CALIBRATION OF THE UVX LNLS STORAGE RING OPTICS USING A LINEAR RESPONSE MATRIX THEORY

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Abstract

We present initial results of the optics calibration project of the LNLS UVX electron storage ring by the use of the response matrix formalism.

1 INTRODUCTION

Motivated by recent works in the literature [1], the Accelerator Physics Group of the LNLS is currently developing a plan for the UVX optics calibration through a linear response formalism known as response matrix theory. The response matrix is a well-behaved function of the machine parameters. Besides optics parameters, the procedure is used to obtain values for the calibration of the orbit correction system, that is, BPM and steering magnet gains.

The response matrix $M_{i,j}^{k,m}$ is defined by the relation

$$\Delta x_j^m = M_{i,j}^{k,m} \Delta \theta_i^k,$$

where $\Delta x_j^m$ is the orbit change in the j-th BPM in the m plane after a small kick $\Delta \theta_i^k$ in the i-th steering magnet in the k plane. In the linear approximation, one assumes small coupling between the planes such that the matrix is close to zero for $k \neq m$. An optics calculation program is used to determine the machine optics functions and the response matrix model. For most practical purposes, the MAD program [2] is a convenient tool. A figure of merit function is then defined

$$\chi^2(\{p\}) = \sum_{k,m,i,j} \left[ \frac{M_{i,j}^{k,m}([p]) - \bar{M}_{i,j}^{k,m}}{\sigma_{ij}^2} \right]^2,$$

where $[p]$ represents all parameters entering in the calibration, $\bar{M}$ is the experimental matrix and $\sigma_{ij}^2$ is the error associated to the matrix elements. From the numerical point of view, the essence of this work is summarized in the following way: given the experimental matrix, find all the corrections, $[\Delta p]$, to all relevant parameters in such a way that the model matrix is as close as possible to the experimental one. As one approaches the minimum, the chi-squared function ideally gets close to the actual number of degrees of freedom of the system.

2 ASSOCIATED ERRORS

In previous works the matrix error is taken as the noise level at each BPM [1]. Given the definition of the experimental response matrix, the error associated to each of its elements is given by the function

$$\sigma_{ij}^2 = \frac{1}{\Delta \theta_i^k} \left[ \sigma_{\text{BPM}}^2 + M_{i,j}^{k,m} \sigma_{\theta(j)}^2 \right],$$

where $\sigma_{\text{BPM}}$ is the noise level of the monitors (admitted the same for all monitors) and $\sigma_{\theta}$ the standard deviation associated to the corrector system (also assumed the same to all steering magnets). This last equation means that, even if the noise level of the BPM were zero (ideal monitors), the experimental matrix would suffer from the uncertainty associated to each corrector strength. However, the noise level of the steering magnets is understandably energy-dependent. The matrix error can be easily measured by taking several measurements of the matrix elements for a given steering magnet and then finding the standard deviation from the average value. For the UVX ring of the LNLS machine, at the energy 1.37 GeV, the corrector system has little effect upon the matrix elements. The same is not true at lower energies, e. g., 300 MeV, where we could find a significant effect of the steering magnet errors (see Figure 1).

![Figure 1: Measured standard deviation of the vertical response matrix for the corrector ACV03A at 300 MeV in terms of the module of the matrix element (effect of the corrector noise for large element values).](image-url)
In this report we will present results in the high-energy range. Therefore, the response matrix error is given by

\[ \sigma_{ij} = \frac{\sigma_{BPM}}{\Delta \theta}. \]

For the LNLS machine, this standard deviation is around 7mm/rad, given initial corrector kicks of 0.2 mrad. For a certain obtained calibration, therefore, the chi-squared value increases as the intensity of the corrector kickers decreases. Beyond a certain level, however, other systematic errors may perturb the corrector kickers. Around 7mm/rad, given initial corrector kicks of 0.2 mrad. For a certain obtained calibration, therefore, the minimum of \( \chi^2 \) such as non-linear effects in the BPM system.

### 3 CHOOSING THE PARAMETERS

Due to numerical limitations, the list of parameters entering in the optics calibration can not include all possible elements. Moreover, the number of parameters should be smaller than the number of available data, if the calibration process is to have statistical value. The LNLS machine has a system of 36 quadrupoles divided into 14 independent families, 23 monitors, 24 and 18 steering magnets for vertical and horizontal correction respectively. We have chosen to calibrate for quadrupole and dipole gradient errors (there are 12 dipoles), steering magnets and corrector gains and eventually the energy-shift [1] associated to the correctors in the horizontal plane. The energy-shift correction at each corrector \( \Delta \zeta \) is easily included since it is given by the simple formula

\[ \Delta \zeta_j = -\frac{\eta_j}{\alpha L}, \]

where \( \eta_j \) is the dispersion function at the j-th corrector, \( \alpha \) is the momentum compaction factor and \( L \) the machine circumference. We said above ‘eventually’ because this energy-shift factor is a function of the model lattice and, therefore, should consistently be accounted for in the model horizontal matrix by the optics calculation program. We report that the inclusion of the corrector energy-shifts as fitting parameters showed little effect upon the final value of the minimum. The final total number of parameters used in the fitting was 131 while the number of data points (elements of the matrix) is 966.

### 4 SEARCHING FOR THE MINIMUM

Compared to other works [1], we have used a slightly different numerical procedure in order to determine the characteristic linear system of equations for the parameter calibration [3]. The set of parameter corrections is obtained by solving the linear system of equations

\[ C_{n,r} \Delta p_r = B_n, \]

with

\[ C_{n,r} = \sum_{k,m} \frac{1}{\sigma_{ij}^2} \frac{\partial M_{i,j}^k}{\partial p_n} \frac{\partial M_{i,j}^k}{\partial p_r}, \]

and

\[ B_n = \sum_{k,m} \sum_{i,j} \left( \frac{M_{i,j}^k}{\sigma_{ij}^2} - \bar{M}_{i,j}^k \right) \frac{\partial M_{i,j}^k}{\partial p_n}, \]

where the index \( n \) runs over all possible parameters. Convergence is attained by singular decomposition methods [3]. The matrix \( C \) may present singular value as a result of physical or numerical degeneracies. A well known example is the fact that multiplying all monitor gains by a certain factor while dividing corrector gains by the same factor leaves the matrix unchanged. The calibration of gains produces therefore relative values, the above mentioned factor should be later adjusted, for instance, by suitably matching the experimental and model dispersion functions.

### 5 SOME INITIAL RESULTS

The response matrix of the UVX-LNLS storage ring was measured several times during the machine study sessions in 1998 at the highest energy of 1.37 GeV and average current of 100 mA. Some of the measurements were performed with sextupoles turned off in order to avoid non-linear quadrupole components in the model lattice. Moreover, coupling effects were also minimized. Initially, we have imagined that quadrupole fitting would be straightforward. Such impressions proved itself naïve in face of the average distance between the experimental and model matrices, 980mm/rad in the vertical plane and 450 mm/rad in the horizontal one. This was far above the expected value of the matrix standard deviation even accounting for kick standard deviations and indicated that the inclusion of BPM and corrector gains was mandatory before quadrupole fitting [4]. The fitting was therefore performed in two stages, BPM and corrector gains first and then quadrupole and energy-shifts. We observed an asymmetry in the value of the response matrix elements as produced by either positive or negative corrector strengths. Further, in order to minimize hysteresis effects, we had conveniently cycled each steering magnet before measuring the matrix elements. This has significantly enhanced measurement repeatability. In Figure 2 we show surface graphics of the difference between the measured and model matrices before and after calibration for the complete set of parameters. After calibration for BPM and corrector gains, the average distance between the model and experiment became 150 mm/rad in the horizontal plane and 400 mm/rad in the vertical plane.
Inclusion of quadrupole coefficient further decreased this distance to 60 mm/rad in the horizontal plane and 61 mm/rad in the vertical one.

The LNLS orbit correcting system underwent an upgrade in the end of 1998, which enabled us to check out improvements in the gains of BPM and correctors [5]. In fact we were able to find (via calibration of gains) an improvement of up to 10% in BPM calibration compared to the situation before the upgrade. Also, before the upgrade, using solely calibration of the gains, we could explain the large asymmetry found in the horizontal dispersion by deviations in the BPM calibration. As seen in Figure 3, the calibration could fit well large values of the dispersion function, while the small discrepancies found in the low values are caused mainly by quadrupole errors (not included in this fit). This figure also show the value of a parameter ($\kappa$) which gives the absolute calibration of BPM gains.

6 CONCLUSIONS

So far the LNLS machine study group has succeeded in establishing a systematic optics calibration process by using the information contained in the response matrix of the ring. The calibration includes the fitting of BPM and corrector gains, energy-shift corrections and quadrupole forces in straight quadrupole and dipoles. In this first period, we have written down the main codes, performed some measurements and debugged the whole processes. The chi-squared minimum value has not been attained yet, indicating the existence of unknown parameters (possibly hidden quadrupole components) not yet included.

Of particular importance is the error analysis of the response matrix. The response matrix error is composed of two different contributions determined by BPM and corrector noises. The influence of correctors is strong in the low energy range (around 300 MeV) while, at high energy, the corrector system has shown little effect. This is important since we intend to map quadrupole field deviations from nominal values for the entire energy range covered by the LNLS machine (from the injection energy, 120 MeV up to 1.37 GeV), and the matrix error has to be dealt appropriately. Results in this direction will be published in the future.

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