ROTATION ANGLE OF A HELICAL DIPOLE

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Abstract

With the collaboration between Brookhaven National Lab. (BNL) and RIKEN, the helical dipole magnets is under construction for RHIC spin project.[1] It is required that the deflection of beam is negligible in the helical dipoles.[2] The relation between the rotation angle in the helical dipole and the cancellation of the integral of transverse field along the beam axis is investigated.[3-5] Then, with the derived optimization method, the optimal length of the full-length helical dipoles is obtained from the measured field of the half-length prototype.

1 A DEFINITION OF THE EFFECTIVE ROTATION ANGLE FOR HELICAL DIPOLES

As a natural extension of the effective magnetic length with the replacement of the axial coordinate z by the phase angle φ of dipole, the effective magnetic rotation angle Δφ can be defined as follows,

$$\Delta\varphi = \frac{1}{B_{10}}\int B_1(\varphi)d\varphi = \frac{1}{B_{10}}\int B_1(\varphi(z))\frac{d\varphi}{dz}dz = \overline{\frac{d\varphi}{dz}}L \quad , \quad (1)$$

where $B_1(\varphi)$ is the amplitude of the dipole field at the angle $\varphi(z)$ with the definition of $\varphi(z=0) = 0$, and $B_{10} = B_1(\varphi=0) = B_1(z=0)$ is the amplitude of dipole field at the center, and L is the effective magnetic length.
According to this definition, it means that the magnetic field of the region where $d\varphi/dz = 0$ does not contribute to the effective magnetic rotation angle.

2 RELATION BETWEEN THE ROTATION ANGLE AND THE CANCELLATION OF THE TRANSVERSE INTEGRATED FIELD

In general, the cancellation of the integral of the field component B_a for the arbitrary direction with the angle φ_0 for the y axis, can be expressed as follows,

$$\int B_a(z)dz = \int B_1(\varphi(z))\cos(\varphi(z)-\varphi_0)dz = 0 \quad . \quad (2)$$

Then, it can be verified that the cancellation of the integral of B_a for the arbitrary direction is equivalent to those for two transverse directions, x and y directions,

$$\begin{cases} \int B_1(\varphi(z))\cos\varphi(z)dz = \int B_y(z)dz = 0 \\ \int B_1(\varphi(z))\sin\varphi(z)dz = -\int B_x(z)dz = 0 \end{cases} \quad . \quad (3)$$

In addition, $B_x(z)$ can be defined as an odd function from the symmetry for the magnet center, z=0, where the dipole field is in the y-direction with $B_y(z=0) = B_{10}$ and $B_x(z=0) = 0$. Therefore, the integral of the x-directional field B_x can be always chosen to be zero for every symmetric helical dipole. Then, it results that the cancellation of the integral of the y-directional field B_y is essential for that of the integrated field for the arbitrary direction.

2.1 Optimization of Rotation Angle

The optimal length or rotation angle of helical dipoles can be calculated for the following general field distribution,

$$\begin{cases} B_1(z) = B_1(z=0) = B_{10}, \quad \frac{d\varphi}{dz} = k, \quad 0 \le z \le z_e \\ B_1(z-z_e) = B_{10}\cdot b(z-z_e), \quad \varphi = \varphi(z-z_e), \quad z_e \le z \le z_f \end{cases} \quad , \quad (4)$$

where the function of $z-z_e$, $b(z-z_e)$ describes the z dependence of the normalized dipole field, and $\varphi(z-z_e)$ also describes the phase angle of dipole in the end regions.
The following equation is required to meet the condition of the zero integrated transverse field,

$$B_{10}\int_0^{z_e}\cos(kz)dz + \int_0^{z_f-z_e} B_1(z-z_e)\cos[\varphi(z-z_e)+kz_e]dz = 0 \quad . \quad (5)$$

Then, the optimal half length z_e of the helical body portion can be calculated numerically from the following equation,

$$\int_0^{z_f-z_e} b(z-z_e)\cos[\varphi(z-z_e)+kz_e]dz = -\frac{\sin(kz_e)}{k} \quad . \quad (6)$$

For the various cases, the optimal rotation angles for the cancellation of the transverse integrated field can be calculated with the deviation from 2π for the general cases, as listed in Table 1.[5]

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Table 1: Optimal rotation angles for the cancellation of the transverse integrated field for various cases. [ref.5]

Case	Helical body*	End**	Rotation angle
#a	k	(no end)	2π
#b	k	k_e = const. = k	2π
#c	k	k_e = const. $\neq$ k	$\neq 2\pi$
#d	k	$k_e \neq$ const.	$\neq 2\pi$

*) k : constant phase rotating rate of dipole in the body
**) k_e : phase rotating rate of dipole in the ends

3 MEASURED RESULTS OF THE HALF-LENGTH PROTOTYPE

For the half-length prototype, the z dependence of the amplitude $B_1(z)$ and the phase angle $\varphi(z)$ of the dipole field was obtained from the magnetic field measurement with the Hall probe at I=105 A and 220 A, as shown in Fig.1.[6-8] In these figures, the central position z = 0 of the magnet is defined as the middle point for the magnetic length. The difference of the magnetic structures in both of non-lead (the left side in Fig.1) and lead ends is not so large.

The rotation angles calculated from Eq.(1) for the helical body and end potions are listed in Table 2. In Table 2, $\Delta\varphi$ [body], $\Delta\varphi$ [non-l], and $\Delta\varphi$ [lead] mean the calculated rotation angles for the helical body, and the non-lead, and lead end portion.

In addition, due to the thermal contraction of helical dipole, the phase rotating rate of the dipole field of the helical body portion (- 0.4 m < z < + 0.4 m), k = $d\varphi/dz$ deviated from the design value at room temperature of 150 deg/m, as shown in #1 and #2 of Table 2. The measured results are almost consistent to k = $2\pi/\{2.4 \times (1 - 0.00415)\}$ = 150.6 (deg/m), expected from the thermal contraction of Al coil bobbin of 0.415 % from the room temperature to 4.2 K.

4 CALCULATION FOR A HELICAL MAGNET WITH FULL LENGTH

On the assumption of the symmetric magnetic structure with identical coil ends that both ends have the same measured field distribution of the lead end at I = 220 A, the length of the half body z_e was numerically optimized from Eq.(6) with the calculational error of about 2×10^{-6} Tm for the integral of B_y, obtaining z_e = 1.000 m. This means that the helical body portion of the prototype magnet from z = - 0.4 m to + 0.4 m, should be elongated by (1.000 - 0.4) $\times$ 2 = 1.200 m for that of the optimal full-length magnet. This result was almost equivalent to other optimization calculations.[3,4]

Furthermore, the non-symmetric full-size helical dipole can be composed from this optimal helical body and the different ends, as #4 in Table 1. The field distribution for B_{10} = 4.0 T of this non-symmetric full-length helical dipole are shown in Figs.2-5.

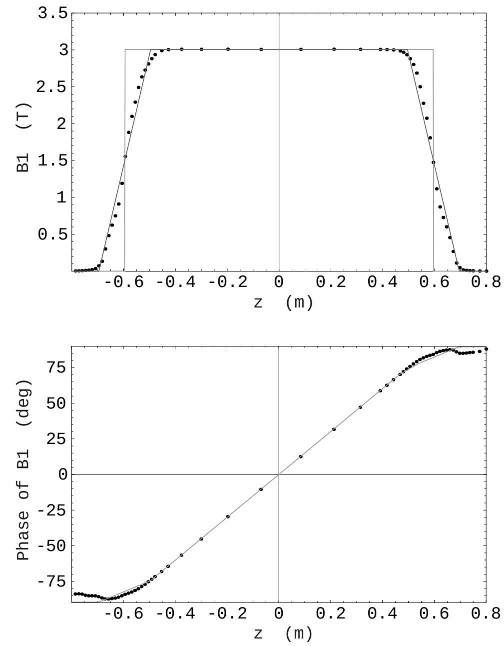

Fig. 1: The amplitude $B_1(z)$ and the phase $\varphi(z)$ of dipole along the beam axis at I=220 A for the half-length prototype.

Table 2: Comparison between the calculated rotation angles of the half-length prototype (#1 & #2) and the optimized full-length helical dipole (#3 & #4)

Half or Full	k (deg/m)	$\Delta\varphi$ [body] (deg)	$\Delta\varphi$ [non-l] (deg)	$\Delta\varphi$ [lead] (deg)	$\Delta\varphi$ [total] (deg)
#1) H (105 A)	150.7	120.3	22.7	23.6	166.6
#2) H (220 A)	150.4	120.4	23.0	23.4	166.8
#3) F (Sym.)	150.6	301.2	23.4	23.4	348.0
#4) F (non-S)	150.6	301.2	23.0	23.4	347.6

In these figures, black dots are shown at every 5.0 mm along the beam axis for the indication of the relation between the same z positions in each plot. The comparison between the integrals of the magnetic field between two cases, the symmetric ends (#3) and the non-symmetric ends (#4) in Table 2, is compiled in Table 3. Especially, the integrated value of B_a for the direction with the angle, φ_0 = + 30 degree is listed, together with those of B_x, and B_y, in Table 3. It results that the deviation from zero of the transverse integrated field become larger for the non-symmetric dipole. It will be also possible to seek the better optimal length for the non-symmetric case with the modification of Eqs.(4)-(6).

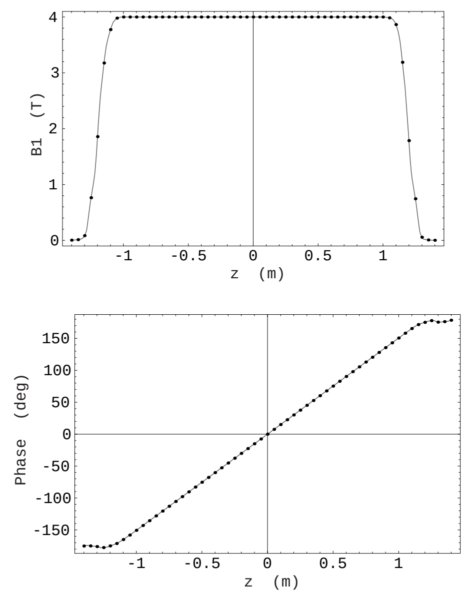

Fig. 2: The amplitude B_1 (z) and the phase angle $\varphi(z)$ of the dipole field along the beam axis of optimal full-length helical dipole with different ends.

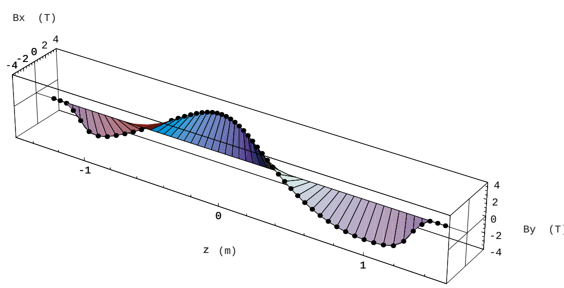

Fig. 3: 3D plot of the dipole field along the beam.

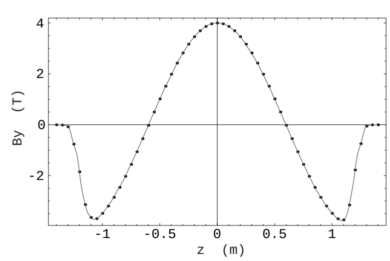

Fig. 4: B_y along the beam axis.

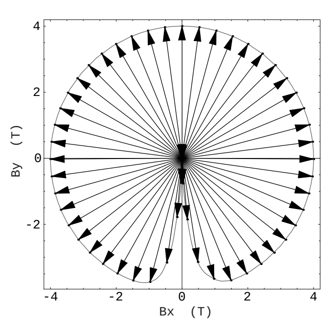

Fig. 5: Plot of (B_x, B_y) along the beam axis.

Table 3: The integrated values of the transverse field along the beam axis.

Case	Int. of B_x (Tm)	Int.of B_y (Tm)	Int. of B_a [$\varphi_0 = 30°$] (Tm)
#3) Symmetric	0	≈ 0	≈ 0
#4) Non-symmetric	$+1.5\times10^{-3}$	$+5.8\times10^{-4}$	-2.5×10^{-4}

5 CONCLUSION

The definition of the effective rotation angle for helical dipoles is proposed, similarly with the effective magnetic length. From the relation between the effective rotation angle and the cancellation of the transverse integrated field, it results that the rotation angle of helical dipoles, which meets the condition of the cancellation of the transverse integrated field, generally deviates from 2π.

In general, the rotation of the dipole field is specified only by the rotation of the coil in the helical body region. On the other hand, the field rotation in the end region is not specified only by the rotation of the coil, but also depends on the end structure of the iron yoke, etc.

It is shown that the length of the helical dipoles can be optimized for the measured (or known) ends with the derived optimization method. In addition, it is revealed that the integrated transverse fields for helical dipoles with the non-symmetric ends become larger than that with the symmetric ends.

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