EXPERIMENTAL STUDIES OF TRANSVERSE BEAM INSTABILITIES AT INJECTION IN THE FERMILAB MAIN RING

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ABSTRACT
Transverse beam instabilities at injection prevent the Fermilab Main Ring from storing higher intensity proton beam. It is essential to understand the nature of existing beam instabilities in order to increase the stored beam intensity. Results of experimental studies are presented and the nature of beam instabilities is depicted. Possible cures are discussed.

I. THE OBSERVED PHENOMENA

The existence of transverse collective instabilities in the Main Ring (MR) has been known for years [1, 2]. The most severe effects occur in the vertical plane because of the smaller width of beam pipe in the vertical direction. Two damper systems were built to combat the instabilities. One is slow damper which covers the frequency from DC to 1.5 MHz. The other is the super damper which is a bunch-by-bunch feedback system using digital technology. Evidence of collective beam instabilities has been shown in Ref[2].

A. Growth rate vs. beam intensity
The growth rate of the amplitude of vertical betatron oscillation clearly shows strong dependence on the total beam in the MR as depicted in Figure 1 and 2.

B. Growth rate vs. batch spacing
The separation between adjacent batch also plays an important role in the growth rate of vertical instability. The beam batches were injected in a successive order. A longer spacing between adjacent batch certainly helps to deter the onset of vertical instability. Each batch contains 84 beam bunches in the data shown in Figure 3 and 4.

C. Growth rate vs. bunch length

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II. DATA ANALYSIS

A. Coupled bunch instability

A qualitative analysis was developed to help identify the unstable mode of coupled bunch instability.

Suppose there are $m'$ batches in the accelerator, each has $\ell$ bunches. Each batch is separated by $g$ rf buckets. The bunch configuration is depicted in Figure 7. The right side of Figure 7 is the cut-away view of the configuration. If we only consider the equilibrium condition, i.e. there is no damping or growth in the amplitude of betatron oscillation. We also use a macroparticle to represent a bunch. Then the transverse motion of the beam centroid for bunch $p$ observed by a beam position monitor (BPM) at a fixed location can be expressed as:

\[
d_p(t) = A_{\beta} Q \sum_{n=\infty}^{\infty} e^{-i \psi(t)} \delta(t - nT - p \frac{T}{M} - \hat{\tau} \cos(\omega_s t + \phi_p))
\]

\[
\psi(t) = q_{\beta} \omega_0 t + \omega_{\xi} \hat{\tau} \cos(\omega_s t + \psi_p)
\]

where $q_{\beta}$ is the fractional part of betatron tune, $\omega_0$ is the angular revolution frequency, $\omega_{\xi}$ is the chromatic frequency, $\omega_s$ is the synchrotron frequency, $\phi_p$ is the initial synchrotron phase, $\psi_p$ is the initial betatron phase, $\hat{\tau}$ and $A_{\beta}$ is the amplitude of synchrotron oscillation and betatron oscillation respectively. $Q$ is the total charge of a beam bunch, $T$ is the revolution period and $M$ is the harmonic number of accelerator. The signal observed by the beam position monitor is given by:

\[
d(t) = \sum_{m=0}^{m'-1} \sum_{p=m+(\ell+g)}^{m'+(\ell+g)-1} d_p(t)
\]

For a closed-loop coupled bunch mode, we can write the coherent phase advance $\theta$ between the adjacent bunches as $2\pi \mu / M$ where $\mu$ is an integer ranging from 0 to $M-1$. Then the frequency spectrum of transverse signal for the multibatch configuration with missing bunches is given by [3]:

\[
D(\omega) = \sum_{n,k=-\infty}^{\infty} A_{\beta} Q \omega_0 (-1)^k J_k((q_{\beta} \omega_0 - \omega + \omega_{\xi} \hat{\tau})) \cdot F(n - \mu, \ell) B(n - \mu, \ell, g, m')
\]

which is just the single bunch result multiplied by two extra factors $F$ and $B$. $F$ is the form factor for a single batch with many consecutive bunches. $B$ is the form factor for the multibatch configuration. When there is only one batch in the accelerator, the value of $B$ converges to one. Therefore, the frequency spectrum of transverse signal with coupled bunch coherence is just shifted by the mode factor $\mu$. The explicit expressions for the form factors are:

\[
F(n, \ell) = \frac{e^{i2\pi n / M} - 1}{e^{i2\pi n / M} - 1}
\]
\[
B(n, l, g, m') = \left| \frac{e^{i2\pi n(l+g)m'/M} - 1}{e^{i2\pi n(l+g)/M} - 1} \right|
\]

Figure 8 shows good agreement between the analysis and measurement data. It was found that \( m = 1 \) gave the best fit. Therefore, the dominant unstable mode is the \( n=1-q \) betatron sideband. Figure 9 is the time domain picture of the vertical betatron oscillation measured with a stripline BPM. The bunch oscillation was recorded up to five revolutions.

**Figure 8**: The frequency spectrum of vertical motion when each batch was only half filled. Both dampers were off. The dashed line is the calculated form factor with \( m = 1 \).

**Figure 9**: The vertical oscillation of 12 equally spaced batches. The horizontal scale is 10 \( \mu \)s/div. Signal was measured by a stripline BPM. Due to the reflected pulse from the downstream end of BPM, the period of oscillation is about 40 \( \mu \)s instead of 20 \( \mu \)s. The frequency of the \( n=1-q \) betatron sideband is around 26 kHz. The revolution period of MR is 21 \( \mu \)s.

### B. Head-tail instability

Various modes of vertical head-tail instability were also observed in the MR. Signals were measured by a quarter wavelength stripline BPM and digitized by a Tektronix digitizer RTD720 running at 2 GSample/sec. The time step used in Figures 10 and 11 is 500 ps/unit.

**Figure 10**: Vertical chromaticity=3.6, bunch length=5 nsec, beam intensity/bunch=1.07 E10 protons.

**Figure 11**: Vertical chromaticity=-40, bunch length=5 nsec, beam intensity/bunch=1.67 E10 protons.

### C. Discussions

The measurements presented here confirm the previous suggestions that MR suffers from the vertical instabilities due to the resistive wall impedance[2]. Measurements of the accelerator impedance also show large resistive impedance in the low frequency range[4]. Design work for a new damper system is currently underway.

### III. REFERENCES