Abstract

Longitudinal echo signals have been produced in the CERN SPS by exciting a proton beam at 120 GeV/c with two short RF pulses separated by a suitable time-delay. The aim of the experiments was to confirm the analytical predictions for beam echoes in the SPS and to probe the applicability of beam echoes for a measurement of the energy distribution and diffusion coefficients in the accelerator. We summarise here the results obtained with bunched and un-bunched beams. For an un-bunched beam, the RF kicks are adjusted to excite the quadrupole mode of the bunch motion and the beam echo response can also be observed as a quadrupole mode.

1 INTRODUCTION

Echo phenomena have been well known in plasma physics for many years [1]. However, the effect has only been recently introduced to accelerator physics and first measurements of the echo signal in a storage ring suggest the possibility of using echo techniques in the beam diagnosis [2][3]. This paper summarises the echo measurements in the CERN SPS with bunched and un-bunched beams.

The echo signal is an interference pattern of two consecutive short RF-pulses. In the case of an un-bunched beam, the echo signals in the CERN-SPS were generated by exciting a coasting proton beam at 120 GeV/c with two short RF pulses using the 200 MHz travelling wave RF system. Without diffusion, one can find an exact solution for the echo response in a coasting beam and one can show that the beam echo appears at a time

\[ t^* = \frac{h_1}{h_2 - h_1} \cdot \Delta t \]  

and \( \Delta t \) the time measured from the second RF kick. \( J_n(x) \) are Bessel functions of the first kind with

\[ x = h_1 \epsilon_2 k_0 \Delta t \]  

and

\[ k_0 = \frac{\omega_0 \eta}{\beta^2}, \quad \eta = \frac{1}{\gamma_k^2} - \frac{1}{\gamma^2} \]  

\( \epsilon_1 \) and \( \epsilon_2 \) are perturbation parameters

\[ \epsilon_i = \frac{\omega_0 T_i}{2\pi} \cdot \frac{eV_i}{E_0}, \quad i = 1, 2 \]  

where \( \omega_0 = 2\pi f_0 \) is the revolution frequency, \( T_1 \) and \( T_2 \) the kick lengths and \( V_1 \) and \( V_2 \) the kick amplitudes of the first and second RF kick respectively. \( \epsilon_1 \) and \( \epsilon_2 \) are the relative energy gains during the first and second RF kick respectively. In the following we will assume that the relative energy gain during the two RF-kicks are small compared to the initial energy spread in the distribution.

The first line in Equation (2) implies that the leading term of the echo response varies with the time separation of the two RF-kicks like a Bessel function of the first kind \( J_1(x) \). The integral in Equation (2) indicates a strong influence of the energy distribution on the shape of the echo signal. In the following we will call it the form factor \( F(\tau) \) of the echo response. The strong cubic dependence of the damping term on \( \tau \) suggests the possibility of measuring even very small diffusion coefficients within a reasonably short time interval.

In the case of a bunched beam, the echo signals in the CERN SPS were generated by applying two short amplitude reductions to the nominal RF voltage of the 200 MHz travelling wave RF system which provides the RF buckets for the proton beam at 120 GeV/c.

2 EXPERIMENTAL SETUP

Fig. 1 shows the experimental setup for the echo measurement and Table 1 lists the relevant parameters for the CERN SPS.

<table>
<thead>
<tr>
<th>( f_0 ) [kHz]</th>
<th>( \eta )</th>
<th>( E_0 ) [GeV]</th>
<th>( f_{RF} ) [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.23</td>
<td>1.8 \cdot 10^{-3}</td>
<td>120</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 1: Machine parameters of the SPS.

3 EXPERIMENTAL DATA FOR AN UNBUNCHEd BEAM

We presented first results of un-bunched beam echo measurements already in [6]. Here, we will only briefly illustrate the effect and demonstrate how the echo measurement
can be used for measuring the diffusion coefficient. Fig. 2 shows a typical echo measurement for a kick amplitude of 500 kV. Each kick lasted for 92 μs (4 turns) and the two kicks have a time separation of 45 ms. Fig. 3 shows the superimposition of 25 such measurements, each having a different time separation between the first and the second RF-kick. The time separation varies from 45 ms to 230 ms. Note that for a time separation of 230 ms the echo response occurs only 2 minutes after the second RF-kick. The envelope of the echo signals in Fig. 3 agrees qualitatively with the expected Bessel function dependence indicated in the first line of Equation (2). However, the later the echo signal appears after the second RF-kick, the larger the divergence of the measured signal from the pure Bessel function envelope in the first line of Equation (2) indicating a non-vanishing diffusion coefficient in (2). For \( D \approx 10^{-13} \cdot s^{-1} \) the measured data agrees very well with the expected behaviour in (2) and corresponds to an emittance growth of

\[
\Delta \sigma_{\Delta p} / \sigma_0 = 10^{-3}
\]

in 115 days.

In [6] it was shown that the echo signal can also be used for measuring the energy distribution function and the energy spread in the beam.

4 MEASURING DIFFUSION COEFFICIENTS

The results of the previous Section indicated the possibility of measuring even very small diffusion coefficients with echo signals. This aspect was analysed in more detail by deliberately applying a noise signal to the 800MHz cavity in the SPS. The noise signal was calibrated by measuring the growth of the energy spread in the beam for different noise amplitudes from the longitudinal Schottky signal [7]. Over a time interval of 100 s, the Schottky signal was sensitive to noise signals which correspond to Once the noise signal was calibrated the noise amplitude was reduced to values where we could no longer observe a growth of the energy spread in the longitudinal Schottky signal. The beam echo could be successfully used to measure noise amplitudes which were at least two orders of magnitude smaller. Fig. 4 shows the measured diffusion coefficients versus the applied noise amplitude in dB. The points with noise amplitudes larger than -45 dB are calculated from measurements using a Schottky signal [7]. All other points are calculated from echo measurements. All data points agree with the analytical estimates and lie on a straight line with slope 1/10 in the double logarithmic plot.

5 BUNCHED BEAM ECHOES

Recent experiments in the CERN SPS extended the echo measurements to the case of a bunched beam. Fig. 5 and Fig. 6 show typical measurements, where the beam was excited with two consecutive quadrupole kicks. Each kick lasted approximately 200 μs. Both kicks in Fig. 5 had an amplitude of −500 kV compared to a nominal RF-voltage of 5.1 MV and were separated by \( \Delta t = 90 \text{ ms} \). The first kick in Fig. 6 had also an amplitude of −500 kV but the second kick was approximately four times smaller than

Figure 1: Schematic setup for the echo measurements.

Figure 2: The measured echo signal on a linear scale. The measured echo response corresponds to an approximately Gaussian energy distribution. The horizontal scale is 0.5 s per division.

Figure 3: The picture shows the superposition of 22 beam echoes for different separation times \( \Delta t \) as a function of time after the second RF kick. The time separation of the two RF kicks varies between 5 ms and 220 ms, resulting in an echo response up to two minutes after the second RF-kick.

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Figure 4: Inferred diffusion coefficients. The vertical axis shows the diffusion coefficients on a logarithmic scale in units of $10^{-11} \text{s}^{-1}$ and the horizontal axis the noise amplitude in dB. Assuming a diffusion process with white noise, one expects a straight line with slope 0.1 in this representation. The points with a noise amplitude larger than $-45 \text{dB}$ are calculated from measurements using the Schottky signal. All other points are calculated from echo measurements.

The un-bunched beam echo measurements in the CERN SPS agree well with the analytical expectations and show that echo signals can be used for measuring beam distributions and diffusion coefficients. Measurements of bunched beam echoes are just starting and a detailed study of this effect is still in progress.

7 REFERENCES