A STUDY OF BEAM-BEAM INTERACTIONS AT FINITE CROSSING ANGLES FOR A B-FACTORY

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Abstract

Feasibility of adopting a finite beam crossing angle at the interaction point of KEKB B-factory has been studied. Various aspects of beam behaviors, such as sensitivities to resonances and development of bunch tails, have been investigated with computer simulations. It is shown that an acceptable operating condition can be found with a suitable combination of machine parameters that are envisioned at KEKB.

I. INTRODUCTION

KEKB is a high-luminosity asymmetric electron-positron (8 ×3.5 GeV) collider for studies of productions and decays of B mesons at \( E_{CM} = 10.5 \) GeV [1] [2]. At KEKB it has been planned to adopt a finite-angle beam crossing scheme (2 × 11 mrad) at its interaction point [3]. This paper reports some recent results of studies that have been done to investigate the beam dynamics in this collision condition. Table I summarizes the KEKB parameters that are pertinent to discussions of beam-beam interactions.

| \( \beta_x \) at the IP | 0.33 | m |
| \( \beta_y \) at the IP | 0.01 | m |
| \( \epsilon_x \) | \( 1.8 \times 10^{-8} \) | m |
| \( \epsilon_y \) | \( 3.6 \times 10^{-10} \) | m |
| \( \sigma_z \) | 0.004 | m |
| \( (\nu_x, \nu_y, \nu_z) \) | (0.52,0.08,0.017) |
| Particles / bunch | \( 1.4 \times 10^{10} \) | electrons |
| | \( 3.2 \times 10^{10} \) | positrons |
| Total number of bunches | 5120 | per ring |

Table I

Working parameter set for the half crossing angle \( \phi = 11 \) mrad, determined from considerations on beam-beam effects, dynamic apertures and others.

A new beam-beam simulation algorithm has been developed for this study [4] [5]. As indicated in Figure 1, the bunches which are colliding at a crossing angle are first Lorentz-transformed into a frame where their momentum vectors appear parallel. In this “head-on” frame a symplectic synchro-beam mapping is applied to calculate the beam-beam forces and their effects on the bunches. When the mapping is finished, the two bunches are Lorentz-transformed back to the laboratory frame, where the beam tracking code takes over the rest of simulation.

The model is fully symplectic in the 6-dimensional phase space, and it incorporates all known effects such as the energy loss due to the traverse of transverse electric fields at an angle, energy loss due to longitudinal electric fields, and effects due to the variation of \( \beta \) along the bunch length during collision (holographic effect).

II. BEAM-BEAM SIMULATION WITH LINEAR LATTICE FUNCTIONS

Dependence of beam sizes and the luminosity on the wide range of machine parameters have been investigated with a simplified lattice model, where the beam transfer through the ring is represented by a linear matrix [5].

A weak-strong beam formalism is used to implement the beam-beam interaction algorithm outlined in the previous section. Typically the strong bunch is longitudinally sliced into 5 slices, and the weak bunch is represented by 50 super-particles. Effects of radiation damping is taken into consideration. The beam-beam collision and revolutions through the ring are simulated for up to 10 radiation damping time. Then the equilibrium beam size is examined. The expected luminosity is calculated from a convolution of the distribution functions of the two beams.

Initial beam parameters are specified so that they would give the design luminosity of \( 1 \times 10^{34} \) cm\(^2\)s\(^{-1}\) or somewhat higher values, with collisions of 5120 bunches per ring in the absence of aberrations and a beam blow-up.

Figure 2 shows a calculated luminosity contour plot in the \( \nu_x - \nu_y \) plane, with the crossing angle of \( 2 \times 10 \) mrad, in the vicinity of the working point: \( (\nu_x, \nu_y) = (0.52, 0.08) \).

Notable observations are summarized as follows:

1. A finite crossing angle at the interaction point (IP) certainly causes a reduction of usable area in the \( \nu_x - \nu_y \) plane, because of synchro-betatron and other resonances.
2. However, when \( \nu_x \) is kept small \( i.e. \) below 0.02, a fair amount of areas in the \( \nu_x - \nu_y \) plane is still free from resonances. This requirement is compatible with the overall KEKB design. Some of such acceptable \( \nu_x - \nu_y \) areas are compatible with the conditions preferred from dynamic aperture considerations.
3. With the beam intensity of a few $\times 10^{10}$ per bunch or below, no intensity-dependent beam blow-up is seen with finite crossing angles.

4. When the $v_y$ is small, and when a resonance free condition of $v_x$-$v_y$ is chosen, the predicted luminosity there is roughly consistent with naive expectations from the geometric and linear effects. Occasionally the simulated luminosity exceeds naive expectations which only consider geometric effects. This is because of effects of the dynamic beta and dynamic emittance.

III. SIMULATIONS WITH THE LATTICE WHICH INCLUDES NONLINEARITY AND ERRORS

The beam-beam simulation algorithm based on the weak-strong model has been incorporated in the beam tracking software SAD at KEK [6]. This provides a tool to study effects of finite crossing angles at the IP, combined with the nonlinearity of the KEKB machine lattice[7] and its possible errors.

Simulations with SAD have been conducted with realistic assumptions on lattice errors. Presence of detector solenoid field and its partial compensation near the interaction point is taken into account. Finite alignment and excitation errors of bend (B), quadrupole (Q), sextupole (SX), and steering correction magnets (ST) are simultaneously considered. Typical magnitudes of assumed errors, which we consider realistic, are summarized as follows:

<table>
<thead>
<tr>
<th>Element</th>
<th>BPM</th>
<th>B</th>
<th>Q</th>
<th>SX</th>
<th>ST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horiz. shift ($\mu$m)</td>
<td>75</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Vert. shift ($\mu$m)</td>
<td>75</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>$x$-$y$ roll (mrad)</td>
<td>0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Field error</td>
<td>0</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Gaussian errors are produced according to the rms values given in the table above. For each series of generated errors, the orbit and tune corrections are done in the tracking code as if it were in an actual machine operation. Then the scale of assumed errors is re-normalized so that the expected vertical spot size $\sigma_y$ agrees with the design value. We call it “error normalization factor” $f$. With such renormalized errors in the machine, the orbit and tune corrections are, once again, performed. The expected luminosity is evaluated by using the beam-beam code, plus the tracking with SAD. Different random seeds used for generating lattice corrections are, once again, performed. The expected luminosity is consistent with naive expectations from the geometric and linear effects. Occasionally the simulated luminosity exceeds naive expectations which only consider geometric effects. This is because of effects of the dynamic beta and dynamic emittance.

It is seen that the lattice nonlinearity and likely machine errors do not lead to fatal degradations of the estimated luminosity.

IV. QUASI STRONG-STRONG SIMULATION

To address issues which may be overlooked in the strong-weak formalism, while not spending a prohibitive amount of CPU time, a quasi strong-strong formalism has been developed. Here, every once in 500 turns of revolution, the average electron and positron bunch sizes are “registered.” During the next 500 turns, a weak-strong model calculations are performed, while this “registered” electron (positron) bunch size is used as the “strong bunch size” for calculating the development of positron (electron) bunch size. Then the “strong bunch sizes” are updated again, and the simulation continues.

Figure 3 shows the expected luminosity as function of revolution number obtained from this simulation. A linear matrix is used to represent the lattice beam transfer. No indications of a bunch core blow-up are seen. Figure 4 shows that the horizontal beam size obtained in the simulation is $\sigma_x = 6.2 \times 10^{-5}$ m. It is somewhat smaller than the nominal value $7.56 \times 10^{-5}$ m. It is consistent with the dynamic beta and dynamic emittance effect.

V. BUNCH TAILS EXCITED BY BEAM-BEAM INTERACTIONS

The presence of non-Gaussian bunch tails causes an extra synchrotron radiation (SR) background to the detector facility. The fractional bunch tail population should be kept less than $10^{-5}$ for $> 10\sigma_x$, and $10^{-5}$ for $> 30\sigma_y$, according to design considerations on SR masks near the interaction point. The bunch tail growth due to beam-beam interactions has been studied with a long-term strong-weak calculation with a linear lattice model. Typically the simulation is done by tracking 50 super particles over $10^8$ turns of revolution. This means 1000 seconds for 50 particles, and 14 hours for a single particle in an actual machine.

Figure 2. Calculated luminosity contour diagram in the case of crossing angle $= 2 \times 11$ mrad. Expected luminosity in the $v_x$-$v_y$ plane is shown. The contour spacing is $10^{33} \text{cm}^{-2}\text{s}^{-1}$.
Figure 3. The expected luminosity as function of revolution number in the quasi strong-strong model calculation.

Figure 4. Behavior of $\sigma_x$ as function of revolution number. The solid line shows the electron bunch size. The broken line shows the positron bunch size.

Figure 5 shows the particle distribution, predicted by this simulation, as function of action variables ($I_x$ and $I_y$) in the horizontal and vertical planes. The canonical beam parameters, as given in Table I, have been used in this case. From this data, the particle population in the bunch tail has been calculated. It was found that the probability that a particle has the vertical amplitude larger than $30\sigma_y$, where $\sigma_y$ is the design bunch size, is approximately $10^{-12}$. Since the bunch population is $O(10^{10})$, no particle is likely to have such a large vertical amplitude. Tails in the horizontal direction have also been studied, and it has been found that its development is much slower than in the vertical direction. Preparations are under way to evaluate bunch tails with tracking calculations which include non-linear effects of the lattice and possible machine errors.

VI. CONCLUSIONS

It is seen that within the simulation studies conducted so far, the design luminosity goal can be achieved with the finite angle crossing of $2 \times 11$ mrad at the interaction point. Naturally this cannot be fully confirmed until operating the real-life machine. As a back-up safety measure, the use of crab crossing scheme to combine with the finite angle collision is being considered [2]. In the meanwhile, more elaborate studies of beam-beam effects will be continued. Some of the major projects include:

- In the tail simulation, nonlinear effects in the lattice should be included in the calculation.
- A strong-strong simulation will be updated so that it evaluates the beam envelopes in each turn, using a Gaussian approximation to calculate beam-beam forces.

References