THE PROPOSAL OF COMPLEX IMPEDANCE TERMINATION FOR VERSATILE HOM DAMPER CAVITY

V.V. Paramonov
Institute for Nuclear Research of the RAS, 117312, Moscow, Russia

Abstract
The proposal of high order mode (HOM) damping using heavily loaded annular-slot resonant cavity has been described in details and some results have been presented [1]. Investigation has shown, that damper absorbs enough large RF power from fundamental mode, leading to reduction in Q factor. Short physical consideration and results of calculations have shown, that introducing of complex impedance termination (RC chain in series) with simple realization evolves lower power absorption at the fundamental frequency. Reduction in absorbed RF power at low frequencies in 8÷9 times is due increasing in absolute value of the loading impedance and introducing of the phase shift between RF current and voltage.

I. INTRODUCTION

The proposal of HOM damping using heavily loaded annular-slot resonant cavity has been described in details and some results have been presented in [1], [2]. A prototype HOM damper [3] for the ferrite tuned KAON TRIUMF booster cavity has been constructed to measure damping of all modes up to 1 GHz. The mode damper has a broad range and damps effectively up to 1 GHz, but absorbs power at the fundamental mode. In this paper we consider some recommendation for optimization of the damper cavity and the proposal [4] of complex impedance termination (CIT) to decrease power absorption at fundamental mode.

II. LUMPED CIRCUIT ANALYSIS

Analysis of the dumping effect for the system main cavity - damper cavity with dumping resistor we will provide using lumped circuit method. This case we shall limit consideration with modes of coaxial type field distribution in the vicinity of the accelerating gap. Most of HOM’s (except, probably, some modes at high frequency near 1 GHz) fulfill this condition. Following [1], equivalent circuit for the system is shown in Fig.1, where \( Z_c(\omega) \) is the output impedance of the main cavity to accelerating gap, \( C_g \) - equivalent capacitance of the accelerating gap, \( R_c \) - equivalent resistance of the RF losses in the main cavity, \( Z_d \) is the impedance of the damper cavity and \( Z_l \) is its loading (in general form complex) impedance.

Let consider, at first, the main cavity (assuming shortening between points A and B at the circuit Fig.1) parameters. The values of \( Z_c, C_g, R \) have to be adjusted to simulate cavity with the same frequency, impedance to the beam \( Z_0 \) and \( Z_0/Q \) value as for accelerating cavity. Condition for resonant frequencies of the main cavity \( \omega_n \) is follows:

\[
Z_c(\omega_n) + \frac{1}{\omega_n C_g} = 0.
\]

Defining the complex impedance presented to the beam at the gap as [1]:

\[
Z_b = \frac{(Z_c + R_c)Z_g}{Z_c + R_c + Z_g},
\]

where \( Z_g = 1/\omega C_g \) is the complex impedance of the gap capacitance, and taking into account (1), one can derive for impedance \( Z_{bn} = Z_b(\omega_n) \) at the resonant frequencies \( \omega_n \) of the system:

\[
Z_{bn} = \frac{1}{\omega_n^2 C_g^2 R_c} - \frac{1}{\omega_n C_g} \approx \frac{1}{\omega_n^2 C_g^2 R_c}.
\]

It is natural to consider the main cavity as the short-circuited copper coaxial line, because original ferrite tuned accelerating cavity is coaxial type one. But for the short-circuited coaxial line \( \omega_n \approx (2m + 1)\omega_1 \), where \( \omega_1 \) is the frequency of the fundamental mode and \( m \) is integer; \( 2m + 1 = n \). For the impedance of the HOM, if \( R_c = const \), one will have (3) \( Z_{bn} \sim n^{-1/2} \). Direct calculations shows for the impedance \( Z_{bn} \) the dependence \( Z_{bn} \sim n^{-1/2} \). To simulate more realistic case, we will assume \( R_c = R_c(\omega) \). This case the impedances of the HOM’s decrease slightly with frequency increasing and dumping problem...
becomes more severe. The values for $Z_c(\omega_1), C_q, R_{c=e}$ were fitted to have $Z_{d1} = 210 \, k\Omega, Z_{d1}/Q_1 = 40 \, O\Omega$ at the frequency $\omega_1/2\pi = 62 M Hz$.

At the second step of the analysis let’s consider the main cavity together with the damper one, supposing loading impedance $Z_l = \infty$. This case pure imaginary (inductive type) impedance of the damper cavity $Z_d = jZ_d$ at the frequency of the fundamental mode is small addition to $Z_c$, leading to the small shift in the resonant frequency (1). There are no increasing in real part of resistance, and, hence, there are no dumping of the impedance $Z_d$. Without the loading resistance the damper cavity is insufficiently small addition to the total surface of the main cavity in the region with low magnetic field and don’t lead to the increasing of the RF power dissipation.

Then consider the case, when the loading impedance $Z_l$ is active resistance $Z_l = R_l$. We can transform parallel chain 'damper cavity - loading resistance' in series one with the same impedance $Z_{d1}$:

$$Z_{d1} = \frac{Z_dZ_l}{Z_l + Z_d} = \frac{|Z_d|^2 R_l}{R_l^1 + |Z_d|^1} + j \frac{|Z_d|^2}{R_l^1 + |Z_d|^1}. \quad (4)$$

At the frequency of the fundamental mode $|Z_d(\omega_1)| \sim 2 \div 4\, Ohm \ll R_l$ and for $Z_{d1}$ good estimation will be:

$$Z_{d1} \sim \frac{|Z_d|^2}{R_l} + j |Z_d|. \quad (5)$$

One can consider the real part of $Z_{d1}$ as the addition to the equivalent resistance $R_{e}$. This addition will lead to increasing in the RF power dissipation at the fundamental mode.

Let’s consider more complicated loading circuit - resistance $R_l$ in series with capacitance $C_l$. We transform total circuit 'damper cavity - loading chain' to the equivalent chain in series with the loading impedance $Z_{d2}$:

$$Z_{d2} = \frac{|Z_d|^2 R_l + j |Z_d|^2}{R_l + \frac{|Z_d|^2}{\omega_1 C_l}} + \frac{|Z_d|^2}{\omega_1 C_l} \quad (6)$$

If we choose the loading capacitance $C_l$ to fulfill the condition $rac{1}{\omega_1 C_l} \gg R_l \sim |Z_d|$ (it is not difficult, for $C_l = 20 \, pF \frac{1}{\omega_1 C_l} \approx 120 \, Ohm$), for the loading impedance $Z_{d2}$ one get:

$$Z_{d2} \sim \frac{|Z_d|^2}{R_l} (\omega C_l)^2 + j |Z_d|^2 (\omega C_l)^2. \quad (7)$$

Comparing (5) and (7), we see reduction of the loading impedance at the fundamental frequency in $(R_l \omega_1 C_l)^2$ times. Following [1], the damping resistance $R_d$ is:

$$R_d = \frac{1}{\text{real}(1/Z_g)} \frac{R_l}{Z_d} \approx \frac{R_l}{|Z_d|^2} |Z_d|^4, \quad (9)$$

if loading circuit is resistance and:

$$R_d \approx \frac{1}{R_l |Z_d|^2 |Z_d|^4}, \quad (10)$$

if loading circuit is RC chain. Comparing (9) and (10), one see increasing of the dumping resistance at the fundamental frequency in $(R_l \omega_1 C_l)^2$ times.

### III. Optimization of the Damping Resistance

For good damper we need in high value of the damping resistance $R_d$ at the fundamental frequency and low value at frequencies of HOM’s. Consideration shows, that introduction of loading capacitance $C_l$ improves the total selectivity of damping circuit in low frequency range. Taking into account $|Z_d| = \sin \omega$, one see $R_d \sim \omega^{-4}$ (9) for original proposal and $R_d \sim \omega^{-6}$ (10) for CIT. It allows us have larger difference in damping effect between fundamental mode and HOM’s. In comparison with active resistance, CIT allows obtain this difference in one order more. With introducing of additional parameter we obtain more flexibility in optimization of the damping circuit.

Let provide simple qualitative analysis to obtain guidance line in choosing of parameters for the damping circuit. Comparing (9) and (10), one see, that loading capacitance reverse effect from loading resistance at low frequencies. It is clear, because for $\frac{1}{\omega_1 C_l} \gg R_l$ the absolute value of loading impedance is determined by capacitative part and RF current in RC chain is shifted in phase with respect to RF voltage. To have at the fundamental frequency high value of dumping resistance, we need in small values of $R_d, C_d$ and $Z_d$ (10). But at high frequency $R_d \sim 1/(Z_d^2 R_d)$ and we can’t take $R_d$ too small. To prevent increasing of $R_d$ due to serious resonance of the damper cavity ($Z_d = 0$), this resonance must be higher than range of damping (so, as in original proposal). The dependence $R_d(\omega)$ exhibits the minimum near frequency $Z_d = 1/(\omega C_l)$. The capacitance $C_l$ loads the damper cavity at the frequency of first resonance, shifting it down, and don’t effect on frequency of the second resonance. It is additional performance, because damper cavity have to be shorter and mode separation additionally improves. With small values of $C_l$ and $R_l$ the minimum of the dependence $R_d(\omega)$ is enough narrow and deep. To damp successfully the first HOM, the damping circuit have to be tuned to have minimum $R_d$ in the range of the frequency changing for this HOM, closer to the low limit of this range. If we will increase $|Z_d|$ at the fundamental frequency, $R_d$ at high frequency will decrease. One can do it by increasing of the slot width in the damper cavity, provoking so more strong coupling with HOM’s of the main cavity. But at the fundamental frequency $R_d$ will decrease too.

These reasons may be considered as recommendations in choosing of the damper cavity option for realization. Another reasons, including technological, have to be taken into account. Because there are analytical expressions for the damping circuit
parameters, optimization of the damping circuit may be passed to computer.

As the example, for one option the effect of the damping circuit is shown at Fig. 2. Parameters of the damping circuit are $C_d = 1.5 C_d$, $R_d = 60 \, \text{Ohm}$. At the fundamental frequency reduction in the impedance is $\Delta Z_b / Z_b \approx 0.07$ for $\omega / \sqrt{\tau} = 62 \, \text{MHz}$. At the lower limit of the fundamental frequency range $\omega / \sqrt{\tau} = 46 \, \text{MHz}$ reduction in the impedance is one order less. With decreasing of the fundamental frequency the damping impedance rises fact. Moreover, at low frequency own RF losses in the main cavity increases due to higher losses in ferrite. The RF power, dissipated in the damping circuit depends on RF voltage, induced on damper cavity and don’t depends on quality factor $Q_0$ of the main cavity. So, with high $Q_0$ relative value in increasing of RF power dissipation will be higher. It is valid for all types of dampers and have to be taken into account under comparison of different types of dampers.

Direct calculation (using codes like SUPERFISH) of the RF voltage induced on damper shows lower RF losses in comparison with lumped circuit results.

Results of this consideration show, that with introducing of CIT RF power dissipation at the fundamental frequency may be decreased significantly and be close to very good results, obtained in [5] (experimentally).

Technical realization of CIT seems not so difficult, because value of the capacitance $C_d$ needed is not so large. At fundamental mode in the gap of the damper cavity there are no large voltages and different design may be realized.

IV. CONCLUSION

Short physical consideration and results of calculations have shown, that introducing of complex impedance termination evolves lower power absorption at the fundamental frequency without deterioration of HOM’s damping.

The author thanks R.L. Poirier and A.K. Mitra for discussion and interest in this work.

References