# **OVERVIEW OF THE LHC DYNAMIC APERTURE STUDIES**

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# Abstract

The dynamic aperture (D.A.) of the LHC is estimated by tracking a thin-lens model of the lattice over a real machine time of 9 seconds. This raw result is then processed to predict the D.A. over longer times and realistic conditions. We discuss the reliability and limits of the method, the phenomena limiting the D.A. and scaling laws used for fast estimates. The long-term dynamic aperture at injection is close to the requirements, with prospects of further improvements.

# **1 INTRODUCTION**

In the LHC, the accumulation of the beam will take 7 minutes. Prior to this operation, the machine optics will have to be tuned. Experience from other colliders indicates that this step may take from 30 minutes to several hours, depending on the previous history of the magnets. The luminosity lifetime at peak energy is expected to be about 10 hours. Consequently the lifetime of a single beam must be significantly larger than one and ten hours respectively to allow efficient operation. Its estimation is however out of reach of present computers. Instead we resort to computing the dynamic aperture (D.A.) which is the largest stable transverse amplitude in a connected domain for a sample of 'typical' particles circulating in a simplified model of the accelerator. Even this computation is very demanding and is limited in practice to about 10 seconds of real LHC time. Using the results of additional studies, especially measurements of the D.A. in the SPS and HERA, the simulation results are extrapolated to realistic times and accelerator characteristics.

# 2 MODEL OF THE LHC MACHINE

The D.A. of LHC at injection is dominated by systematic multipoles in the superconducting dipoles. The sextupole  $b_3$  and decapole  $b_5$  are corrected by small coils at the end of each dipole. After correction with an accuracy better than 10%, these multipoles do not limit the D.A. The other allowed systematic multipoles are weak by design and contribute little to the D.A. The non-allowed multipoles, in particular  $a_4$ ,  $b_4$ ,  $a_3$  may have significantly large bias inside a production line and limit seriously the D.A. For the time being we assume eight independent production lines, each one being used to equip one arc. The average values of the multipoles are drawn from a Gaussian distribution cut at  $1.5\sigma$ . This severe cut reflects the capability of measuring and correcting trends during the fabrication.

At injection, the multipole errors considered are those

measured about 15 minutes after the end of magnet cycling. The consequences of the decay of the persistent currents and their spread from magnet to magnet, the ramp-induced multipoles related to the inter-strand resistance of the superconducting cable and the ramping rate have not been taken into account so far. Our scaling laws indicate that they should not degrade the D.A significantly even though they produce an increase of linear coupling.

The model of the machine includes a realistic description of the misalignment errors and linear imperfections. The closed orbit and linear coupling are corrected with the methods available in control rooms to 1 mm rms and a residual closest tune approach of less than 0.001. The fractional betatron tunes are chosen to be .28/.31. Tune scans have yet to be performed for the new optics and error table.

# 3 METHODS FOR CALCULATING THE DYNAMIC APERTURE

The evaluation of the D.A. is limited mainly by the computing resources and our ability to analyse the large quantity of data generated by tracking codes. Most of the choices about the methods result from these limitations.

Each real magnet is represented by one thin lens, providing symplecticity and fast integration. In collision, a finer model of the low- $\beta$  triplet quadrupoles (localization of the imperfections and edges properly represented) is yet to be implemented.

Sixty different imperfect machines are tracked to estimate the lower bound of the stable motion (D.A.) with a confidence level of 95%. A coarse tracking is done first to localize the D.A., followed by fine mesh tracking just below the coarse D.A. estimate.

To ensure a realistic sampling of initial conditions, 90 initial  $x_i$  and  $y_i$  are chosen within a range of  $3\sigma$ , with  $x_i/y_i = 1$  for equal emittances and  $\beta$ -functions in the two planes. The momentum deviation is initialized to 1.7  $\sigma_e$ . Studies [1] showed an insignificant loss of D.A. when going closer to the bunch edge. The three corresponding phases are initialized to zero as they naturally sample their phase space from turn to turn. Each initial condition is in fact a pair of two near-by particles ('Lyapunov pairs') whose distance is used to detect the onset of a chaotic motion. In the course of tracking, the variation of the amplitude ratio  $x_i/y_i$  is usually small. A coarser tracking is occasionally done for five different amplitude ratios and used to estimate the loss of D.A. with respect to  $x_i/y_i = 1$ . A computer cluster made of 10 powerful workstations (the NAP Project [2]) is dedicated to tracking with either Sixtrack [3] or MAD [4]. The estimate of the 'conventional' D.A. as defined above, takes less than 2 days of computation.

Certain machines are tracked up to  $10^6$  or  $10^7$  turns (10 minutes of LHC time ). The survival plots are fitted to a conjecture law to extrapolate to longer times. The onset of chaos is used as a qualitative indication and may represent a pessimistic estimate of the D.A. The effect of the ripple of the magnetic fields was studied for 60 machines [5] and the result is included in the interpretation of the D.A. provided by tracking.

For a fast evaluation of the D.A., the concept of D.A. per multipole  $d_n$  turns out to be surprisingly effective. The D.A. is tracked for a relevant value of one multipole n one at a time [6]. For a  $\alpha$ -times larger value of multipole n, the equation of motion remains invariant if the amplitudes (and the D.A.) are divided by  $\alpha^{1/(n-2)}$ . The D.A. for a set of multipoles is obtained by combining the  $d_n$ 's following the conjecture in [7]:

$$d_n(b_n = \alpha) = \frac{d_n(b_n = 1)}{\alpha^{1/(n-2)}} \qquad \frac{1}{d^4} = \sum_n \frac{1}{d_n^4} \quad (1)$$

So far, these scaling laws are verified to better than 10% by tracking in all cases considered.

# **4 TARGET DYNAMIC APERTURE**

Table 1: Relation between required and target D.A.'s.

Source or Uncertainty	Impact	D.A. in $\sigma$
Target D.A. at $10^5$ turns		12.0
Finite mesh size	-5%	
Initial to average amplitude	-5%	
Amplitude ratio $x_i/y_i$	-5%	
Extrapolation to $4 \ 10^7$ turns	-7%	9.6
Time dependent multipoles	-10%	
Ripple	-10%	7.8
safety margin	-20%	6.2

The halo of the injected beam will be cut at  $3\sigma$  in the injector. To allow for injection errors and orbit drifts, the primary collimator will be positioned at  $6\sigma$  [8]. To avoid perturbing the collimation by non-linearities, we require the D.A. to be at least  $6\sigma$ . This corresponds to observed good conditions in SPS and HERA. The particles in the beam secondary halo survive for up to 100 turns and may reach an amplitude of  $9\sigma$ . The amplitude smear must then stay within limits to maintain a good collimation efficiency. In Table 1, the target D.A.for tracking is computed back from the required actual D.A.. The safety margin reflects the discrepancy between calculated and measured D.A.'s as observed in the SPS and HERA. The target D.A. from tracking should be  $12\sigma$ . It could reduce to  $10.5\sigma$  if time dependent effects and safety margins are not independent and combined quadratically.



Figure 1: Distribution of D.A.'s

#### 5 RESULTS AND ANALYSIS

#### 5.1 Dynamic aperture

The latest results [5] were calculated for LHC version 4.3 [8] which has a lattice without tune split and a cell phase advance very close to  $90^{\circ}$ . The imperfection table 9607 was used. Figure 1 shows that the minimum D.A. is  $9.5\sigma$ .

### 5.2 Survival plot



Figure 2: Fitting law for the averaged D.A.

After proper averaging in phase space [9], an inverse logarithmic law fits well the LHC D.A. data (Figure 2) and seems to allow a reasonable extrapolation from  $10^5$  to  $4 \times 10^7$  turns. A further loss of 7% can be predicted.

#### 5.3 Contributions to the onset of unstable motion

The linear imperfections meaning corrected closed orbits (1 mm rms), a corrected betatron coupling and a residual  $\beta$ -beating of 10%, account for a loss of 5% of D.A.

Figure 3 shows that the amplitude detuning is only correlated with the D.A. when it is sufficiently large ( $\delta Q > 0.005$  at the D.A.).

The main limitation of the D.A. comes from  $a_4$  and  $b_4$ . Figure 4 shows that the D.A. is significantly increased by correcting the average values of  $a_4$  and  $b_4$  with additional coils at the end of each dipole. The difference coupling resonances  $(2Q_x - 2Q_y \text{ and } Q_x - Q_y)$  driven by these multipoles appear well correlated with the D.A. results [10],



Figure 3: Relation between D.A. and amplitude detuning  $\partial Q/\partial J$  in mm<sup>-2</sup>

suggesting that an improvement will be obtained from the new optics with a tune split.



Figure 4: Effect of  $a_4$ ,  $b_4$  on the D.A.

#### 5.4 Influence of the second beam at injection

The beams cross at an angle of  $\pm 150 \ \mu$ rad and are separated by  $\pm 1$ mm in the orthogonal plane. This gives an almost uniform separation of about  $7\sigma$ . Optionally the scheme can be rotated by  $45^0$  to minimize the beam-beam tune footprint. Tracking has revealed a clear reduction of the  $10^5$ -turn D.A. from over  $10\sigma$  to less than  $7\sigma$  (Figure 5) with 4 interaction regions. In the new version 5 of the LHC optics, an additional quadrupole allows a reduction of the  $\beta_{max}$  in the triplet quadrupoles, thereby increasing the beam separation to  $11\sigma$ .

#### 5.5 D.A. at collision energy

The aim of the first tracking campaign carried out at collision energy was to evaluate the requirements for the field quality in the low- $\beta$  triplet. For that purpose, an extreme situation with four physics insertions ( $\beta^* = 0.5$  m) and the nominal crossing angle of  $\pm 100\mu$ rad was considered.

Tuning	D.A. (σ)	
$\beta^* = 6 \text{ m}$	37.7	
$\beta^*=0.5~\mathrm{m}$	10.0	



Figure 5: D.A. with/without a second beam at  $7\sigma$ 

The beam-beam effect was disregarded. With the first estimates of the multipole field errors in the low- $\beta$  triplet, 6D tracking [11] shows clearly that the D.A. is only limited by the triplet errors and more precisely by a large  $b_{10}$  component which will be reduced by design.

# 6 CONCLUSIONS

Given the cost of safety margins, considerable care is being taken to describe in the most realistic way the LHC magnetic fields and the particle dynamics. The D.A., computed by element-by-element tracking, appears to be just sufficient at injection. We aim to gain a safety margin of 20% by reducing or correcting the dominant errors  $a_4$ ,  $b_4$ and  $a_3$ . The effect of the systematic normal and skew octupoles could be reduced in the new optics with tune split. At collision energy, without beam-beam effect, the D.A. is limited to  $10\sigma$  by the  $b_{10}$  of the low- $\beta$  quadrupole which will be reduced.

#### 7 REFERENCES

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