

# FIELD PROPAGATION EFFECTS AND RELATED MULTIBUNCH INSTABILITY IN MULTICELL CAPTURE CAVITIES

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Field propagation effects during the filling and re-filling of the cavity are not harmful for a relativistic beam accelerated in a multicell standing wave cavity. On the opposite the beam dynamics of a non relativistic beam injected into a capture cavity can be significantly affected by the interaction with other modes of the fundamental passband, under beam-loading conditions. In fact, the dominant beating provided by the passband mode nearest to the accelerating  $\pi$ -mode introduces fluctuations on the accelerating voltage. The phase slippage occurring in the first cells, between the non relativistic beam and the lower modes, produces an effective enhancement of the shunt impedances, which are usually negligible for a relativistic beam in a well tuned cavity. In some cases simulations show a significant oscillation of the energy spread and transverse normalized emittance along the bunch train. As an example, we have applied our computational models to the beam-loading problem in the Tesla Test Facility (TTF) superconducting capture cavity.

## I. INTRODUCTION

In addition to single bunch effects, induced in particular by space charge forces, multi-bunch effects due mainly to RF field propagation inside the cavity, will affect the quality of a non relativistic beam accelerated by standing wave structures. The study of these beam loading effects, which could limit the performances of injectors involving SW cavities, led to the development of numerical codes [1]: HOMDYN, which includes space charge effects and transverse motion and MULTICELL, which involves the longitudinal motion.

After a cavity filling time, the cavity is periodically refilled by RF power during the bunch to bunch interval. Although the generator frequency is set close to the accelerating  $\pi$ -mode, all the modes of the  $TM_{010}$  pass-band will be excited. Recently, in the aim to study the beam loading effect in the superconducting cavity TESLA for a relativistic beam, the multi-mode problem was solved by using systems of first order differential equations [2] or Laplace transforms [3].

In this paper, we use however a different approach, by directly solving the differential equations relative to each usual mode of the pass-band, provided that an intermode coupling term is taken into account. It can be shown in fact [4] with the help of the theory of coupled resonators, that the usual modes of the pass-band, found in the steady-state regime, are coupled through the external Q of the first cell, where the coupler is located, and not through the intrinsic wall losses of the cells.

The excitations  $Z_m$  of these normal modes of index m are then found by solving the following system (1) of coupled differential equations:

$$\ddot{Z}_m + \frac{\omega_o}{Q_o} \dot{Z}_m + T_{1m} \frac{\omega_o}{Q_{ex}} \sum_k T_{1k} \dot{Z}_k + \omega_m^2 Z_m = \frac{T_{1m}}{N\epsilon} \frac{d}{dt} \int_{S'} (\vec{n} \times \mathbf{H}_n(\vec{r}, t)) \mathbf{E}_{an}(\vec{r}) dS + \frac{1}{\epsilon} \frac{d}{dt} \int_{V_{cav}} \mathbf{J}(\vec{r}, t) \mathbf{E}_a^m(\vec{r}) dV \quad (1)$$

The first driving term represents the generator current, while the second integral over the whole cavity represents the beam interaction with the mode m and will be computed during each bunch passage. The coefficient  $T_{nm}$  is the normalized excitation of mode m ( $m=1, M$ ) at the center of cell n ( $n=1, N$ ). The evolution of the M field amplitudes during the cavity filling and refilling driven by the generator current, together with the perturbation due to beam-loading, are found by numerical integration of system (1) coupled to the beam equations of motion. Since we are particularly interested in the evolution of amplitude and phase envelopes of the RF fields of the fundamental pass-band, which are slowly varying functions, this second order system can be easily transformed to a first order differential equations system [1].

## II. RELATIVISTIC BEAM

In a well tuned cavity interacting with a relativistic beam, the average accelerating field vanishes for all modes, except of course for the  $\pi$ -mode. However, since the  $\pi$ -mode is coupled to the other excited modes through the  $Q_{ex}$ , some fluctuations remain.

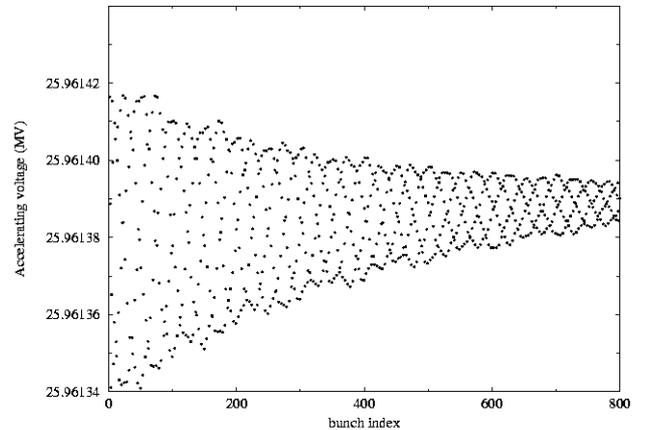


Figure 1 : Accelerating voltage evolution during 800 bunches

Figure 1 is a plot of the total accelerating voltage during the entire TESLA beam passage in a 9-cells SC cavity (bunch charge: 8 nC, bunch spacing: 1  $\mu$ s, accelerating gradient: 25 MV/m), and points out the residual oscillations, mainly caused by the mode of the pass-band nearest to the  $\pi$ -mode, decaying according to the mode time constants  $\tau = 2Q_{eX} / \omega$ . The induced bunch-to-bunch energy spread is nevertheless very small, from  $3 \cdot 10^{-6}$  at the beginning to  $0.5 \cdot 10^{-6}$  at the end.

### III. NON-RELATIVISTIC BEAM

With a non-relativistic beam the situation changes completely: the effects of the modes lower than the  $\pi$ -mode do not cancel any more. The phase slippage occurring in the first cells, between the non relativistic beam and the lower pass-band modes, produces an effective enhancement of the shunt impedances, which is usually negligible for a relativistic beam in a well tuned cavity. Furthermore, since the beam phase is slipping all along the structure, the field jumps and the detuning due to the off-crest beam vary from cell to cell. Some appreciable fluctuation of the output energy during the beam pulse is then expected. This multi-bunch energy spread is here estimated for the SC capture cavity of the low charge injector I of the Tesla Test Facility [5], (bunch charge: 37 pC, bunch spacing: 4.615 ns, accelerating gradient: 10 MV/m).

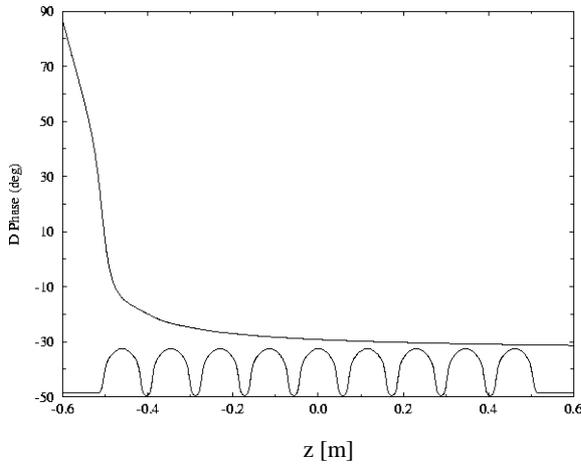


Figure 2 : Beam phase shift along the SC capture cavity (the cavity profile is also shown)

Figure 2 shows typical plots of the beam phase with respect to the RF wave for a single bunch crossing the 9-cell capture cavity. An injection energy of 240 KeV was assumed. Before reaching a stable value, the phase shift varies rapidly especially in the first cells. The total frequency detuning is about 110 Hz, i.e. almost one third of the cavity bandwidth.

Figure 3 shows the evolution of the energy gain on a short-time scale (1000 bunches), with the coupler linked to the cell n.1 or the cell n.9. The accelerating voltage exhibits fluctuations with a main beating due to the nearest  $8\pi/9$  mode spaced 0.76 MHz apart from the  $\pi$ -mode.

The beam loading effect is different according whether the power coupler is located upstream or downstream with respect

to the beam. The shapes of the oscillations are similar and the multi-bunch energy spread amounts to  $9 \cdot 10^{-4}$  in both cases.

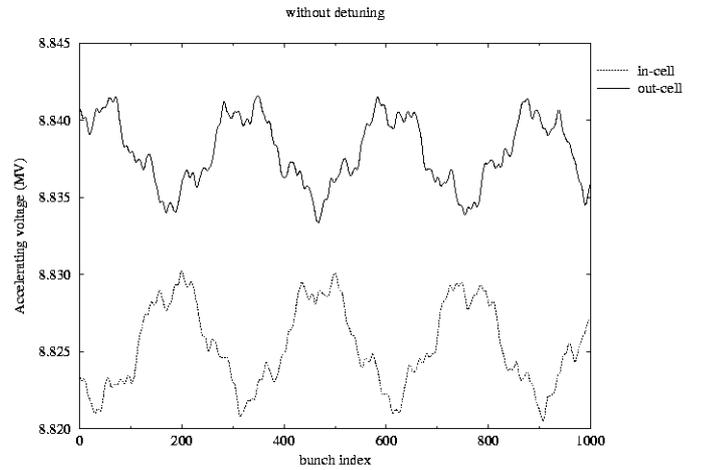


Figure 3 : Energy gain evolution during 1000 bunches

Figure 4 shows the accelerating voltage evolution on a longer time-scale, like the TTF beam pulse duration of 0.8 ms (about 173 333 bunches with injector I). The average compensation of the beam loading, by adjustment of the different parameters (generator, beam voltage or injection time), is not possible, resulting in a large slope on the cavity voltage at the beginning or at the end of the beam pulse.

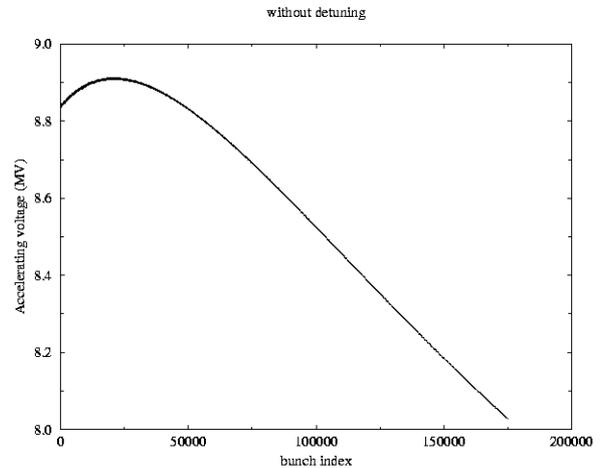


Figure 4 : Energy gain evolution on a long time-scale

### IV. BEAM DETUNING COMPENSATION

When a beam is running off-crest, a cavity detuning, in addition to the critical coupling, is generally introduced [6] in order to cancel the reflected RF power and thus to minimize the RF power fed by the klystron. In the steady-state regime and for critical coupling, the tuning angle must be set to the RF phase with respect to the beam phase according to:

$$-\tau \Delta\omega_{\pi} = \tan \psi = \tan(\phi_{rf} - \phi_b)$$

The generator and the cavity voltages are then in-phase. Furthermore, it will be shown later that this beam-detuning compensation will decrease the cavity voltage fluctuations. The phasors diagrams are drawn on Figure 5, without (a) and with (b) cavity detuning.

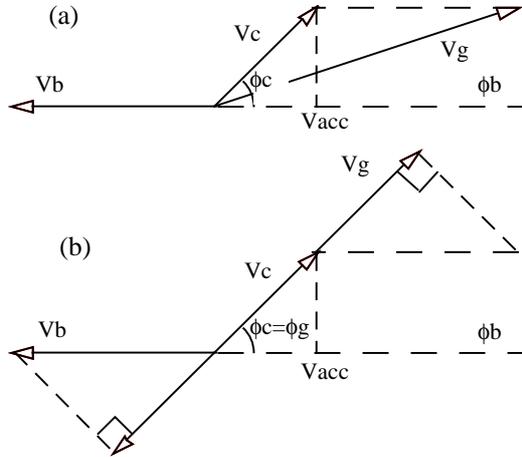


Figure 5 : Phasors diagram without (a) and with cavity detuning (b)

In order to have a constant accelerating voltage during the beam pulse, by balancing the rising generator voltage and the beam voltage, the beam should be injected after the beginning of the RF power pulse with a delay  $t_0 = \tau \text{Ln} 2$  with cavity detuning or  $t_0 = \tau \text{Ln}(V_g \cos \phi_g / V_b)$  without cavity detuning. Both conditions, however, neglect the other modes than the  $\pi$ -mode

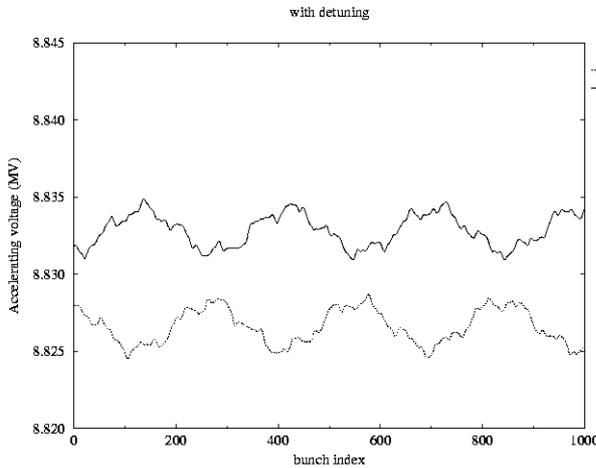


Figure 6 : Energy gain evolution with cavity detuning, solid line: out-cell, dotted line in-cell.

Figure 6 shows the energy gain with a cavity detuning of  $-110$  Hz. The phase of the generator is assumed to track the cavity phase, at least during the field rise time. We note that the accelerating voltage fluctuations are about two times lower, giving a multi-bunch energy spread of  $4 \cdot 10^{-4}$ .

Figure 7 shows the accelerating voltage evolution on a longer time-scale: an average beam loading compensation can be obtained when the proper cavity detuning is introduced. We

note again that the voltage oscillations decay with the time constants of the other modes of the pass-band.

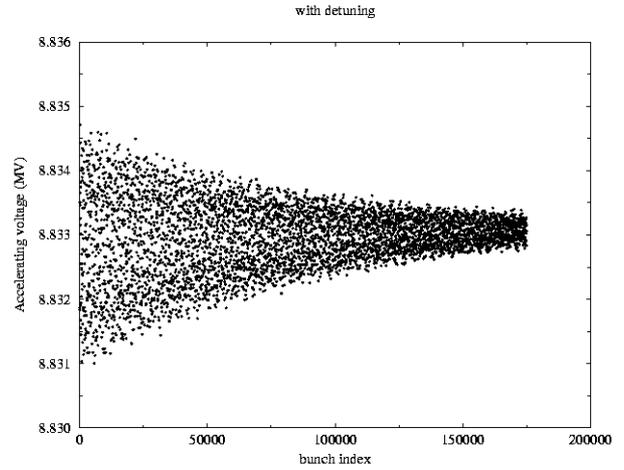


Figure 7 : Energy gain evolution on a long time-scale with cavity detuning

## V.CONCLUSIONS

Although the propagation effects are not harmful for a relativistic beam accelerated in a multicell cavity, they have to be taken into account with non relativistic beams. In the latter case, the modes other than the  $\pi$ -mode introduce larger cavity voltage beatings. For the low charge TTF injector, the resulting bunch-to-bunch energy spread will be lower than 0.1% in any case. This value is nevertheless very small in comparison with the single-bunch energy spread of 3% , found with PARMELA simulations [5]. The impact on the transverse dynamics is also small: an rms normalized emittance fluctuation of 1% has been computed [1]. The TESLA cavity geometry is thus well suited to the capture section of the TTF injector.

With a larger number of cells or a smaller cell-to-cell coupling, stronger effects would have been obtained. On the other hand, we could imagine larger energy spreads induced by more critical beam parameters, like the bunch charge or the input energy.

## VI.REFERENCES

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