PROPERTIES OF A TRANSVERSE DAMPING SYSTEM, CALCULATED BY A SIMPLE MATRIX FORMALISM

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Abstract

In a synchrotron, proton beams with injection steering errors perform coherent betatron oscillations, possibly of large amplitude. The oscillations may be damped by using a system of a beam position monitor and a variable, fast kicker combined in a feedback loop to form a 'transverse damper'. The system of ring, beam and damper can be modeled by iteration of a matrix mapping once per turn. This paper reports the calculation of damping rates, and coherent tune shifts by analytic solution of the recursions. Two cases are treated: (i) kick proportional to beam displacement; and (ii) 'bang-bang' damping in which, above a certain threshold, the kick depends only on the sign $(+/-)$ of the displacement. We demonstrate (under certain conditions) that the 'bang-bang' scheme provides a linear damping of the amplitude and no tune shift, and (for the same peak power) is faster than the conventional proportional damper which produces an exponential damping with time.

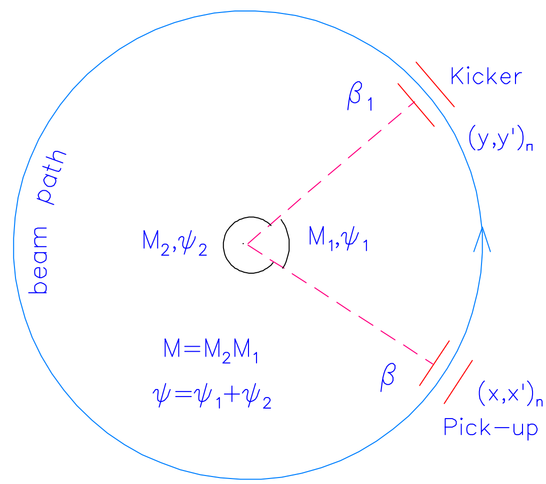

Figure 1. Schematic of a damping system.

I. INTRODUCTION

The aim of a damping system is to reduce the betatron oscillation of a beam as fast as possible. The damper may be designed to reduce injection errors, or to combat coherent instability; often the damper services both aims and its performance is a compromise: the effect of the kick is small compared with the displacement and it takes many repeated kicks to bring the beam on axis. If the oscillation amplitude is not reduced in a short period of time, then nonlinear effects which tend to accumulate with time, can dilute the emittance and reduce the beam quality. In fact, if filamentation is great enough the coherent motion 'washes out', the dipole signal vanishes and damping stops. A further concern, is that growth rate of a coherent instability is proportional to displacement; and if the condition for instability occurs during injection, the initial errors can be large. For these reasons it is important to provide fast damping. Further, if a damper intended to combat instability (later in acceleration) is used to reduce injection errors, its response will saturate for large amplitudes; and we should still like to find the damping rate.

A. System model

We shall use the single particle model of coherent beam motion and linear optics to illustrate the working of the system and derive its properties. Figure 1 shows the essential components of a damping system. Assume, as is inevitable in practise, that a beam has been injected into the synchrotron with some error; and that it oscillates about the closed orbit. The oscillation can be damped by reducing the net divergence with a fast kicker. For simplicity, the kicker is taken as a thin element that changes only the divergence but not the displacement at its location. Due to power limitation of the kicker, the divergence is reduced a small amount each revolution, and many kicks are required. Because the direction of the divergence can be different each time, the kick direction has to be adjusted accordingly. A beam position pick up (P.U.) at ψ_1 betatron phase advance up stream is used to provide this feedback information.

B. Kick schemes

We shall consider only two possible kick schemes: (i) the kick is proportional to upstream displacement, and (ii) the kick magnitude is constant but the sign comes from the sign of the beam displacement. The second scheme is simple to arrange: the kicker is powered by a constant supply whose output polarity is adjusted each time to damp the oscillation. This is known as 'bang–bang' damping. For the proportional kick, the P.U.-signal is used to drive a linear amplifier that powers the kicker. This eliminates the possibility of 'over kicking' the particle, as is unavoidable with a constant magnitude kick. In reality a linear amplifier will saturate at some peak power, and so above a certain threshold displacement, the magnitude of the kick becomes constant. Consequently, with this arrangement, the time sequence of kicks is a combination of proportional or constant. However, for simplicity, we only derive the damping characteristics of either purely proportional or purely constant-magnitude kicks.

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II. PROPORTIONAL KICK

Consider first the case of a proportional kick, which leads to exponential damping of the betatron oscillation. We will outline the derivation of the coherent tune shift and the dependence of the damping coefficient on the beta function and phase advance. We adopt a vector notation in which the first component is the displacement and the second the divergence.

In Figure 1, $\mathbf{x}_n$ denotes the coordinates of the particle at the pick–up after n revolutions and likewise $\mathbf{y}_n$ at the kicker. The magnitude of the kick is

$$\Delta y_n' = -k x_n , \tag{1}$$

where k is the kick strength proportionality constant. The displacement is assumed to be unchanged. The coordinates of the particle on the next turn become

$$\mathbf{x}_{n+1} = \mathbf{M}'\mathbf{x}_n = \{\mathbf{M}_2(\mathbf{M}_1 + \mathbf{K})\}\mathbf{x}_n \ , \tag{2}$$

where $\mathbf{M}_1$ is the linear transfer matrix from the P.U. to the kicker, $\mathbf{M}_2$ is that from the kicker (around the far side of the ring) back to the P.U., and $\mathbf{K}$ is the kick matrix. The modified one–turn map, $\mathbf{M}'$, is linear; so applying the map n times to the intial coordinates $\mathbf{x}_0$, we have

$$\mathbf{x}_n = (\mathbf{M}')^n \, \mathbf{x}_0 \ . \tag{3}$$

Equation 3 is a system of 2 linear homogenous equations with constant, real coefficients and has solutions of the form

$$\mathbf{x}_n = \lambda^n \mathbf{e}, \tag{4}$$

where λ and $\mathbf{e}$ are corresponding eigenvalue and eigenvector of $\mathbf{M}'$. Because $\mathbf{M}'$ is 2×2 and real, the two eigenvalues and eigenvectors come in complex conjugate pairs. Let us write $\lambda = \exp(\alpha + i\mu)$ and $\mathbf{e} = \mathbf{u} + i\,\mathbf{v}$ with α, μ and **u**,**v** real. The complete solution can be written as

$$\begin{aligned} \mathbf{x}_n &= e^{n\,\alpha}\{c_1\,(\mathbf{u}\cos n\mu \,-\, \mathbf{v}\sin n\mu) \\ &\quad +\ c_2\,(\mathbf{u}\sin n\mu \,+\, \mathbf{v}\cos n\mu)\} \ , \end{aligned} \tag{5}$$

where c_1 and c_2 are real constants. The oscillation is exponentially damped and has a modified one–turn phase advance μ. The damping coefficient α is given by

$$e^{2\alpha} = 1 - k(\beta\beta_1)^{1/2}\sin\ \psi_1, \tag{6}$$

where β and β_1 are the beta function at the pick–up and at the kicker respectively, and ψ_1 the relative phase advance between them. μ is given by

$$\cos\ \mu \ = \ \frac{2\cos\ \psi - k(\beta\beta_1)^{1/2}\sin\ \psi_2}{2[1 - k(\beta\beta_1)^{1/2}\sin\ \psi_1]^{1/2}} \ , \tag{7}$$

where ψ_2 is the phase advance from the kicker, around the far side of the ring, to the P.U. and ψ is the unperturbed one–turn phase advance without damping.

III. CONSTANT MAGNITUDE KICK

When the magnitude of the kick is the same each time, $\mathbf{x}_n$ is given by a nonlinear recursion. The mapping contains the summation over all previous revolutions of the function sgn(x_k), and does not admit an exact solution in closed form. However, we will show that to first order and under a certain phase advance, the damping of the amplitude is linear with turn number and there is no coherent tune shift.

We start with the one–turn map modified by the constant magnitude kick:

$$\mathbf{x}_{n+1} = \mathbf{M}(\mathbf{x}_n + \text{sgn}(x_n)\boldsymbol{\Delta}), \tag{8}$$

where $\boldsymbol{\Delta}$ is the equivalent of the kick transformed upstream to the pick–up and $\mathbf{M}$ is the unperturbed one–turn map of the ring. The map of $\mathbf{x}_n$ after n revolutions is thus

$$\mathbf{x}_n = \mathbf{M}^n\mathbf{x}_0 + \sum_{k=0}^{n-1} \text{sgn}(x_k)\mathbf{M}^{n-k}\boldsymbol{\Delta} \ . \tag{9}$$

The solution $\mathbf{x}_n$ can be written in terms of the eigenvalues and eigenvectors of $\mathbf{M}$. After some algebra, the complete solution of the displacement can be written

$$\begin{aligned} x_n &= C\cos(n\psi + \phi) \\ &+ D\cos(n\psi + \phi)\sum_{m=0}^{n-1}\text{sgn}(x_m)\cos(m\psi + \Delta\phi) \\ &+ D\sin(n\psi + \phi)\sum_{m=0}^{n-1}\text{sgn}(x_m)\sin(m\psi + \Delta\phi), \end{aligned} \tag{10}$$

where C, ϕ, are determined by the initial conditions and D, $\Delta\phi$ are determined by the kick strength and ψ_1 .

When the kick is independent of amplitude, one can profitably think of the damping as occurring not by changing the divergence, but by changing the closed orbit (C.O.) each turn to bring it closer to the displaced beam. Given that it is only the C.O. which changes we should expect no tune shift. Hence to first order, we can substitute the unperturbed oscillation $x_m = C\cos(m\psi + \phi)$ in sgn(x_m) on the right hand side of equation (10). In order to evaluate the sum, we approximate sgn$[x_m]$ by $\cos(m\psi + \phi)$. After summation, collecting like terms gives four sinusoidal terms of equal phase advance per turn. Hence, to first order there is no coherent tune shift of the damped oscillation and so the trial solution is self-consistent. Neglecting the two sinusoidal terms whose amplitudes are constant and small compared to the initial beam amplitude, the damped oscillation decreases linearly as n. For the special case in which ψ_1 is an odd multiple of $\pi/2$ and the derivative of the beta function at the P.U. is zero, the damped oscillation can be written as

$$x_n \approx (A - n\Delta)\cos(n\psi + \phi), \tag{11}$$

where A is the modified initial amplitude and the linear damping rate

$$\Delta = \sqrt{\beta\beta_1}\,|\Delta y'|. \tag{12}$$

depends on the constant magnitude kick.

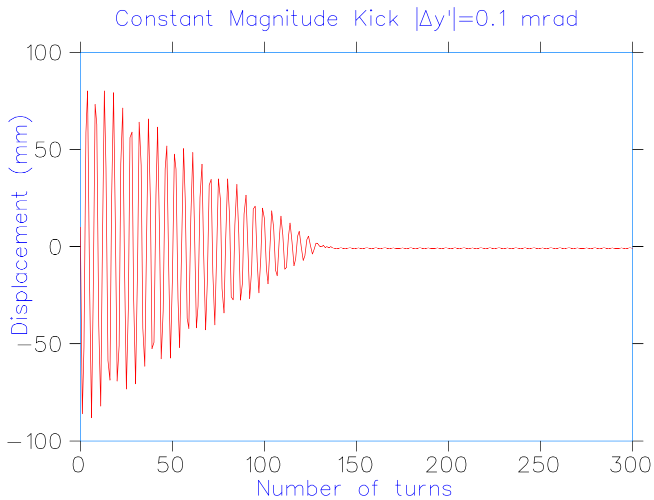

Figure 2. Linear damping by constant magnitude kick.

IV. SIMULATION OF DAMPING

We have computer simulated a damping system with constant magnitude kick to compare the results with the linear damping rate given in equation (12), and also to confirm that there is no coherent tune shift for any phase advance between P.U. and kicker. A damper subroutine was written and incorporated into the multiparticle-tracking injection-simulation code ACCSIM. The optics assumed was that of the KAON Factory Accumulator ring. As a check, the code was used to simulate the case of a proportional kick, where the damping coefficient and tune shift can be compared with exact expressions (6 and 7); and the accuracy was found to be satisfactory.

Figure 2 shows the displacement of the particle as registered by the pick–up as the oscillation is being damped with a constant magnitude kick of $|\Delta y'| = 0.1$ mrad. The large initial displacement was chosen to highlight the linear decay of the amplitude. However, to allow Fourier analysis and extraction of the tune, we have chosen $|\Delta y'| = 0.005$ mrad which damps an amplitude of about 100 mm in a few thousand turns. The results tabulated below are for five different phase advances ψ_1 between the pick–up and the kicker, including the special case of $\pi/2$. All five cases exhibit linear damping and, in all cases, the tune shift is less than 5×10^{-5}, which is the resolution limit of the FFT. This confirms that there is no tune shift.

Table I : Simulations for constant magnitude kick.

$\frac{\psi_1}{2\pi}$	β_1 (m)	$2\pi\Delta\nu$ $\times 10^{-6}$	Damping rate (10^{-3} mm/rev)
0.2347	5.262	-10.0 ± 0.1	-21.93 ± 0.05
0.2500	6.033	0.0 ± 0.10	-23.63 ± 0.05
0.3141	15.891	40.0 ± 0.1	-35.34 ± 0.05
0.4508	9.303	50.0 ± 0.1	-8.81 ± 0.05

For the special case $\psi_1 = \pi/2$, we can compare the actual damping rate to that given by the approximate formula (11). The formula gives a linear decay rate of 37.14×10^{-3} mm/rev and the actual rate (from simulation) is $(23.63 \pm 0.04) \times 10^{-3}$ mm/rev. The discrepancy is large, and is due to the approximation of sgn(x_m) by $\cos(m\psi + \phi)$ in the derivation. Given that $|\text{sgn}(x_m)| \geq |\cos(m\psi + \phi)|$ and kicks in the simulation are larger than in the approximate summation, it may surprise that the simulated damping rate is smaller. However, with a constant kick, there will be times when the kick is too large which results in temporary antidamping. However, if the kick is scaled as $\cos(n\psi + \phi)$ the resulting damping is more effective, because there is less over kicking. Hence the formula (12) slightly over estimates the damping rate, but can estimate the kick requirement $\Delta y'$ if used with care.

V. PERFORMANCE CONSIDERATIONS

According to the expression (6) for the exponential damping coefficient, the pick–up and the kicker should be placed as close as possible to where the beta function has its maximum values and the relative phase advance should be ideally an odd multiple of $\pi/2$. This arrangement produces the fastest damping because the displacement of the particle is greatest at the P.U. and this, in turn, leads to large proportional kicks. The same arrangement also works well for the case of damping with constant magnitude kick; because the given kick makes the largest possible change to the closed orbit.

If the amplifier has infinite power resources, then obviously exponential damping is faster than linear damping. In reality, the amplifier will saturate and so for the same peak power linear damping is often faster. For an oscillation amplitude at the peak power limit, and proportional kicking, the number of revolutions n_p required to damp the amplitude to $1/e$ is:

$$n_p \;=\; 2/[k\sqrt{\beta\beta_1}]\;. \tag{13}$$

For the same amplitude and peak power, and constant magnitude kicking,

$$n_c \;=\; 1/[k\,e\sqrt{\beta\beta_1}]\;. \tag{14}$$

Accordingly, linear damping is $\simeq\; 2e$ faster than exponential damping for the same peak power. A drawback of linear damping is that it does not damp the amplitude down to zero. To achieve this, some final stage exponential damping is necessary.

VI. CONCLUSION

We have derived the characteristics of damping systems with purely proportional kick and purely constant magnitude kick under certain conditions. We have found that proportional kick produces exponential damping and induces a coherent betatron tune shift; whereas the constant magnitude kick produces linear damping and no coherent tune shift. We have also considered the damping performance of these two types of kick with practical power supply constraints and found that linear damping is faster.

A. Acknowlegment

We thank Fred Jones for promptly writing and installing the damper subroutine in his ACCSIM code and we greatly appreciate his enthusiasm for this work.