On the Higher Order Mode Coupler
Design for Damped Accelerating Structures

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Abstract
In this paper we will discuss mainly damped structures and class them into three types. The procedures to make a higher order mode (HOM) coupler design have been given, and some useful analytical formulae, which can be used to determine the coupling apertures’ dimensions and the equivalent loaded quality factors, have been derived. It is shown that few couplers to dampe a detuned structure is not at all efficient.

I. Introduction
For the linear colliders working in the multibunch mode the long range wakefields have to be properly controlled by using detuned, damped or damped detuned structures depending on the concrete machine design parameters. In this paper we try to class damped structures into three types and to make a detail discussion about how to determine the coupling aperture dimension of a HOM coupler and how to estimate its effect on the damped HOM. In section 2, the first type damped structure is discussed, where each cavity is damped by HOM couplers. In section 3, the second type damped structure is defined as a section of constant impedance structure is damped by a HOM coupler. The essential difference between the first and the second type damped structures is demonstrated. To estimate the effect of this HOM coupler an equivalent quality factor has been introduced and formalized. Finally, in section 4, damped detuned structures are discussed and it is shown that few couplers to dampe a detuned structure is not at all efficient.

II. The First Type of Damped Structure
We consider a single pill-box resonant rf cavity resonating at a resonant mode, for example the TM$_{110}$ mode. The quality factor corresponding to this mode is $Q_{110}$. If this cavity is loaded with some waveguides, the loaded quality factor $Q_{L,110}$ is defined as

$$ Q_{L,110} = \frac{Q_{0,110}}{1 + \beta_{N,110}} $$

(1)

where $\beta_{N,110}$ is called the coupling coefficient between waveguides and the resonant cavity corresponding to the TM$_{110}$ mode. Where the subscript $N$ denotes the number of the waveguides. If the cavity is loaded with four waveguides distributed uniformly around the cavity’s cylindrical surface, from the general expression derived in refs. [1] and [2] one finds that $\beta_{4,110}$ is TM$_{110}$ mode polarization independent and can be expressed analytically as

$$ \beta_{4,110}(l) = \frac{\pi Z_0 \kappa k_{110} l^4 e^{-2\alpha_c l} J_1^2 (u_{11})}{144 (ln(4l/w) - 1)^2 A B R R_{s,110} (R + h) J_2^2 (u_{11})} $$

(2)

where $k = 2\pi / \lambda_{110}$, $k_{110} = k (1 - (\lambda_{110}/2A)^2)^{1/2}$, $\alpha_c = (2\pi / \lambda_{110}) ((\lambda_{110}/2l)^2 - 1)^{1/2}$, $R_{s,110} = (\pi \mu_0 / \sigma \lambda_{110})^{1/3}$, $\mu_0$ is the magnetic permeability and $\sigma$ is the electric conductivity, $A$ and $B$ are the width and the height of the four rectangular coupling slots with $l$ parallel to the magnetic field, and $t$ is the wall thickness between cavity inner surface and waveguide. Now, if many of this kind of waveguide loaded cavities are coupled together, one obtains the first kind of damped accelerating structure under the condition that the fundamental mode is not damped. For the TM$_{11}$ mode passband one can say that the loaded $Q$ is almost $Q_{L,110}$. If $K_0 Q_{L,110} < 2$ there will be no coupling between cavities for the TM$_{11}$ mode [3], where $K_0$ is the coupling coefficient for the TM$_{11}$ mode passband.

III. The Second Type of Damped Structure
Now we consider the second kind of damped structure which is essentially different from the first. Given a constant impedance disk-loaded structure of length $l$ as shown in Fig. 1, one can calculate the passband of the TM$_{11}$ mode (which is the most dangerous mode for multibunch operation). We define $\theta_{1,i}$ as the phase shift of the TM$_{11}$ mode passband at which the phase velocity equals to the velocity of light. Assuming now a charged particle is injected into the structure off axis at a velocity close to that of light, one knows that this particle will lose its energy to the TM$_{11}$ mode mainly at synchronous phase shift $\theta_{1,i}$, and generates behind it so-called wakefield $W_{z,11}(s)$, where $s$ is the distance between this exciting particle and a following test particle (of course it will generate wakefields oscillating at other
The third type of damped structure is to damp a detuned structure by few HOM waveguides. We will first discuss it without adding HOM coupler and assuming that cavities’ dimensions are different from each other adiabatically. The TM$_{11}$ modes dispersion curves of this detuned structure are shown in Fig. 3. When a particle traverse the structure off axis it will deposit part of its energy on TM$_{11}$ mode oscillating at the frequencies mainly from $f_1$ to $f_n$ as shown in Fig. 3, where the subscript $n$ is the total number of different cavities in this detuned structure of length $L$. The deflecting force felt by a test particle can be expressed as

$$F_{\perp,11} = \sum_{i=1}^{n} 2qK_{\perp,11} t_{i} L \frac{\sin(\omega(\theta_{1,i})s)}{s} \exp(-\frac{\omega(\theta_{1,i})s}{2Q_{0,11,i}})$$

with

$$K_{\perp,11} = \left( \frac{k_{1,1}c}{\omega(\theta_{1,1})} \right)^{1/2}$$

$$v_{p}(\theta_{1,i})$$ is the angular frequency of the travelling TM$_{11}$ mode at the phase shift of $\theta_{1,i}$. $Q_{0,11}$ is the quality factor of the structure working at the angular frequency $\omega(\theta_{1,i})$. It is obvious that when $v_{p}(\theta_{1,i}) = 0$ there is no damping effect (only the coupler cavity is damped). For those who are used to think in terms of loaded quality factor to judge the damping effect of this HOM coupler, we will use an exponential function $Le^{-\omega(\theta_{1,i})s}$ to simulate function $D(s)$ in eq. 5 by choosing $\beta_{eq}$ as

$$Q_{\perp,11} = \frac{Q_{0,11}}{1 + \frac{Q_{0,11}}{Q_{\perp,11}}}$$

The second kind damped structure is used, for example, in TESLA linear collider project.
where $k_{1,i}$ is the TM$_{11}$ mode loss factor corresponding to the $ith$ cavity, and it can be calculated analytically [2]

$$k_{1,i} = \frac{a_i^2 w_{11}^2}{4\pi e_0 \hbar^2} \alpha_{11} \left( \frac{2R_i}{a_i} J_1 \left( \frac{u_{11}}{R_i} a_i \right) \right)^2$$  \hspace{1cm} (12)

$$\eta_{1,i,i} = \frac{2}{\theta_{1,i,i}} \sin \left( \frac{\theta_{1,i,i} h}{2D} \right)$$  \hspace{1cm} (13)

If we consider an uniformly detuned structure with its iris radius adiabatically decreasing from $a_1$ at the beginning to $a_n$ at the end, the deposited energies can move inside the structure instead of oscillating locally. The motions of the TM$_{11}$ modes’ energies inside the structure can be classified into three different ways. For the modes deposited at the beginning of the structure they will oscillate locally since they could not propagate downstream. For the modes generated at the end of the structure (from $f_m$ to $f_n$, $v_g(\theta_{1,i,i}) < 0$, $i = m, \ldots, n$), however, the energies will propagate upstream and be trapped finally somewhere in the downstream cavities where the group velocities corresponding to these frequencies are equal to zero. Finally, the energies of the modes between $f_1$ and $f_m$ will propagate upstream until they are reflected by the first cavity. The reflected energies will move downstream and finally be trapped in the cavities where the group velocities are zero.

To damp the TM$_{11}$ modes in a detuned structure via a few HOM couplers is the main idea of the so-called damped detuned structure currently adopted in the S-Band linear collider main linac design [5]. To demonstrate the behaviour of a HOM coupler let’s assume that at the beginning of a detuned structure a HOM coupler (four waveguides) is installed and is matched to the $ith$ TM$_{11}$ mode. The equivalent loaded quality factor corresponding to the $ith$ mode can be expressed as

$$Q_{L,i} = \frac{Q_{0,11,i}}{1 + Q_{0,11,i} \left\langle \frac{1}{w_{11}}(\theta_{1,i,i}) \right\rangle^2}$$  \hspace{1cm} (14)

where $\left\langle \frac{1}{w_{11}}(\theta_{1,i,i}) \right\rangle$ is the average group velocity of the $ith$ mode travelling from $z = z_i$ to $z \approx 0$ where the HOM coupler is located. The damping effects of this HOM coupler on the other modes, however, are less efficient and different from one to another due to mismatching. The reflected HOM energies by this mismatched HOM coupler will be trapped somewhere in the structure. It seems that the first type damping scheme is the most suitable choice for a damped detuned structure (SLAC damped detuned structure is a good example [6]).

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References


