LONGITUDINAL INSTABILITY DUE TO THE RANDOMNESS OF HIGHER MODE FREQUENCIES OF RF CAVITY

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Abstract

The resonance frequencies, shunt impedances and Q-values for the higher order modes in our designed cavity are calculated by the computer codes URMEL and MAFIA. The longitudinal instabilities for the higher order modes are calculated by the eigen mode analysis of Sacherer's integral equation. The growth rates and the tune shifts in the electron storage ring are calculated by considering the randomness of the higher mode frequencies of a cavity due to the construction errors. The results with the construction errors are compared to those of without error cases for the dipole mode and quadrupole mode.

1 THE EIGENVALUE EQUATION AND INSTABILITIES

The method of analysis of longitudinal instabilities is that of Sacherer's [1] whose main result is an integral equation for the eigen mode and eigenfrequencies of the longitudinal bunch oscillations. Our method of analysis is based on Zotter's formalism [2, 3] which is the eigenmode analysis of Sacherer's integral equation without mode coupling for a Gaussian beam and Chin's method [4]. Sacherer's integral equation can be reduced to the eigenvalue equation by expanding the radial mode function in orthogonal polynomials and the eigenvalue equation is written by

$$\det(M - \delta\nu I) = 0 \qquad (1)$$

where the matrix M is the interaction matrix which is no longer diagonal and $\delta\nu$ is the relative complex tune shift. The infinite number of solutions for the eigenvalue is specified by the radial modes and azimuthal modes. The real part of the coherent frequency shift gives the real coherent mode frequency shift and the imaginary part gives the instability growth rates. The complex tune shifts of coherent oscillations for a single resonator impedance can be calculated by solving the eigenvalue equation. For the calculations of the growth rates and the tune shifts in the our designed storage ring, we have to consider the randomness of higher mode frequencies due to the construction errors of the cavities. Each impedance peak can contribute to both the growth rate and the damping rate of the Robinson instability according to the position of the impedance peak with respect to the harmonics of the revolution frequency. The Monte Carlo method is used to calculate the average values and rms spreads of the complex tune shifts due to both the accelerating mode impedance and randomly distributed higher mode impedances.

Table 1: Main parameters for calculations

Energy	2 GeV
Average machine radius	26.866 m
Synchrotron tune	0.0059
Revolution frequency	1.78 MHz
Momentum compaction factor	0.0024
Bunch length	0.01 - 0.1 cm
Harmonic number	282
RF frequency	501.96 MHz
Peak voltage	1.5 MV
Cavity length	40 cm
Cavity diameter	46.03 cm

2 THE TUNE SHIFTS AND GROWTH RATES WITH STATISTICAL ANALYSIS

The instability by the statistical analysis have been studied by many authors [4, 5, 6]. Especially, Chin took the impedance of a cavity, not a single cell to take into account the actual fields seen by particles. The resonance frequencies, shunt impedances and Q-values for the higher order modes in our designed cavity with the length of 40 cm, diameter 46.03 cm, peak voltage 1.5 MV and frequency 501.96 MHz and R/Q 58.89 ohm are calculated by the computer codes URMEL [7] and MAFIA [8]. We need random numbers with a distribution that is different from a uniform one. The general approach to obtain a set of random numbers with a given distribution is to start with a uniform set and change it into the one required. We assume that the spread in the higher mode frequencies due to the construction errors of cavities is a Gaussian with standard deviation σ. The magnitude of σ at resonant frequency f_r is estimated by using the formula of the pill box cavity. Thus the standard deviation σ is given by the relative construction error ϵ as $\sigma \sim \epsilon f_r$. We assume the relative construction error of the designed RF cavity is 0.5×10^{-3} for these calculations. The beam parameters used in our calculations are the energy of 2 GeV, average machine radius of 26.866 m, momentum compaction factor 0.0024, betatron tune 6.29, revolution frequency 1.78 MHz, synchrotron tune 0.0059 and bunch length 0.01 to 0.1 cm as shown in Table 1.

The results for the tune shifts versus the bunch length are shown in Fig. 1 and 2 for two azimuthal modes of the dipole where the bunches move rigidly as they execute longitudinal synchrotron oscillations and quadrupole where the bunch head and tail oscillate longitudinally out of phase and three radial modes. Fig. 1 shows the relationship between the tune shifts and the bunch length for the azimuthal mode m=1(dipole mode) and the radial modes of k=0, 1 and 2.

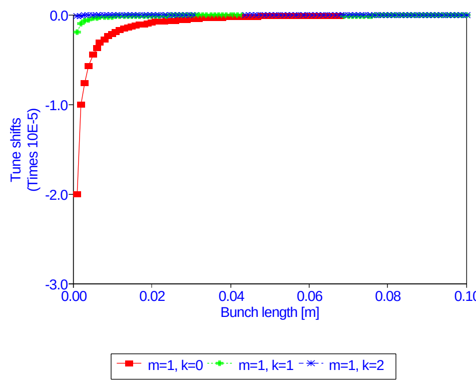

Figure 1: The relationship between the tune shifts and the bunch length for m=1 and k=0, 1 and 2.

Fig. 2 shows the relationship between the tune shifts and the bunch length for the azimuthal mode m=2(quadrupole mode) and the radial modes k=0, 1 and 2. In Fig.1 and 2, Tune shifts are not sensitive to bunch length as bunch lengthen in both cases of the dipole and quadrupole modes.

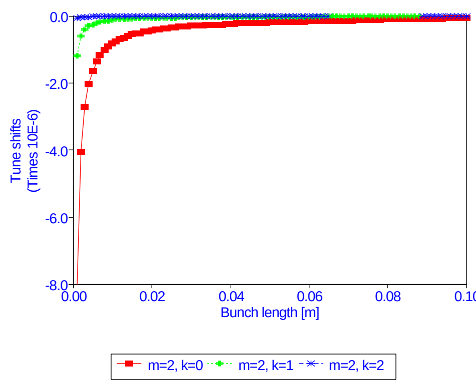

Figure 2: The relationship between the tune shifts and the bunch length for m=2 and k=0, 1 and 2.

Fig. 3 and 4 show the comparison of the growth rates and the tune shifts for with and without errors in the case of m=1 and k=0. Fig. 5 and 6 give the comparison of the growth rates and the tune shifts for with and without errors in the case of m=2 and k=0. The growth rates with and without errors give the maximum error deviation of about a factor of ten by the statistical analysis. For the tune shifts with and without errors, the maximum error deviations are increased by a factor of four. We have calculated the growth rates and the tune shifts for the several hundred cases and considered several data to get the maximum deviation of the tune shifts and growth rates of coherent synchrotron oscillation due to randomly distributed RF cavity impedances.

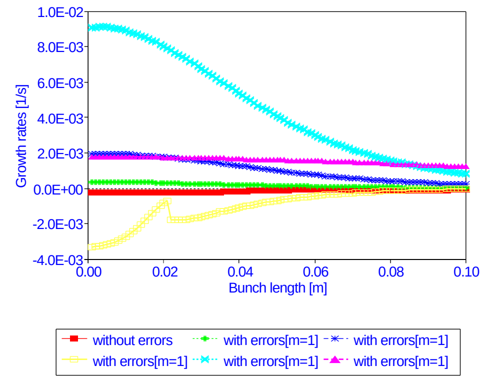

Figure 3: The relationship between the growth rates and the bunch length with and without errors for m=1 and k=0.

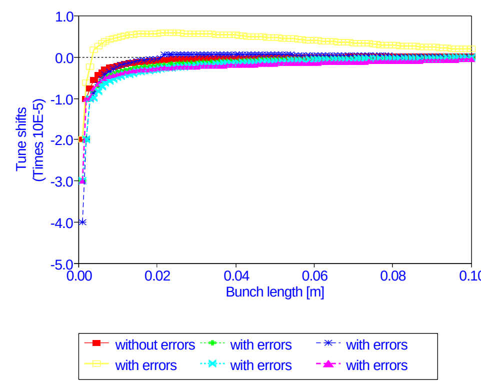

Figure 4: The relationship between the tune shifts and the bunch length with and without errors for m=1 and k=0.

The longitudinal radiation damping rate at 2 GeV storage ring is about 222 sec^{-1}, large enough to stabilize the dipole and quadrupole modes for various azimuthal modes. The contributions of higher modes in these calculations are not sensitive to the spread of resonances due to the construction errors of the designed cavity.

3 CONCLUSION

The longitudinal instabilities for the higher order were calculated by the eigenmode analysis of Sacherer's integral equation. The growth rates and the tune shifts in the electron storage ring were calculated by considering the randomness of the higher mode frequencies of a cavity due to the construction errors. Tune shifts were not sensitive to bunch length as bunch lengthen in both cases of the dipole and quadrupole modes. The results with the construction errors were compared to those of without errors for the dipole mode and quadrupole mode. The growth rates with and without errors gave the maximum error deviations of about a factor of ten by the statistical analysis. For the tune shifts with and without errors, the maximum error deviations are increased by a factor of four. But the growth rates, even in any cases, were small enough compared to the ra-

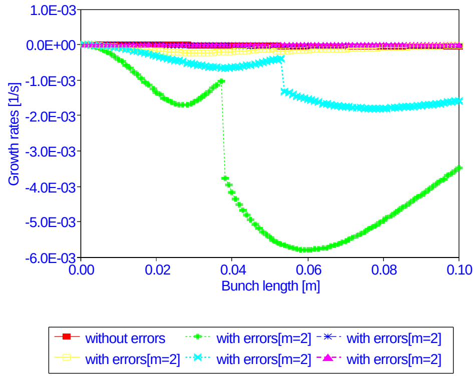

Figure 5: The relationship between the growth rates and the bunch length with and without errors for m=2 and k=0.

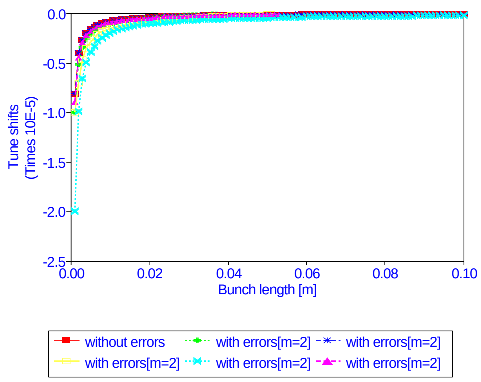

Figure 6: The relationship between the tune shifts and the bunch length with and without errors for m=2 and k=0.

diation damping time of 4.5 ms in our storage ring. The results also indicated that the deviations of the growth rates and the tune shifts due to higher modes of cavities were not sensitive to the construction errors of cavities if they are about 0.5×10^{-3}.

4 REFERENCES

[1] F. J. Sacherer, IEEE Trans. **NS-24**, 1393 (1977).

[2] B. Zotter, CERN SPS/81-18, 1 (1981).

[3] B. Zotter, CERN/ISR-TH/80-03, 1 (1981).

[4] Y. Chin, KEK 81-20, 3 (1982).

[5] C. Pellegrini and M. Sands, PEP-258, 1(1977).

[6] C. Y. Yao and A. Chao, SLAC-PUB-2680, 1(1981).

[7] T. Weiland, Nucl. Instr. Methods, **NS-216**, 329(1983).

[8] The Mafia collaboration(1988).