# THEORETICAL STUDY OF POWER RELATIVISTIC AMPLIFIERS FOR ELECTRON BEAM BUNCHING \*

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## Abstract.

Two schemes of electron-positron linear colliders with energy in the TeV range based on the two beam accelerator (TBA) concept are being studied now [1,2]. In a TBA the first accelerator-driver produces a high current electron beam. The beam must be bunched along all collider length..There are some schemes for generating of such high-intensity drive beam bunches. One of the schemes for generating of high-intensity drive beam bunches was tested in [3], where the Choppertron used transverse velocity modulation of a beam. Another idea for generating drive beam is to use the bunching in the free electron laser (FEL) [4]. It was shown in [5,6], that a high degree of bunching can be rather easily achieved in a short FEL amplifier.In [7,8,9], the first direct observation of the bunching of a relativistic electron beam, produced by a high power FEL interaction, was presented..

This work is devoted to the computer simulation and comparison of the bunching of a relativistic electron beam produced by a FEL and by a travelling wave tube (TWT). The calculation results of the electron beam bunching in the FEL are also compared with the experimental results in [7,8]. With the help of rather simple model it is shown that one can obtain the electron beam bunching in the TWT at a shorter length than in the FEL because the space gain in the TWT is higher than in the FEL.

#### **1 BEAM BUNCHING EQUATIONS**

The following equations describe the self-consistent spatial problem of the movement of a relativistic electron beam in a microwave electromagnetic field, which can be used both for a TWT and for a FEL in the Compton regime:

$$\frac{d\gamma_{j}}{dZ} = -\kappa a_{s} \sin \psi_{j}, \quad (j = 1, 2, ..., M)$$

$$\frac{d\theta_{j}}{dZ} = \frac{1}{\beta_{\parallel j}} - \frac{1}{\beta_{ph}}, \quad \frac{da_{s}}{dZ} = \eta < \sin \psi_{j} >$$

$$\frac{d\varphi}{dZ} a_{s} = \eta < \cos \psi > \qquad (1)$$

Here M is the number of electrons (macroparticles),  $\gamma_j$  is the  $j^{th}$  electron energy  $E_j = m_0 c^2 \gamma_j$  units of  $E = m_0 c^2$ ;  $m_0$  is the electron rest mass, c is the light velocity,  $Z = z\omega_0/c$  is the dimensionless longitudinal coordinate, and  $\omega_0$  is the microwave frequency. The value  $\theta_j$  is the phase of the  $j^{th}$  electron relative to the electromagnetic field;  $\varphi$  is the phase of the microwave complex amplitude ( $\hat{a} = a_s e^{i\varphi}$ ),  $\psi_j = \varphi + \theta_j$  is the total ponderomotive phase. The brackets in the equations (1) denote an average over the bucket. The value  $a_s = eE_s/m_0\omega_0c$ is the dimensionless amplitude of the microwave electric field,  $E_s$  is the amplitude of the microwave electric field.

The parameter  $\kappa$  is the microwave coupling coefficient. Its value depends on the type of used device. The parameter  $\eta = \left(\frac{I_b}{I_A}\right) \cdot \frac{2k}{N}$ , the constant  $I_A = m_0 c^3/e$ 

 $\cong$  17 kA; *N* is the norm of the electromagnetic wave[10];  $\beta_{\parallel j}$  is longitudinal electron velocity and  $\beta_{ph}$  is the microwave phase velocity.

# 2 SIMULATION OF BUNCHING PROCESS IN FREE ELECTRON LASER

To obtain the system of differential equations for simulation of the bunching process in a FEL with a helical wiggler we have used the corresponding equations [10,11], taking into account the effective frequency shift  $\omega_p$  connected with the plasma wave in a beam. It is similar to the accounting of a plasma wave in the resonance condition in [7,8]. Finally we get:

$$\frac{dw_j}{dZ} = \kappa_{1j}a_s \sin \psi_j \qquad (j = 1, 2, ..., M)$$

$$\frac{d\sigma_{j}}{dZ} = -\frac{(\kappa_{w} + \kappa_{s}) \cdot c}{\omega_{0}} + 1 + \frac{\omega_{p}}{\omega_{0}\beta_{z}} + \frac{1 + u_{w} + u_{w}u_{s}\cos\psi_{j}}{2\gamma_{0}^{2}(1 - w_{j})^{2}}$$
$$\frac{da_{s}}{dZ} = \eta_{1} < \frac{\sin\psi_{j}}{1 - w_{j}} > , \frac{d\varphi}{dZ}a_{s} = \eta_{1} < \frac{\cos\psi_{j}}{1 - w_{j}} > , (2)$$
where  $\kappa_{1j} = \frac{a_{w}}{2\gamma_{0}\gamma_{j}} = \frac{\kappa_{0}}{1 - w_{j}}$ . Here  $w_{j} = 1 - E_{j}/E_{0}$ 

is the relative change of the  $j^{th}$  electron energy;  $k_w = 2\pi/\lambda_w; \lambda_w$  is the wiggler period;  $k_s$  is the axial wavenumber for the waveguide resonance mode inside

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the wiggler;  $\gamma_0$  is the electron initial energy;  $\beta_z$  is the longitudinal dimensionless velocity of the resonant particle, determined from the relations:  $\beta_z = \sqrt{\beta_0^2 - \beta_\perp^2}$ ,  $\beta_\perp / \beta_z = a_w / \gamma_0$ ;  $k_0 = a_w / 2\gamma_0^2$ and  $\omega_p = \sqrt{\frac{4\pi n_e e^2}{m_0}} \cdot \frac{1}{\gamma_z \gamma^{1/2}}$  is the relativistic plasma frequency;  $n_e$  is the electron beam density and  $\gamma_z = 1/\sqrt{1 - \beta_z^2}$ . The parameter  $a_w = eB_w \lambda_w / 2\pi m_0 c^2$ , where  $B_w$  is the wiggler magnetic field amplitude. The coefficient  $\eta_1 = \left(\frac{I_b}{I_v}\right) \cdot \frac{2\kappa_0 \gamma_0}{N}$ .

For our simulation we used the electron beam and FEL parameters from [7,8]: electron beam energy ~2.2MeV, electron current inside the wiggler ~ 500 A, electron beam radius  $\sim 0.5$  cm, wiggler period 12 cm, wiggler field  $B_w = 1.1$  kG, microwave frequency  $f_0 = 3.5 \cdot 10^{10}$  Hz ( $H_{11}$  mode). In [7,8] the initial microwave power of 10 kW was injected into the circular waveguide in the  $H_{11}$  mode. The corresponding microwave electric field amplitude  $a_s \cong 4.4 \cdot 10^{-4}$ . Then following the we obtain parameter values:  $\kappa_1 \cong 0.022$ ,  $\eta_1 = 2.5 \cdot 10^{-4}$ , (the norm of wave  $N = N_{11} \cong 24.3$  in our case). We define the bunching parameter B in the same manner as in [7]:  $B = |\langle e^{i\psi} \rangle|$ .

We also simulated the initial energy spread in the electron beam with the help of 2000 electrons which had been distributed in 40 phase points from  $\theta = 0$  through  $\theta = 2\pi$  and in every point 50 electrons had the gaussian distribution upon the relative energy with dispersion  $\sigma$ .

Fig.1 shows the results of the calculations of the FEL microwave power and their comparison with the results obtained in [7,8]. To take into account the wiggler adiabatic entrance in [7,8], we suppose the exponential power growth to start at 48 cm from the wiggler beginning. This point corresponds to zero length in Fig.1. As one can see from Fig.1, these results are in a rather good agreement. Comparing the distribution of the microwave power along the wiggler obtained experimentally in [7,8], one can draw a conclusion that the difference observed between the maximum microwave power and the calculated one for monoenergetic beam in ~10 times may be partially caused by the initial energy spread in the electron beam. Fig.2 shows the calculated spatial evolution of the

bunching parameter *B*. The vertical lines in Fig.2 correspond to the time dependence meassurements of the correspond to the time dependence meassurements of the bunching parameter along the beam [7]. The regular wiggler part ends at the length approximately equal to 170 cm. Our further along the wiggler calculations are not sufficiently correct. So comparing the obtained curve corresponding  $\sigma = 5\%$  and the experimental results one

can suppose that the low experimental bunching parameters in [7] ( $B \sim 0.1$ ) were due to the beam energy



Figure: 1 The FEL microwave power versus the interaction length.

spread influence. The maximum bunching is reached in the region of maximum micromave power values.

## 3 SIMULATION OF BUNCHING PROCESS IN TRAVELLING WAVE TUBE

To simulate the electron beam bunching in a TWT amplifier based on the corrugated waveguide, we used the following system of differential equations:

$$\frac{dw_{j}}{dZ} = \kappa_{2}a_{s}\sin\psi_{j}, \quad (j = 1, 2, ..., M)$$
$$\frac{d\theta_{j}}{dZ} = \frac{1 - w_{j}}{\sqrt{(1 - w_{j})^{2} - \gamma^{-2}}} - \frac{1}{\beta_{ph}}, \quad (4)$$

$$\frac{da_s}{dZ} = \eta_2 < \sin \psi_j >, \quad \frac{d\varphi}{dZ}a_s = \eta_2 < \cos \psi_j >,$$

where  $\kappa_2 = \frac{\pi l_0}{\gamma_0 d}$ ,  $l_0$  - is the corrugation amplitude and

*d* is the corrugation spatial period [12]. The parameter  $\eta_2$  is the following:  $\eta_2 = \left(\frac{I_b}{I_A}\right) \cdot \frac{2k_2\gamma_0}{N_{01}}$ , where  $N_{01}$  is the norm of  $E_{01}$  type wave in the TWT.

We chose the following electron beam and  $E_{01}$  type electromagnetic wave parameters for our simulation:-electron beam energy ~2.2 MeV, electron current inside the TWT~ 500 A, electron beam radius ~ 0.5cm, microwave frequency  $f_0 = 17 \cdot 10^9$  Hz ( $\lambda \sim 1.76$  cm), initial microwave power in TWT=10 kW. The parameter d value was found from the dispersion curve of the corrugated wave guide having  $l_0 = 1mm$ ,

radius ~1.8 cm and when  $\beta_{ph}$  ~ 0.982. The dispersion curve was calculated with the help of the code URMEL. In our case



Figure: 2 The FEL bunching parameter versus the interaction length.

d was ~5.8 mm;  $\kappa_s d = \frac{2\pi}{3}$ . Then we have  $\kappa_2 \approx 0.102$  and  $\eta_2 \approx 3.2 \cdot 10^{-3}$  (the norm of wave  $N_{01} \sim 10$ ). For the initial microwave power ~ 10 kW in the TWT the corresponding dimensionless amplitude of the microwave electric field  $a_s \sim 6.8 \cdot 10^{-4}$ .



Figure: 3 The TWT micrrowave power versus the interaction length.

Fig.3 represents the calculated evolution of the microwave power as a function of the interaction length

along the TWT. The two curves correspond to the initial electron beam energy 2.2 MeV and 1 MeV.

One can also see from Fig.3 that the bunching length in the TWT decreases essentially when the initial beam energy is diminished. The dependence of the microwave power at the TWT exit on the energy spread (2.2 MeV) is rather weak. This dependence becomes sufficient when the initial electron beam energy decreases down to 1 MeV. As one can see from Fig.1 and Fig.3, the bunching length of the electron beam with energy 2.2 MeV in ~ 1.5 times less in the TWT than in the FEL. The energy spread in the bunches is larger in the TWT than in the FEL.

## **4 CONCLUSIONS**

Our simulations showed that a high degree of bunching of the electron beam having the energy in the range 1 - 2 MeV can be rather easily achieved in a short travelling wave tube.

The electron beam (2.2 MeV, 500 A) bunching occurs in a TWT at the distance in ~1.5 times shorter than in a FEL. It is connected with the fact that the space gain in the TWT is much higher than that in the FEL. But the bunch energy spread is larger in the TWT than in the FEL. The bunching effect depends weakly on the initial energy spread in the TWT and greatly in the FEL.

The TWT bunching efficiency decreases with the initial electron beam energy increase. Thus the TWT may be efficiently used for the electron beam bunching in the energy span 1 MeV through 2 MeV when the TWT length may be made of 0.5 m through 1 m.

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