Abstract

This paper describes dynamic aperture limitation of VEPP-2M in round beam operation due to chromaticity correction sextupoles and a way of lattice optimisation. First order canonical perturbation theory is used to estimate influence of sextupoles on dynamic aperture. The estimates are compared with tracking results.

1 INTRODUCTION

Electron-positron collider VEPP-2M is a machine operating in the energy range 2E from 0.36 to 1.38 GeV. Its reconstruction to the round beam operation mode is in progress to test the round beam concept experimentally [1]. The existing and proposed beam and optics parameters are summarized in the Table 1.

The existing and proposed beam and optics parameters are shown in Table 1.

Table 1: Comparative parameters of VEPP-2M beams for existing flat beams option (“wiggler-on”) and round beams option. (Energy 510 MeV)

Because of significant changes in the machine lattice and increase of natural chromaticity, the chromaticity correction scheme has to be seriously reviewed.

2 VEPP-2M LATTICE PROPERTIES

The present magnetic lattice of VEPP-2M has four mirror-symmetric periods with low $\beta^*$ -functions in interaction regions (IR), which are provided by two quadrupole doublets. Chromaticity correction is performed by 16 sextupoles of two families $S_x$ and $S_z$ situated near the corresponding quadrupoles (the dispersion is non-zero in the IRs). Due to symmetry and betatron phase advance between members of one family close to $\pi$, this scheme is well-compensated and the dynamic aperture is determined by other reasons.

The main idea of VEPP-2M lattice modification to round beam mode is to replace the quadrupole doublets in the IRs by SC solenoids accomodated inside the detectors (fig. 1). As far as the dispersion is now zero over the IR, the compensation of the betatron tune chromaticities is possible only in the arcs. Unfortunately, the remaining there eight sextupoles can not compensate relatively high natural chromaticity ($\gamma \frac{\partial \nu_z}{\partial \nu_z} - \gamma \frac{\partial \nu_x}{\partial \nu_x} = -13$) without significantly reducing dynamic aperture.

The way out was found in installing one additional “compensating” sextupole family $St$.

3 NONLINEAR RESONANCES

The design operating point for the round beam mode is $\nu_x = \nu_z = 3.1$. Thus, the nearest sextupole resonances are:

$$\begin{align*}
3 \cdot \nu_x &= 10 \\
3 \cdot \nu_x &= 9 \\
2 \cdot \nu_x + \nu_z &= 9 \\
2 \cdot \nu_x + \nu_z &= 10
\end{align*}$$

Each resonance amplitude is defined by the relevant harmonic of the Hamiltonian [2]:

$$H(J, \psi, \theta) = \sum h_{m_x, m_z, n} \cdot e^{i(m_x \psi_x + m_z \psi_z + n\theta)}$$
Here \( J \) is the action variable, \( \psi \) is the betatron phase, \( \theta = s/R_0 \); \( R_0 \) is the gross radius. For sextupoles:

\[
h_{3,0,n} = \frac{R_0^2}{24 \cdot B \rho} \cdot \int \frac{ds}{2\pi} \mathcal{S}_{x} \beta_{x}^{3/2} e^{i(3\chi_x + n\theta)}
\]

\[
h_{1,0,n} = \frac{3R_0^2}{24 \cdot B \rho} \cdot \int \frac{ds}{2\pi} \mathcal{S}_{x} \beta_{z}^{3/2} e^{i(3\chi_z + n\theta)}
\]

\[
h_{1,\pm2,n} = \frac{3R_0^2}{24 \cdot B \rho} \cdot \int \frac{ds}{2\pi} \mathcal{S}_{x} \beta_{z}^{3/2} e^{i(3\chi_z \pm 2\chi_x + n\theta)}
\]

Where \( S = \partial^2 B_z/\partial x^2 \) is the sextupole gradient, \( \chi_{x,z}(s) = \int ds/\beta_{x,z} - \nu_{x,z}s \). Now we can apply the first order canonical perturbation theory [3]. If we consider one isolated sextupole resonance, for example \( \nu_x = 3 \) (detuning \( \epsilon = \nu_x - 3 \)), with the amplitude \( h \), the averaged Hamiltonian in the variables \( J \) and slow phase \( \Phi = \psi_x - 3\theta \) is:

\[
H = \epsilon \frac{R_0}{2} J_x + hJ_x^{3/2} \cos(\Phi)
\]

The dynamic aperture limitation is given by the Hamiltonian stationary points:

\[
\frac{\partial H}{\partial \Phi} = 0 \quad \frac{\partial H}{\partial J_x} = 0
\]

From these equations we obtain \( J_{th} = \frac{4}{\eta}(\frac{R_0 \epsilon}{2|m|})^2 \). And \( X_{max} = \sqrt{J_{th} / \beta} \) is the limit of the stable motion. One easily sees that the lower is \( |h| \), the higher is \( X_{max} \). In fact, the reality is much more complicated, as all the rest harmonics are not zero. But in our case this simplification is applicable because this resonance prevails (see table 2). Of course we should take into account unharmonicity of the ring \( \frac{\Delta \nu}{\nu_{max}} \).

### 4 LATTICE OPTIMISATION

As it has been already mentioned, we needed an additional sextupole family to improve the lattice properties. Position for this family was chosen in accordance with the lattice functions and available place in the existing ring.

A good place was found in the "technical" drift (fig 2), where dispersion is non-zero, and the \( \beta \) – functions vary strongly enough (\( \beta_1/\beta_2 = 5.7 \)).

The sextupole strengths were optimised to obtain the lowest Hamiltonian harmonics. Total unharmonicity of the ring is negative if we take into account sextupoles and non-linear edges of quadrupoles and solenoids (fig 3).

![Figure 3: Betatron tunes versus amplitude.](image)

Thus, the most attention was paid to the harmonic \( h_{1,0,3} \).

Table 2 shows comparison of different resonances amplitudes for two and three sextupole families.

<table>
<thead>
<tr>
<th>( 3\nu_x )</th>
<th>( \nu_x = 3 )</th>
<th>( 3\nu_x = 9 )</th>
<th>( 2\nu_x - \nu_x = 3 )</th>
<th>( 2\nu_x + \nu_x = 9 )</th>
<th>( 2\nu_x + \nu_x = 10 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_x,S_z )</td>
<td>24.8</td>
<td>201.5</td>
<td>52.1</td>
<td>56.3</td>
<td>114.2</td>
</tr>
<tr>
<td>( S_x,S_z,St )</td>
<td>10.1</td>
<td>151.7</td>
<td>39.5</td>
<td>20.1</td>
<td>62.7</td>
</tr>
</tbody>
</table>

Table 2: Resonance amplitudes (relative units).

The final parameters of the chromaticity correction sextupoles are:

| Family | Length (m) | \( \frac{d^2 H}{d
\eta^2} \) (T/m^2) |
<table>
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<tr>
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<tbody>
<tr>
<td>( S_x )</td>
<td>0.0395</td>
<td>205</td>
</tr>
<tr>
<td>( S_z )</td>
<td>0.0505</td>
<td>-456</td>
</tr>
<tr>
<td>( St )</td>
<td>0.09</td>
<td>-835</td>
</tr>
</tbody>
</table>

Table 3: Sextupole parameters.
5 RESULTS

Application of the third chromaticity correction family has strongly reduced all the sextupole harmonics. After optimisation, the dynamic aperture was calculated by tracking a particle through the lattice with averaging over the initial betatron phase. As the tracking has shown, the dynamic aperture increased from $7.8\sigma$ with two families to $14.5\sigma$ with three, which is considered to be satisfactory for the machine operation. The calculated dynamic aperture shape is shown in fig. 4.

![Figure 4: Dynamic aperture (tracking data).](image)

6 ACKNOWLEDGEMENTS

The authors wish to thank E.A. Perevedentsev for enlightening discussions and help in compiling this text.

7 REFERENCES


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