INSERTION OF HELICAL SIBERIAN SNAKES IN RHIC*

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I. HELICAL SNAKES AND SPIN ROTATORS

For polarized protons in RHIC, two Siberian snakes and four spin rotators per ring will be used [1]. Snakes, 180˚ apart, and with their axis of spin precession at 90˚ to each other, are an effective means to avoid depolarization through resonances. Spin rotators, in pairs, rotate the spin from the vertical to the horizontal and back at an interaction. They are needed to study proton collisions with the spin in the horizontal plane.

We adopted a solution with four identical helical magnets for snakes and rotators [2]. Our choice is dictated by distinctive advantages of helical over transverse magnets. (i) they are modular, (ii) the maximum orbit excursion is smaller, and (iii) independent from the separation of magnets, (iv) they allow an easier control of the angle of the spin precession axis. To use the standard RHIC dipole cryostats, the length of the devices is limited to 12 meters. We have chosen 2.4 m as the length of each module.

An analytical approximate calculation of the properties of a four helical magnet was made [3]. Since we have found that the fringe field is important both for orbits and spin rotation, a systematic study by numerical integration of the equations of motion and of spin through the magnetic field was performed [4], using the field expression first described by Blewett and Chasman [5]

$$\begin{cases} b_x \approx \left[-1-\frac{1}{4}\left(3u^2+v^2\right)\right]\sin kz + \frac{1}{2}uv\cos kz \\ b_y \approx \left[1+\frac{1}{4}\left(u^2+3v^2\right)\right]\cos kz - \frac{1}{2}uv\sin kz \\ b_z \approx -\sqrt{2}\left[v+\frac{1}{4}\left(u^2 v+v^3\right)\right]\sin kz - \sqrt{2}\left[u+\frac{1}{4}\left(u^3+uv^2\right)\right]\cos kz \end{cases}$$

with $b = \dfrac{B}{B_0}$ $\quad \begin{cases} u = kx\ \sqrt{2} \\ v = ky/\sqrt{2} \end{cases} \quad k = \dfrac{2\pi}{\lambda}$.

For the fringe, we assumed that the field decays as 1/cosh in a distance equal to half the magnet aperture.

In the the program, the equation of motion

$$\frac{d\beta}{dt} = \beta \times \Omega \qquad \Omega = \frac{e\mathbf{B}}{m\gamma}$$

and the spin precession equation

$$\frac{d\mathbf{s}}{dt} = C_1 \mathbf{s} \times \Omega + C_2 (\beta \cdot \Omega)\mathbf{s} \times \beta,$$

with $C_1 = 1 + G\gamma \quad C_2 = -\dfrac{G\gamma^2}{1+\gamma} \quad G = \frac{1}{2}g - 1$

are integrated. The results described in the following are obtained by numerical calculation,

Rotator parameters are given in Table I . Orbit and spin are shown in Figs. 1A and 1B. To compensate the field integral including fringe, the angle of rotation of the field in each helix is less than 360˚. So, a particle entering the magnet on axis will emerge on axis. In RHIC, collisions will be done at energies from γ = 27 to the maximum γ = 250. Since the rotator beam line is at an angle ϕ = 3.674 mrad with the adjacent interaction straight, after the rotator the spin precedes further. To obtain a proper polarization at the interaction, the spin should emerge from the rotator at an angle $G\gamma\phi$ with the rotator axis. The field to provide a longitudinal polarization at the interaction is shown in Fig. 2.

Snake parameters are in Table II. Orbit and spin are shown in Figs. 3A and 3B, for the condition of the axis of spin precession at an angle of 45˚ with the beam direction. This angle is slightly adjustable by varying B1 and B2 to compensate for effects of solenoidal fields in the detectors.

Table I. Parameters of the helical magnets at injection (γ=27).

Spin Rotator				
Length [m]	Field[a] [tesla]	Field rotation[b]	Field orientation[c]	Max orbit [mm]
2.40	2.047	+345˚	97.5˚	24.1(hor) 10.0 (vert)
2.40	2.654	-345˚	82.5˚	
2.40	2.654	+345˚	97.5˚	
2.40	2.047	-345˚	82.5˚	
Siberian Snake				
2.40	1.191	+345˚	7.5˚	14.7 (hor) 31.5 (vert)
2.40	3.864	+345˚	187.5˚	
2.40	3.864	+345˚	7.5˚	
2.40	1.191	+345˚	187.5˚	

[a] For longitudinal polarization (Rotators). For precesson axis at 45˚ (Snakes).
[b] "+", right-handed helix. "-", left-handed.
[c] Angle of field with the vertical at magnet's entry.

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II. LINEAR COUPLING EFFECTS

Transfer matrices up to the third order have been numerically calculated by tracking many particles with random initial coordinates and performing a polynomial fit of the final vs. the initial conditions. The dependence of the numerical matrix on input parameters has been systematically checked. The fit results proved insensitive (< 1%) to most of the parameters varied. Only a center offset of 3 cm, unrealistic, produced variations greater than 1%. The values are also in excellent agreement with differential algebra results [6], and in reasonable agreement with the analytical approximation

The linear coupling effect of helical snakes and rotators in RHIC is calculated with a one turn linear map [7]. In the model, Fig. 4, we place snakes and spin rotators in their lattice position, and connect their locations by phase space rotations.

From the one-turn matrix

$$T = R_{61} \times \sigma \times R_{56} \times \pi \times R_{45} \times \pi \times R_{34} \times \pi \times R_{23} \times \pi \times R_{12} \times \sigma$$

with the phase space rotation between points i and j

$$\begin{bmatrix} R_{ij}^x & 0 \\ 0 & R_{ij}^x \end{bmatrix},$$

the linear coupling effect, i.e. the distance of minimum tune separation $\Delta Q_{\min}$ is derived. We write the 4x4 one-turn matrix T as $\begin{bmatrix} M & m \\ n & N \end{bmatrix}$. Then, in the particular case when $Q_x = Q_y = Q_0$ (fractional tune) , obtain

$$\Delta Q_{\min} = \frac{1}{2\pi} \frac{\sqrt{\det H}}{\sin(2\pi Q_0)},$$

where $H = m + n^{\dagger}$.

With Q_x =28.185, Q_y =29.185 and only 2 snakes, the results for the linear model are in Table II. The prediction from the analytical and numerical models are within a factor 2, due to different representation of the fringe field. Table II also shows the coupling when we add 4 rotators.

III. CONCLUSIONS

Spin rotators and Siberian snakes for RHIC can be built using 4 helical magnets obtained, by twisting, from the cosine dipoles. We found that the fringe fields are important. In the calculations we have used a plausible model for the fringe. However, only magnetic measurements on the prototypes presently being built will allow a final optimization. The linear coupling at injection, $\Delta Q_{\min} < 10^{-2}$, is well within the range of the RHIC decoupling system. At storage, the coupling introduced by the devices ($\Delta Q_{\min} < 10^{-4}$) is negligible.

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Table II. Linear coupling in RHIC at injection (γ= 27).

2 snakes $\Delta Q_{\min}$	analytical 0.00157	numerical 0.00289
2 snakes + 4 rotators $\Delta Q_{\min}$	no fringe fields 0.00303	with fringe fields 0.00383

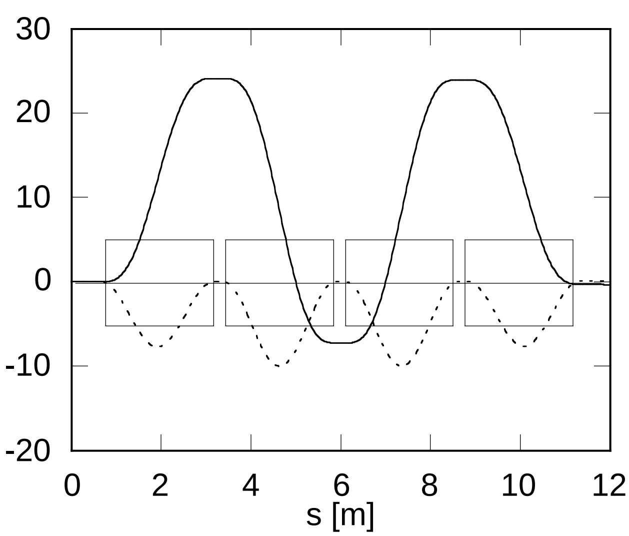

Fig. 1A. Spin rotator. Orbits [mm]. γ =27.

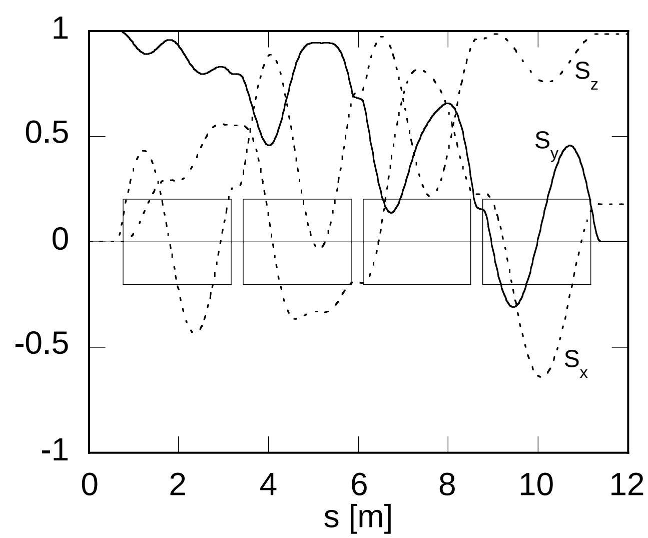

Fig. 1B. Spin rotator. Spin precession (γ =27).

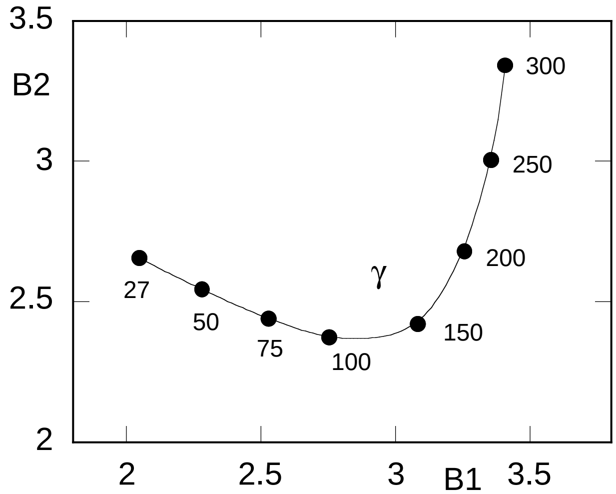

Fig. 2. Spin Rotator. Field [Tesla] to achieve longitudinal polarization in the interaction, vs γ.

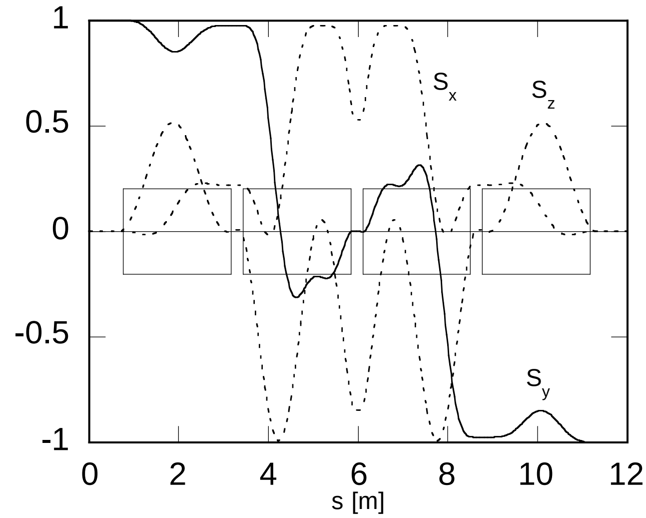

Fig. 3B. Snake. Spin precession. $\gamma = 27$.

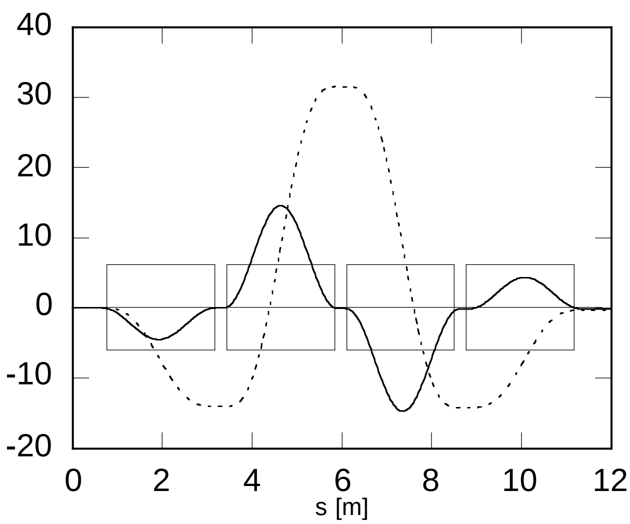

Fig. 3A. Snake orbits [mm]. $\gamma = 27$.

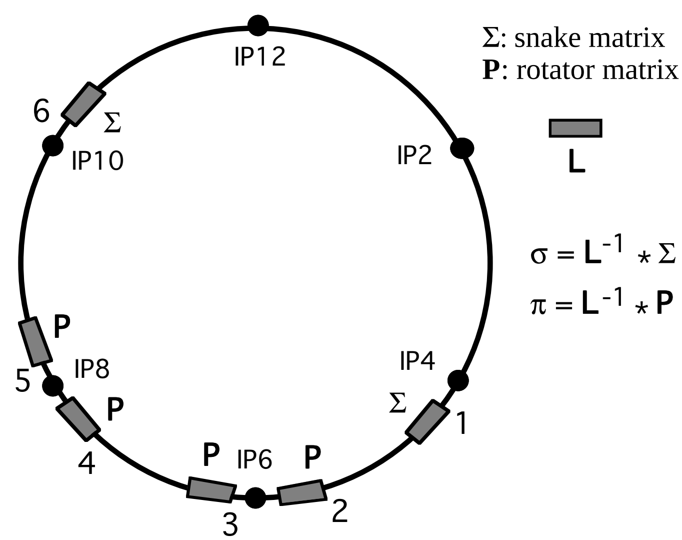

Fig. 4. Model of RHIC for the one-turn matrix.

V. REFERENCES

[1] Conceptual Design for the Acceleration of Polarized Protons in RHIC (T. Roser, Coordinator). May 24, 1993, revised September 3, 1993.
[2] V.I. Ptitsin and Yu.M. Shatunov, 3.rd Workshop on Siberian Snakes. Upton, NY, Sept. 12-13, 1994, p.15.
[3] E.D. Courant, RHIC/AP/47
[4] J.P. Blewett and R. Chasman, J.Ap.Phys. 48 (1977) 2692
[5] A. Luccio, 3.rd Workshop on Siberian Snakes, *loc. cit.*, p. 193
[6] S. Peggs, IEEE PAC Conference 1983, p.2460
[7] N. Malitsky, RHIC/AP/51