CONTROL OF TRAPPED ION INSTABILITIES
IN THE FERMILAB ANTIPROTON ACCUMULATOR

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Significant progress has been made in the control and understanding of trapped ion induced instability in the Fermilab Antiproton Accumulator. In addition to a clearing electrode system, a novel method of clearing Hydrogen ions by lightly bunching the beam with RF has been developed. These ion clearing techniques have eliminated trapped ion induced instability as a limiting factor in antiproton source performance. A brief description of these techniques and why they work is presented in this paper.

I. INTRODUCTION

The residual gas in the Fermilab Antiproton Accumulator is ionized by coulomb interactions with the antiprotons in the beam at a rate of one unit of charge per antiproton every 2 to 5 seconds. The production process imparts substantially less than thermal transverse energy to the resultant ions [1]. The ions, therefore, will have a Maxwellian velocity distribution at the temperature of the vacuum chamber. Virtually all the ions produced in this manner are trapped in the space charge potential well of the antiproton beam where they will oscillate at a frequency which, among other things, depends on the mass and charge of the ion (see Equation 3).

Approximately half of the ions produced are hydrogen. The mass and charge of hydrogen ions (H2+ and H+) is such that, over a large range of beam sizes and intensities, the frequency of ion oscillations about the center of the beam will coincide with the low order betatron resonance frequencies of the beam. The motion of hydrogen ions will therefore drive coherent oscillation of the beam [2].

II. MEASUREMENT OF TRAPPED ION EFFECTS

It is difficult to directly observe any parameter of the ion motion in the beam. What is usually observed are beam phenomena caused by ion motion. The impact of trapped ion motion on global accumulator beam parameters such as transverse emittance and betatron tune, have been described elsewhere [3],[4]. More directly connected to the ion motion is the beam coherent oscillation spectrum induced by the motion of ions in the beam. A determination has been made of which beam oscillation modes the trapped ions couple to. Measurements of the transverse beam transfer function have also been made at the beam dipole betatron resonance frequencies both in the presence of trapped ions and without trapped ions.

A. Beam Coherent Oscillation Spectrum

The transverse coherent oscillation spectrum of the antiproton beam induced by the motion of trapped ions is shown in Figure 1.

![Figure 1: Ion driven horizontal coherent oscillation spectrum.](image)

The spectrum includes both slow and fast wave dipole oscillation. There is also an indication of quadrupole motion (i.e. the 3 – 2q and 4 – 2q lines). This spectrum was measured using transverse dipole pickups and differentially driven electronics which attenuate common mode signals, such as quadrupole oscillation, by at least 27 dB. Therefore, it can be concluded that the amplitude of the quadrupole motion is very large. Transverse beam quadrupole oscillation is in fact an expected and serious consequence of the presence of trapped ions [5].

B. Beam Transfer Function Measurements

The motion of ions in the beam and the coupling of the ions to the dynamics of the beam is non-linear [2]. The non-linearity is manifest in the transverse beam transfer function. Figure 2 shows the transverse response of 52 mA of protons circulating in the antiproton accumulator. Due to the positive charge of the beam, no trapped ion effects are expected.

By contrast, Figure 3 shows the transverse response for 68 mA of antiprotons. The double peaked characteristic is believed to be due to the influence of trapped ions on the response of the beam to a transverse stimulus. This effect is enhanced if any of the various ion clearing mechanisms are made less effective or turned off (see Figure 4). This characteristic is observed up to very high frequencies (up to ~100 MHz).

A self consistent theory which explains the details of the ion modified beam transfer function is still under development [6]. The double peaked feature may be qualita-
tively understood by considering the non-linearity of the ion motion in the beam.

![Figure 2: Beam transfer function at the \((2 - q)f_{\text{rev}}\) resonance with 52 mA of protons in the accumulator.](image)

The equation of motion, to second order in \(r/\sigma\), for an ion of charge to mass ratio \(Z/A\) in the presence of this field is:

\[
Am \ddot{r} = ZeE(r)
\]

\[
\ddot{r} + \left[ \omega_p^2 + \left( 1 - \frac{1}{4} \frac{\sigma^2}{\sigma} \right) \frac{r}{\sigma} \right] \ddot{r} = 0
\]

\(m_p\) is the proton mass. \(\omega_p\) is the ion linear bounce frequency, and is given by:

\[
\omega_p = \sqrt{\frac{Z e^2 |\lambda|}{4 \pi \varepsilon_0 m_p \sigma^2}}
\]

All higher order terms in Equation (2) are even powers of \(r/\sigma\). Thus, large amplitude motion occurs with a smaller oscillation frequency than the zero amplitude motion.

During a beam transfer function measurement the beam is driven near its resonant frequency by a network analyzer. The ensuing transverse oscillation of the beam drives the ion motion at that frequency to larger amplitudes [7] causing a downward shift in the ion oscillation frequency. This depletes the ion population oscillating near the beam resonance. The effect of the network analyzer drive is to temporarily increase the transverse stability of the beam by depopulating the ion distribution near the beam resonance.

III. CLEARING TRAPPED IONS BY RF MODULATION OF BEAM

To date, the most successful strategy for dealing with trapped ion instabilities in the Fermilab Antiproton Accumulator has been to eliminate the ions from the beam. There are two primary techniques in routine use which accomplish this. The first technique is the use of clearing electrodes placed at most of the longitudinal minima in the beam space charge potential [4]. The clearing electrodes allow operation of the Antiproton Accumulator up to antiproton intensities of approximately 130 mA.

The second ion clearing technique is to destabilize the ion motion by modulating the line charge density of the beam by the application of a small amount of RF voltage. Ion clearing with RF has permitted stable operation the Accumulator with antiproton intensities as high as 220 mA (the intensity limit has not yet been reached).

![Figure 4: Beam transfer function at the \((2 - q)f_{\text{rev}}\) resonance with 132 mA of antiprotons in the accumulator.](image)
system on. Relatively little stability is gained by further turning up the RF voltage. This feature of RF ion clearing is not quantitatively understood.

A. Stability of Small Amplitude Ion Motion

In order to understand the effect of a small RF modulation of the beam on the ion motion it is instructive to begin with small amplitude motion such that the non-linear terms in Equation (2) are negligible. To account for the bunching of the beam, λ, in Equations (1) and (3) is replaced with

\[ \lambda = \lambda_0 (1 - \delta \cos \omega_{asy} t) \]

where \( \delta \) is the fraction of the beam in the RF bucket, and \( \omega_{asy} = 2\pi f_{rev} \). The RF harmonic, \( h \), is 2 for the results presented in this paper. With this modification Equation (2) becomes:

\[ \ddot{r} + \omega_s^2 (1 - \delta \cos \omega_{asy} t) r = 0 \]  

(4)

Making the change of variable \( x = \omega_{asy} t / 2 \), the linearized equation of motion becomes:

\[ \frac{d^2 x}{dx^2} + (a - 2q \cos 2x) x = 0 \]  

(5)

where\[ a = 4 \left( \frac{\omega_s}{\omega_{asy}} \right)^2 \] and \( q = \frac{a \delta}{2} \)

(6)

Equation (5) is the Mathieu equation. A pertinent property of the Mathieu equation is that the motion is unbounded for certain regions in \((a,q)\) parameter space. The condition that ion motion is unstable at the \((n - q) f_{rev}\) resonance of the beam for all values of the bunching parameter \( \delta \) is given by:

\[ a = 4 \left( \frac{n - q}{h} \right)^2 = 1, 4, 9, \ldots \]  

(7)

This condition is not met at the three modes most relevant to ion induced instability (see Figure 1). Accordingly, small amplitude ion motion is stable.

B. Monte Carlo Analysis of Trapped Ion Motion

In order to investigate the nature of relatively large amplitude ion motion a Monte Carlo simulation of the ion motion was written which numerically solves the non-linear equations of motion in a vacuum chamber of finite radius. The simulation generates the initial ion positions according to a Gaussian distribution with the same \( \sigma \) as the beam. The beam \( \sigma \) is chosen to give a linear ion bounce frequency equal to the frequency of a beam resonance. The initial ion velocities are generated according to a 300˚K Maxwellian distribution. A simulated ion is tracked until it has either undergone many thousands of oscillations or is lost to the vacuum chamber wall.

The results of the Monte Carlo suggest that RF modulation of the beam has the effect of shrinking the amplitude range for stable ion motion. Deeper RF modulation results in a smaller stable phase space area for ion motion (see Figure 5).

Figure 5: Width of the ion initial amplitude distribution for stable simulated ion motion as a function of applied RF voltage. At 250 Volts, approximately 92% of the beam is bunched.

The Monte Carlo also shows that RF clearing is ineffective for clearing heavy ions (\( \frac{\lambda}{\Lambda} < \frac{1}{2} \)). Cleared hydrogen ions will therefore be replaced by heavier, less harmful, ion species. Any pockets of ions which are not cleared by the clearing electrode system will be partially neutralized by ions which do not adversely impact beam stability.

IV. REFERENCES


