Electromagnetic Field Vector Components
Precise Measurements in Accelerating Structures

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Abstract

Precise method for resonator electric or magnetic vector components values and their space positions measurements, based on application of photosemiconductor plate with different configuration lighted images, formed by projections forming and measuring optical system of amplitude modulated light radiation, is presented. The optical system for 433 MHz RFQ accelerating structure is realized by means of serial produced micro-alignment telescopes; the method allows to descriminate the field axis fluctuations on micron level and provides several percents and tenths of percents precision for accelerating efficiency and modulation period measurements respectively.

I. INTRODUCTION

An electromagnetic field distribution measurements in accelerator resonant structures are usually carried out by perturbation method, the data processing gives the field vector modulus as averaged volumetric value on the perturbation object. The measurements accuracy is limited in principle by the object carrying system distortions and unperturbed resonant frequency reading inadequacy. Operative frequency rise as well as applied fields complication will cause inadmissible growth of these inaccuracies. Electro-optical principle is proposed to exclude these errors and to realize different vector components measurements of electric or magnetic fields distributions, the method and system development for RFQ structure is considered.

II. ELECTRO-OPTICAL PRINCIPLE

Perturbation object for the method is designed as a high resistivity photosemiconductor flat plate, that can be installed inside the resonator on a thin filament as before. Light radiation of amplitude modulated source passes through a controller of the light beam spatial position and lights up desired configuration region on the plate surface in required position. The resonant frequency difference between the readings in unlighting amplitude modulation half-cycle and in the next lighting one will determine the field in perturbated region. Electric field vector components measurements can be carried out by thin strip light configurations, oriented along the plate normal magnetic field component a closed-loop configuration is suitable; average properties can be determined by a spot of accordent space. A normal to electric field lighting minimizes the dark and light readings difference (that vanish for infinitesimal thickness of the normal), in quasistatic field it is equipotentials detection without quantity measurements.

Carrying system distortions are entirely excluded, because the perturbation is formed by exact straightforward light beam. The second mentioned error is excluded almost completely by the dark reading in any measurement point, because high-speed (acusto-optical, e.g.) devices allow to decrease amplitude modulation cycle up to the doubled transient duration. For all this, the precise measurement problem is reduced to implementation of correlated with the resonant mode light configurations.

III. RFQ MEASUREMENTS

The field symmetry axis can be detected by equal thin strips lighting on the round plate (II), fig.1. By (1) and (2) couple removing in OY direction the equal dark-light frequency differences for each strip can be achieved, i.e. $E_y$ components in (1) and (2) regions are equal, the axis coordinate $y_0$ is geometrical center of the couple; similar $OX$ removing of (3),(4) gives $x_0$ coordinate. Vanes curvature is determined by the strips turn refer $O_0x$ and $O_0y$ untill maximum (but a.m. equal) dark-light differences for (1'),(2'); (3'),(4') will be obtained – fig.1 presents symmetrical $x$ bend; for a single element distortion the geometrical centers will not form straight lines under $OX$ displacing of (1),(2) couple, (3),(4) – in $OY$ direction. So, that kind positions research yields the field axis coordinates and symmetry distorting causes all information.

Figure 1: Lightings for RFQ measurements.

Obtainable precision analysis is conducted at known [1] electric field in the bore with modulation period $l_m$, accelerating efficiency $\theta$, inner radius mean $r_\theta$:

$$E_x = U (\Lambda - 1/r_\theta^2), \quad \Lambda = 2\theta \kappa \sin \kappa z I_1(\kappa r)/\kappa r,$$
$$E_y = U (\Lambda + 1/r_\theta^2), \quad r = \sqrt{x^2 + y^2}, \quad (1)$$
$$E_z = (U/\pi) 2\theta \kappa \cos \kappa z \cdot I_0(\kappa r); \quad \kappa = \pi/l_m.$$

Spatial selectivity is defined by frequency deviations ratio of interfluent and separated components, e.g. $l_1$ length, $a_s \times a_s$ cross-section strip (4) $E_x$ selectivity in (1) field according
to [App.] is \( \Pi_\perp = (\delta f_{E_x}/\delta f_{E_e}) \leq 4 \cdot 48(a_s/2l)^2 (\ln 2l/a_s - 1)(r_0 h/w)^2 \) - the cylinder curvilinear perimeter is equal to square one. In the structure with \( \theta \in [0.003; 0.05], l_m \in [4; 20] \, \text{mm}, r_0 = 3.5 \, \text{mm} \) and operating frequency \( f_n = (k_n/2\pi \sqrt{\varepsilon_0 r_0}) = 433 \, \text{MHz} \) for \( 0.1 \times 0.1 \times 2.5 \, \text{mm} \) strip the averaged selectivity is \( \Pi_\perp < 2 \cdot 10^{-3} \), that is greatly less than instrumental resolution.

The cavity analysis in the form of coupled shortcircuated sector radial waveguide sections, loaded by end capacitances, gives according to [App. (11)] the deviation

\[
\left| \frac{\delta f_{E_x}}{f_{E_e}} \right| = M \left( \frac{M}{r_0^2} + 1/4 \left( \frac{M}{r_0^2} \right)^2 \right), \quad (2)
\]

\( M \approx 5.52 \cdot 10^{-5} \). An error in desired equality of deviations at RF phase measurements is determined by minimal phase count discrete \( d\varphi_m \), and (2) result yields the strip displacement resolution \( dx_0 \approx 2r_0 (\pi MQ)^{-1} d\varphi_m \), \( Q \) - quality factor. For \( d\varphi_m = 2 \cdot 10^{-5}, Q = 5 \cdot 10^3 \) it is \( dx_0 \approx 1.6 \, \text{mkm} \), that cause the development of precise optical system for the images forming inside small aperture, lengthy (\( L = 1445 \, \text{mm} \)) bore without vanes lighting – the plate excitation by reflection phone is inadmissible. Autoreflective image forming in convergent rays of telescopic objective can be effected by micro-alignment telescope only by placing the light source beyond the graticule on the eyepiece side.

The system fig.2 comprises (4) and (5) micro-alignment telescopes [2]. Collimated image former (1) contains transparency (2) with adjustable in independent square directions transparent region; projective image (4) forms the image along its optical axis with displacements possibility by optical micrometers.

Figure 2: Images forming and measuring system.

Another telescope (5) is interconnected by pentaprism with wedge (3) for the image dimensions measurements on the plate (II). Alignment on the base datum targets (6),(7) sets optical axes of the former and both telescopes in coincidence; the resonator is fixed on the datum axis by (4) viewing on removable transparent targets (8),(9); and datum line of sight can be ascertainment always by (5) viewing on (7) target after (3) removing. Transparency (2) is equipped by rotary device with 30\(^\circ\) count accuracy. The system allows to form rectangular images with variable 0.05...3 mm sides, total error of the image coordinates is \( \pm (3 + 2D) \, \text{mkm} \), \( D \) - image distance, \( M \), but the systematic error can be excluded, [2]. So, the strip position inaccuracy is 3.5 mkm and the field axis coordinates real error \( \approx 4 \, \text{mkm} \). The structure fine tuning properties \( \delta \varphi_m \) are defined by the minimum dark-light deviation position detection under radial strip lighting refer to determined axis. According to [App.], this position corresponds to minimal integral value in (6) equation at the current distribution (7) for (1) field. Direct variation yields the minimum deviation angle \( \alpha \) between \( Ox \) and the strip

\[
\alpha = \frac{\arccos (G\theta \sin \kappa)}{2}; \quad G = \frac{2(\kappa r_0)^2}{\pi} \int_0^\kappa F(x) \, dx,
\]

\[
F(x) = kx - kl \sin kx \csc kl, \quad \text{and it is enough to measure } \alpha \text{ under the plate removing along the structure for } a^{\alpha_{\max}}, a^{\alpha_{\min}} \text{ values and } \alpha = \frac{\pi}{4} \text{ positions determination, because}
\]

\[
\theta = (\cos 2a^{\alpha_{\min}} - \cos 2a^{\alpha_{\max}})/(2G)^{-1}. \quad (3)
\]

Two counts phase method of \( \alpha \) measurement with rms error \( (\delta \theta/\phi) = 2 \cdot 10^{-4} \) in (1) field gives inaccuracy \( (\delta a_{\alpha}/a) \leq 1.7 \cdot 10^{-4} \), but the phase dependence is not symmetric refer \( \alpha \) (except \( \alpha = \pi/4 \)), and systematic inaccuracy \( (\delta a_{\alpha}/a) \leq 4.4 \cdot 10^{-6} \). Spatial selectivity is ample as before, sensitivity decrease is 2.5...10\(^{-3}\) only. The angle reading rms error of the optical system is 1.4...10\(^{-4}\), that determines real precision: for longitudinal coordinate count error \( \approx 5 \cdot 10^{-3} \), the modulation period inaccuracy is \( (\delta l_m/l_m) \leq 7.1 \cdot 10^{-3} \), and (3) result gives accelerating efficiency with \( (\delta \theta/\theta) \leq 1.3 \cdot 10^{-2} \) error.

Asymmetric positioning of \( R_d \) radius, \( T \) thickness, \( \varepsilon \) dielectric constant plate will distort investigating field (on fig.1: \( F \) is fastening point, \( \sqrt{2b} \) – sag to 3 quadrant). But only transverse field components inside the plate are used, \( l_m \), accuracy under \( \alpha = \pi/4 \) position determination in 1 and 3 quadrants is not changed, because \( \delta a_{\alpha} \) rise even to two orders due to sensitivity lowering will not exceed determinative value. The \( \theta \) error electrostatic estimating for \( R_d = l, \varepsilon = 10, T = 0.1 \, \text{mm}, b = 0.05 \, \text{mm} \) gives \( (\delta \theta/\theta) \leq 6 \cdot 10^{-3} \), the efficiency error grow up to 1.5...10\(^{-2}\).

Photosemiconductor characteristics analysis by [App.] method yields optimized ratio of the active light-dark conductivity: for \( \varepsilon \leq 10 \) the ratio is 125...130, and CdS or CdSe materials suit perfectly well.

IV. CONCLUSION

Electro-optical principle and its application for RFQ structures precise measurements method have been developed. Designed optical system provides several micrometers accuracy not only for the field axis coordinates, but for geometrical shapes of the accelerating bore forming elements also – autoreflection method is effected by contact bore targets or master gauges fastening on the same filament. Developed balance technique (that excludes a field quantity measurements) for the structure fine tuning could be used for precise measurements in other types cavity resonators.

APPENDIX. Thin cylinder formfactor for RF nonhomogeneous fields

Use of retarded potential \( \tilde{A} \) field operator singularity for axially symmetric \( l \) length \( a \) radius longitudinal current \( \tilde{I}(x) \) yields on its circumference in \( \rho, \phi, x \) local coordinates

\[
\tilde{A} = N(\rho) \cdot \tilde{I}(x) |_{x_\alpha}, \quad (4)
\]

where \( N(\rho) = (1/2\pi) \ln(2/k\rho) + C_i(kl) - (\sin kl/kl)], \gamma = 0, 577... - \text{Euler’s constant}, C_i(kl) - \text{integral cosine}. \) In two-component case (4) result inaccuracy will not exceed \((a/2l)^2\)
even for equal longitudinal and transverse field components, that follows, e.g., from ellipsoid depolarization tensor principal values [3]; however, the practically used disposing along the supposed field vector will supplement decrease of inaccuracy to 3...4 orders. Boundary conditions in external \( E' \) field lead to

\[
\frac{\partial I}{\partial x} = -\frac{i \omega \varepsilon_0 \varphi}{N(a)}; \quad \frac{\partial \varphi}{\partial x} = E'_x - I(x) \left( i \omega \mu_0 \frac{N(a)}{a} + z \right),
\]

where \( z \) - line active resistance of the cylinder material, \( \varphi \) - scalar potential of the field. Now a resonaor \( \nu \)-mode \( \{ E_{\nu}, H_{\nu} \} \) with \( \omega_\nu \) resonant frequency, \( Q_\nu \) quality factor is excited by some source \( S \) together with the cylinder \( j \) current density, and magnetic \( h_\nu \), electric \( e_\nu \), fields amplitudes equations are

\[
h_\nu(\omega_\nu^2 - \omega^2) + \frac{i \omega \omega_\nu h_\nu}{Q_\nu} = S - \frac{\omega_\nu}{Q_\nu} \int (\vec{j}, \vec{E}_\nu) \, dv,
\]

boundary values are \( I(0) = I(l) = 0 \) for distant from the cavity walls cylinder. So defined \( I(x) \) distribution is defined at \( E'_x = e_\nu E_{\nu x}(x) \) substituting in (5) equations:

\[
\frac{\partial^2 I(x)}{\partial x^2} + k^2 \left( 1 - \frac{i \omega \varepsilon_0 z l}{k^2 \frac{N(a)}{a}} \right) I(x) = \frac{\omega_\nu^2 \varepsilon_0}{\omega \mu_0} E_{\nu x}(x) h_\nu(x),
\]

where \( \tau = k_\nu l \). The end contiguity \( (\beta \to \infty) \) yields \( K_{\nu} \approx 4 K_0 \) for \( k \ll 0.1 \) and than \( K_{\nu} \) decreases up to \( K_0 \) for \( \beta \to 0 \) with the cylinder moving off, (10) general relation corresponds to electrostatic model [4].

Practically interesting results of nonhomogeneous field analysis are obtained at polinomial representation, e.g. for \( E_{\nu \nu} = A_2 x^2 + A_1 x + A_0 \) the method gives

\[
K_\nu = K_0 \left( \frac{\pi \eta + \frac{2}{3} \eta s_2 + \frac{2}{3} s_4}{\eta(1 + \eta)} - \eta(\eta + 2) \right),
\]

where \( \eta = 1 + (4 s_1 - 1) \left( \frac{\xi}{1 + \xi} \right)^2; \quad \xi = \frac{A_1 l}{2 A_0}, \quad \eta = \frac{A_2 l^2}{4 A_0 (1 + \xi)} \)

- normalised variability,

\[
s_1 = \frac{\pi^2}{3} + \pi \cot \frac{\pi}{3}, \quad s_2 = \frac{2}{3} \pi + \pi \cot \frac{\pi}{3} - \frac{2}{3} \pi - \frac{2}{3} \frac{2}{\pi^2} - 1,
\]

\[
s_3 = \frac{1}{2} - \frac{2}{\pi^2} + \frac{\pi}{\tau} - \frac{\tau}{\pi^2}, \quad s_4 = 1 - \frac{4}{\pi^2} - \frac{\tau}{\pi - 2 \tan \frac{\pi}{3}},
\]

and \( K = K_{\nu}, x_0 = \frac{1}{2} \) in form (8).

References


