NONLINEAR BEAM DYNAMICS STUDY AT THE VEPP-4M

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Abstract
Nonlinear dynamics of transverse beam motion has been studied experimentally at the VEPP-4M electron-positron collider. One aspect of nonlinear beam behaviour described in this paper is the amplitude-dependent tune shift near nonlinear resonances. The measurement results are presented and compared with the theoretical prediction.

1 INTRODUCTION
Despite the progress in explanation of nonlinear phenomena in circular accelerators, there still is a gap between computer simulations or analytical predictions and reality. To reduce this gap, many dedicated experiments have been performed in both hadron and lepton machines in recent years. Rather complete and well-prepared reports can be found in Ref.[1],[2].

As it was recently found [3], the dynamic aperture of VEPP-4M is strongly affected by magnetic field nonlinearities. The measured value of the dynamic aperture does not follow the lattice model with nonlinear components computed from direct magnetic measurements. In order to explain this discrepancy, nonlinear motion features were measured extensively at the VEPP-4M storage ring in 1995-1996.

This paper concerns the study of nonlinear detuning under various experimental conditions. The amplitude-dependent tune shift was studied for both sextupole and octupole perturbation using a FFT spectrum of coherent beam oscillation excited by fast kicker. The experimental data agree quite well with the tracking simulation and model prediction.

2 HARDWARE DESCRIPTION
The VEPP-4M storage ring is a 6 GeV racetrack electron-positron collider with a circumference of 366 m. The study was performed at an injection energy of 1.8 GeV. To produce coherent transverse motion, the beam is kicked vertically or horizontally by pulsed electromagnetic kickers. Oscillation of the beam centroid and beam intensity are measured turn-by-turn with a beam position monitor (BPM) SRP3 for up to 4096 revolutions. The rms displacement resolution is $\sigma_{x,z} \approx 70\mu$m in a 1 to 5 mA beam current range.

For a theoretical prediction the following sources of magnetic field nonlinearity were taken into account:

1. 32 vertical and horizontal sextupole corrections distributed along the magnets in the arcs (two families, $DS$ and $FS$).
2. Lumped sextupoles $SES2$, $NES2$ and $SES3$, $NES3$ located symmetrically around the interaction point.
3. Quadratic field component produced by the arc magnet pole shape (two families, $SSF$ and $SSD$).
4. Octupole correction coils incorporated in the arc magnet main coils (32 corrections, two families, $SRO$ and $NRO$).

Because of high beta-function values ($\approx 120$ m), a bulk of natural chromaticity of the ring is produced by the final focus quadrupoles ($\approx 50\%$ in a horizontal plane and $\approx 60\%$ in a vertical plane). This chromaticity is locally compensated by the $SES2/NES2$ and $SES3/NES3$ sextupoles. Hence, we can expect that the influence of these sextupoles on the nonlinear dynamics should be emphasized.

3 AMPLITUDE-DEPENDENT TUNE SHIFT
Coherent beam oscillation is fired by several kicker pulses with different amplitudes, and tune was extracted from a FFT spectrum of 1024 revolutions. To avoid decoherence and various damping mechanisms, a special algorithm is developed to extract beam displacement from first 30-50 revolutions. The accuracy of the tune measurement is better than $2 \cdot 10^{-4}$. Before kick measurement the following preparatory adjustments and calibrations are made:

1. Beta-functions are measured in the $SRP3$ pickup station: $\beta_{x} = 12$ m, $\beta_{z} = 4$ m (model values are $\beta_{x} = 13.2$ m, $\beta_{z} = 4.5$ m).
2. Tune-current dependence is measured ($\delta\nu_{x} = -3 \cdot 10^{-4}$ mA$^{-1}$, $\delta\nu_{z} = -1.3 \cdot 10^{-3}$ mA$^{-1}$) and taken into account. To reduce this effect, in every kick series the beam intensity is dropped down for less than 0.3 mA.
3. The linearity and absolute kick amplitude calibration is made by scrapers with accuracy better than 0.1 mm.

![Figure 1: Typical amplitude dependence of the betatron tune.](image-url)

Figure 1: Typical amplitude dependence of the betatron tune.

For both octupole and sextupole perturbation, the nonlinear tune shift is proportional to the squared initial beam...
A general 2D form of the amplitude-dependent tune shift can be expressed as (a second order approximation):

\[
\Delta \nu_x (a_x, a_z) = C_{11} \cdot 2J_x + C_{12} \cdot 2J_z, \\
\Delta \nu_z (a_x, a_z) = C_{21} \cdot 2J_x + C_{22} \cdot 2J_z,
\]

where \( C_{nm} \) depends on particular perturbative potential. The measured and estimated coefficient values are listed in Table:

<table>
<thead>
<tr>
<th>( C_{nm} \cdot 10^4 ) (m(^{-1}))</th>
<th>Theory</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{11} )</td>
<td>50</td>
<td>3900</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>-840</td>
<td>-1400</td>
</tr>
<tr>
<td>( C_{21} )</td>
<td>-840</td>
<td>-1750</td>
</tr>
<tr>
<td>( C_{22} )</td>
<td>-830</td>
<td>-1400</td>
</tr>
</tbody>
</table>

The difference in theoretical and experimental \( C_{11} \) made us explore systematically the horizontal nonlinearity of the ring. The later may be induced by octupole and/or sextupole (in second order) errors that we did not consider in our model simulation. To distinguish, which one defines \( C_{11} \) in our case, we used the difference between the determination of the octupole and sextupole tune shift. For an octupole potential the horizontal tune shift is independent on an initial tune value:

\[
\Delta \nu_x^{(o)} (J_x) = \frac{dx}{ds} \int_{0}^{C} O(s) \frac{\partial^2 x}{\partial s^2} ds + o(J_x^2),
\]

where \( C \) is the machine circumference and \( O(s) = (d^2 B_z(s)/dx^2)/B_0 \) is the effective octupole strength. Sextupole tune shift on the contrary depends on an initial tune value in a resonant way and can be written as

\[
\Delta \nu_x^{(s)} (J_x) \simeq -J_x \cdot 36 \frac{A_{3m}^2}{3 \nu_x - m} + o(J_x^2),
\]

where \( A_{3m} \) is the azimuthal harmonic of the sextupole Hamiltonian.

The measured horizontal tune shift as a function of an initial tune \( \nu_x \) in the vicinity of the resonance \( 3\nu_x = 26 \) is shown in the top of Fig.2. To fit the computed curve with the data points, we should move the curve in positive direction by a value \( \Delta \nu_x^{(o)} / 2J_x \approx 3500 \text{ m}^{-1} \) independently of initial tunes. We can propose that this value is the octupole contribution to the total tune shift.

To verify validity of this assumption, first, we controlled octupole perturbation by the octupole correctors \( SRO/NRO \), distributed along the ring arcs. Changing their excitation current from 0 A to -0.5 A provides a decrease of the average level of the \( C_{11} (\nu_x) \) for all the unperturbed working points \( \nu_x \) to a magnitude \( \Delta \nu_x^{(o)} / 2J_x \simeq 1700 \text{ m}^{-1} \), while its resonant behaviour remains the same.

Next, we reduced the sextupole driving term responsible for the resonance \( 3\nu_x = 26 \). The excitation current of the \( SES2/NES2 \) sextupoles was decreased from 8 A to 4.4 A and the relevant sextupole harmonic became twice as less. Uncompensated chromaticity was corrected by the sextupole coils in the regular arc magnets. One can see that the average level of the detuning retains, while the sextupole contribution is reduced. It is clearly seen from the resonance stopband \( \Delta \) that was defined as a distance between the points where the beam lifetime became as low as 300÷400 s.

A detailed tracking study points out to the final focus (FF) quadrupoles \( EL1/EL2 \) as a most probable source of the octupole error. Otherwise we should suppose an unrealistically high nonlinear error in regular arc quadrupoles. It was shown in Ref.[4], that quadrupole edge fields can produce large detuning; however, in our case the relevant contribution to the \( C_{11} \) coefficient is ten times as less as the measured one. That is why we suspected that the octupole error was distributed in the FF quadrupoles.

Following this indication, we have done a set of measurements. 1. According to (1), first we measured quadratic...
dependence of the octupole detuning on $\beta_x$. The excitation current in the quadrupoles $EL1/EL2$ was changed, the tune point was adjusted back by the arc quadrupole magnets, the chromaticity was compensated and closed orbit distortion was corrected. The results of the measurement are presented in Fig.3 where $C_{11}$ is shown as a function of $\beta_x^2$ in the FF quadrupoles. From this result we can estimate the value of the octupole errors as follows: $O \simeq 0.5 \text{G/cm}^3 = 8.1 \text{m}^{-4}$.

2. Employing steering coils around the interaction region gives a possibility of measuring integrated magnetic field distribution by an electron beam. We can produce a local symmetric or antisymmetric orbit bump in the horizontal plane as it is shown in Fig.4, keeping closed orbit distortion in the rest of the ring within 0.5 mm, and measure the betatron tune shift caused by the

magnetic nonlinearities. In case of the symmetric bump the main contribution to the tune shift is provided by the chromatic sextupoles $SES/NES$, located inside the bump. In case of the antisymmetric bump the sextupole contribution is substracted and measured betatron tune shift as a function of orbit displacement $x$ in the FF quadrupoles unambiguously demonstrates presence of integrated octupole nonlinearity (Fig.5).

Using least square fitting of this curve one can easily extract the octupole contribution to the gradient error. In our case it gives us the octupole error in the FF quadrupole $O \simeq 0.5 \text{G/cm}^3$ that agrees well with the previous estimation.

3. Due to the symmetry of the vector magnetic potential, a dodecapole component $B_5$ is an inherency of a quadrupole field. For an ideal quadrupole magnetic field is represented as

$$B_z = B_1x + \frac{1}{3!}B_5x^5 + ...$$

Hence the octupole component in a quadrupole lens is proportional to the dodecapole and squared closed orbit distortion $O \propto B_5 x_{co}^2$. To check it, we made a symmetric local bump in the FF region and study the horizontal amplitude-dependent tune shift as a function of $x_{co}$. The result is depicted in Fig.6. Estimation of the dodecapole value gives $B_5 \simeq 0.2 \text{G/cm}^3 \simeq 2.4 \cdot 10^4 \text{m}^{-6}$.

![Figure 4: Closed orbit bump to measure the integrated magnetic field distribution by electron beam around the interaction point (IP). Left - antisymmetric, right - symmetric.](image)

![Figure 5: Betatron tune shift as a function of orbit deviation in the FF quadrupoles in case of the antisymmetric bump.](image)

![Figure 6: $C_{11}$ coefficient as a function of the COD in the FF quadrupoles.](image)

These measurements seem to point out the FF quadrupoles as a probable source of strong horizontal detuning due to octupole field error $O \simeq 0.5 \text{G/cm}^3$ (1.8 GeV). Unfortunately, direct field measurement gives only half of this value and the reason for this discrepancy was not understood yet.

4 CONCLUSION

We studied amplitude-dependent tune shift at the VEPP-4M electron-positron collider. The measurements were performed by the single turn-by-turn BPM technique. The experiments indicate that the features of our nonlinear system are strongly influenced by octupole perturbation that does not follow from the model lattice representation. All measurement results agree well with the theoretical prediction if we assume small (about 0.5 G/cm²) octupole error in the final focus qudrupoles. Unfortunately, direct magnetic measurement provides only one half of the required value.

5 REFERENCES