB).



***tanh***

***sech***

***exp***

**TRUST THE  
MATHEMATICAL  
TRANSISTOR**

***log***

***cosh***

***sinh***