Beam-Based Alignment of Sextupoles with the Modulation Method

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Abstract
Alignment of sextupoles is vitally important for rings such as B-factory and damping rings. A precision of 100μm or less is required to produce low emittance and large dynamic aperture. Beam-based alignment is crucial for such precision. We present a new technique to measure the magnetic center of sextupole magnet using beam. It is based on the so-called “K-modulation”[2], [3] which has been applied to quadrupoles. We use auxiliary coils on sextupoles which are connected to produce a quadrupole field. By modulating the quadrupole field with some frequency, we observe a sizable modulation of closed orbit whose amplitude is proportional to the offset of beams from magnetic center, thus giving information on the center of the sextupole. In the experiment we could measure the center with a precision of less than 50μm. The present method is applicable to sextupoles connected in series to a common power supply because it induces no voltage to main coils. We report on the experimental results and discuss on the effects of eddy current, asymmetry of chamber and eventual precision.

I. INTRODUCTION
Small emittance in the vertical plane is required to maintain high luminosity in future accelerators such as B-Factories, damping rings for linear colliders. In these machines vertical misalignment of sextupoles has large contribution to the vertical emittance. It is very difficult to keep the alignment of 100μm against drift of floor levels even though it could be done with high-tech in the commissioning stage. Beam-based alignment (BBA) has been proposed to overcome such difficulties. Several techniques has been proposed for the BBA: for example, π-bump method[1] and K-modulation[2].

K-modulation method has been applied only to quadrupoles[3]. In this paper we discuss on an application to sextupoles and results of a test experiment in TRISTAN. In section II we describe principle of the method. Section III gives detailed discussion on the influence of eddy current in the vacuum chamber, which limits maximum modulation frequency. Section IV gives results of experiment in TRISTAN and discussions on ultimate precisions of the method and on an application to B-Factories.

II. PRINCIPLE OF THE METHOD
In the application of the modulation method, the point is that the kick due to sextupole is too small to detect since it is quadratic to the beam offset at the sextupole. To overcome this we introduced additional quadrupole windings on the sextupole and applied the “K-modulation” to these auxiliary coils. By exciting the auxiliary coils (Fig. 1) numbered 1, 3, 4, and 6, for example, quadrupole field can be produced. Although higher multipoles, mainly octupole, are also excited, quadrupole is still dominant near the origin. If its strength is modulated with some frequency the beam receives, depending on its position, a sizable kick with the same frequency. The amplitude in the resulting closed orbit modulation is proportional to the beam offset, thus giving information of the magnetic center. The merits of this method are: (1) the frequency domain analysis gives high signal to noise ratio, (2) we only need dedicated pickups sensitive to the orbit modulation and (3) it is applicable to sextupoles connected in series to a common power supply because it induces no voltage to main coils. A possible demerit is that the present method gives merely the center of the quadrupole field produced by the selected poles not the center of the sextupole field which is generated by all of the poles. Difference between them arises mainly from construction errors of poles. They can be checked by comparing the results with other configuration of poles, (1,2,4,5) and (2,3,5,6).

III. EDDY CURRENT EFFECTS
Since the vacuum chamber has high electric conductivity eddy current driven by the modulated field has significant contribution to the total field.

Consider the equation for the magnetic field in the material with conductivity σ,

$$\nabla^2 B = \mu_0 \sigma \frac{\partial B}{\partial t}. \quad (1)$$

Here assuming a very low frequency, several ten Hertz at most, we neglected the displacement current term. The total field can be expanded in perturbation series of ω:

$$B = B_0 + B_1 + B_2 + B_3 + B_4 + \cdots, \quad (2)$$

Figure 1. Geometry of the vacuum chamber and the sextupole for TRISTAN. Auxiliary coils on the poles were excited to produce the quadrupole field.
where $B_n = \tilde{B}_n \exp \text{i} \omega t$ is the external modulation field and $B_n$ ($n = 0, 1, \cdots$) satisfies following Eqs.

\begin{align*}
\nabla^2 B_1 &= \mu_0 \sigma \frac{\partial B_0}{\partial t} = \mu_0 \sigma \text{i} \omega B_0,
\end{align*}

\begin{align*}
\nabla^2 B_2 &= \mu_0 \sigma \frac{\partial B_1}{\partial t}, \cdots.
\end{align*}

From Eqs. (3) and (4) we can rewrite $B_n$ as

$$B_n = (\text{i} \omega)^n \tilde{B}_n \exp \text{i} \omega t.$$  

Real and imaginary part of the total field are given by

\begin{align*}
\Re B &= \tilde{B}_0 - \omega^2 \tilde{B}_2 + \omega^4 \tilde{B}_4 - \cdots, \quad (6)
\end{align*}

\begin{align*}
\Im B &= \tilde{B}_1 - \omega^2 \tilde{B}_3 + \omega^4 \tilde{B}_5 - \cdots. \quad (7)
\end{align*}

We should note that these relations hold for arbitrary geometry and boundary conditions of beam pipe. Eq. (6) implies that in the low frequency the real part (in-phase component) of field always has the form of parabola as a function of the frequency. The strength of high order term depends on the geometry of vacuum chamber.

**Figure. 2.** Horizontal position of the field center as a function of modulation frequency.

**Figure. 1** shows the geometry of the beam pipe and the poles of the sextupole magnet of TRISTAN which we used in the experiment. The beam pipe is made of aluminum with race-track shape and at each side a water channel or a slot for heating is attached. The beam pipe has a large left-right asymmetry, thus inducing large asymmetric eddy current. It is enhanced by the fact that the cooling (heating) channels are in the strong modulated field. The left-right asymmetry in the eddy current induces strong dipole field in y-direction ($B_1$), making the center of quadrupole field shifted in x-direction. The field $B_1$ is, fortunately, retarded by 90 degree giving no in-phase component. The in-phase component comes from next order term ($B_2$). In order to estimate the shift in the center of modulated field in self-consistent manner we made a two-dimensional calculation of the field using a computer code. In the calculation four coils wound on poles (1,3,4, and 6 in Fig. 1) were excited to produce a quadrupole field of 0.032 T/m. The eddy current was calculated under the condition that the total current should be zero in the cross section of the beam pipe. Fig. 2 shows the horizontal position $x_0$, at which in-phase component of the vertical field $B_y(x, y = 0)$ disappears, as a function of frequency. Filled circles are for the case without misalignment of beam pipe while the squares and open circles are for the horizontal misalignment of +1 mm and -1 mm, respectively. In any case parabolic functions of $\omega$ fit the results very well.

**IV. EXPERIMENTAL**

We have installed auxiliary coils on each pole of a existing sextupole magnet of TRISTAN with length of 0.58 m. The cross section of the magnet is sketched in Fig. 1. We excited the auxiliary coils on poles (1,3,4,6) by 40 Ampere-turns(AT) with frequencies ranging from 0.1 Hz to 20 Hz, while the excitation of main coils were kept 615 AT. Beam energy was 8 GeV. Resulting modulation of closed orbit was detected with strip lines in the ring. In the measurement of horizontal(vertical) orbit we used one of strip-line electrodes attached to the horizontal(vertical) side of the chamber. Such selections of the electrodes enable us to separately observe the orbit motion in the horizontal and the vertical plane whenever the beams stay around the central region of the beam pipe. Output signal was processed with the electronic circuit which has been used for the BPM system of TRISTAN[4]. Using an FFT analyzer(HP3562A) we analyzed the pickup signal together with the driving signal that modulated the power supply of the auxiliary coils. We thus obtained both the amplitude and the phase of the cross-spectrum between the two.

**Figure. 2** shows an example of spectrum measured with the horizontal electrode. Upper part of the figure shows the power spectrum while the lower part shows the real part of the cross spectrum. Modulation frequency was 0.4 Hz. We made a horizontal bump orbit of 0.9 mm at the sextupole and resulting modulation amplitude of the orbit was about 4 $\mu$m at the pickup electrodes. Signal-to-noise ratio was 20 dB in this frequency range. Fig. 4 shows the dependence of the in-phase component of the signal.
The beam position was scanned by changing height of the local bump orbit at the sextupole while the beam position was measured with a nearest BPM to the sextupole. Distance between BPM and the sextupole was 18 cm. Fitting straight lines to the data we get the BPM offset. From the figure we clearly see that the measured BPM offset depends on the modulation frequency. Fig. 5 shows the dependence of the offset on the frequency. Fitting a parabola to the data, we obtained the offset for the zero frequency with precision of $25\mu m$. The coefficient of the $f^2$ term is greater by a factor of 4.5 than that of field calculation. This difference could be ascribed to the presence of lead shields in between poles, which was not taken into account in the calculation and possibly could enhance the asymmetry because they were packed not orderly between the pole gaps. Another reason for the difference may be water cooling pipes which bypass the magnet outside the return yoke so that it breaks the condition that the summation of eddy current is to be zero.

**DISCUSSION**

We could change the modulation frequency up to 20 Hz without any interference to the main power supply. This is because the induced voltages in the main coils cancel each other as described in section II.

Since the BPM offset for zero frequency is obtained through extrapolation from the data points of higher frequency it is desirable to use lower frequencies as possible in order to get higher precision. In the measurement we did not use frequencies less than 0.2 Hz by the following reason: Firstly there was very large background in the lower frequency part of the spectrum that is not seen in Fig. 1 since the signal was measured with AC-coupling. Secondly lower frequency implies longer time, however, there may be an orbit drift during a single measurement, which will directly degrade the precision. Another approach to get high precision is to make the beam pipe more symmetric: In the design of KEKB, a project of B-factory at KEK, symmetric beam pipe is proposed to take a profit of high frequency although high conductivity of the chamber (copper), unfortunately, induces stronger eddy current than that of TRISTAN.

Uncertainty of construction errors limit the ultimate precision of the method. A conceivable solution is to make a measurement of the difference between the center of the quadrupole filed and that of sextupole filed before the magnet is installed into the ring. Thus the ultimate precision will be limited by the precision of measurement on the filed centers.

**V. CONCLUSION**

New modulation-method of beam-based measurement on the field center of sextupole was proposed. Eddy current effects were considered in detail. Test experiment demonstrated that the precision is less than $50\mu m$.

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**References**

[1] S. Kamada et al., to be published.