A STANDARD FODO LATTICE WITH ADJUSTABLE MOMENTUM COMPACTION *

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Abstract
An existing lattice made of identical FODO cells can be modified to have adjustable momentum compaction. The modified lattice consists of repeating superperiods of three FODO cells where the cells have different horizontal phase advances. This allows tuning of the momentum compaction or \( \gamma_t \) (transition) to any desired value. A value of the \( \gamma_t \) could be an imaginary number. A drawback of this modification is relatively large values of the dispersion function (two or three times larger than in the regular FODO cell design). This scheme also requires an additional quad bus for the modified cells.

1 INTRODUCTION
Particles travel along the reference orbit in an accelerator ring with momentum \( p_0 \) and period of revolution \( T_0 \). If they have a momentum deviation \( \Delta p \), the time of the arrival at a point of observation will be different. An offset in the revolution period \( \Delta T \) is given by:

\[
\frac{\Delta T}{T_0} = \left( \frac{1}{\gamma_t^2} - 1 \right) \frac{\Delta p}{p_0},
\]

where \( \alpha \), the momentum compaction, is a property of the lattice, and \( \eta = \alpha - \gamma_t^{-2} \) is called the phase-slip factor, \( \gamma_t \) is the Lorentz relativistic factor for the on-momentum particle. The momentum compaction factor is a measure of the path length difference between the off-momentum particle and the on-momentum particle. The transition energy \( \gamma_t \) is the energy at which \( \eta \) vanishes, i.e. it equals \( 1/\sqrt{\alpha} \).

In many accelerators \( \gamma_t \) lies in the acceleration range. We shall show that an existing FODO lattice can be modified so as to make \( \gamma_t \) either very large or even imaginary (negative \( \alpha \)). This could be used to for example to avoid having to cross transition, or to make zero momentum compaction negative. Teng [6] shows that a straight section with a phase advance of \( \pi \) can make the dispersion closed orbit negative at dipoles. Iliev [3] and Guignard [4] use a harmonic approach, where the betatron function is modulated to produce negative values of the momentum compaction by way of resonance conditions. We have reported earlier [5] and [8] the use of flexible-momentum compaction lattices to minimize dispersion values.

2 NORMALIZED DISPERSION FUNCTION
The dispersion function \( D \) needs to be adjusted through the FODO cell to obtain a different integral of its values through dipoles. Because the dispersion function satisfies a second order inhomogeneous differential equation of motion [7] it is useful to use the normalized dispersion function with components \( \xi \) and \( \chi \) as previously defined [5]:

\[
\xi = \sqrt{\beta_x} D' - \frac{\beta_x'}{2\sqrt{\beta_x}} D, \quad \chi = \frac{1}{\sqrt{\beta_x}} D,
\]

where \( \beta_x \) and \( \beta_x' \) are respectively the horizontal betatron amplitude function and its derivative [7], \( \xi \) and \( \chi \) are projections of the normalized dispersion vector.

3 FODO CELLS WITH ADJUSTABLE MOMENTUM COMPACTION
As an example we modify the lattice of the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, where the dipole length is \( L_d = 9.45 \text{m} \) and the

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Figure 1: Courant-Snyder functions in the three RHIC cells under normal operating conditions.
cell length is \( L_c \simeq 29.6m \). In our example we use super-
periods of three cells as previously proposed by Guignard
[4]. Figure 1 shows the Courant-Snyder functions for three
cells under the standard operating conditions.

We chose a point of reflection symmetry of the orbit
functions at the middle of the three cells. We modify the
quadrupole strengths so as to obtain a negative momen-
tum compaction. The quadrupole strengths which make the
momentum compaction negative, with \( \gamma_t = i358 \), are
presented in Table 1.

<table>
<thead>
<tr>
<th>Q. Strengths</th>
<th>( \kappa(1/m) )</th>
</tr>
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<tbody>
<tr>
<td>( \kappa_{QDA} )</td>
<td>0.05648</td>
</tr>
<tr>
<td>( \kappa_{QF} )</td>
<td>0.07330</td>
</tr>
<tr>
<td>( \kappa_{QD} )</td>
<td>0.09634</td>
</tr>
<tr>
<td>( \kappa_{QFC} )</td>
<td>0.13057</td>
</tr>
</tbody>
</table>

The normalized dispersion plot for this case is shown in
figure 2.

Figure 3 shows the orbit functions \( \beta_x, \beta_y \) and D in the
modified cells (the centers of symmetry are at QDA and
QFC).

We see that the penalty paid for making the dispersion
negative is almost a doubling of the maximum \( \beta_x \) and \( \beta_y \)
function and of the dispersion function. In addition the tunes \( \nu_x \) and \( \nu_y \) are changed substantially.

4 CONCLUSION

The momentum compaction of the standard FODO cell lat-
tice could be adjusted by the modulating the betatron func-
tions to any desired values with the drawback of larger val-
ues of the dispersion and betatron functions. A range of dis-
ersion function offsets, obtained by the quad adjustments,
falls within twice of the optimum FODO cell dispersion
values. The maximum values of \( \beta_x \) and \( \beta_y \) are less than two
times the values at optimum betatron tunes (\( \nu = \pi/2 \)). The
beam size was less than \( \sqrt{2} \) larger. This report shows that a
resonance condition ([4], [3]) was not necessary to achieve
different values of the momentum compaction within stan-
dard FODO cells. We used an existing FODO lattice to
accommodate the momentum compaction value, but we do
not recommend it the lattice of a new accelerator.

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