Abstract

Operating the SSC low energy booster (LEB) in a counterphasing mode is necessary because the low total ring voltage (25 kV) required at injection for adiabatic beam capture would otherwise result in cavity multipacting. Each cavity requires greater than 15 kV gap voltage (there are eight single-gap cavities in the LEB) to be free of multipacting. The analysis of the cavities' behavior in this mode and under time-varying beam loading is presented. It is shown that fast feedback loops with moderate gain are necessary to operate with the available tetrode power. The Robinson instability and the required power and phase histories are discussed.

I. INTRODUCTION

During the adiabatic capturing phase in the SSC LEB the total rf voltage needs to be lowered to 25 kV to achieve the desired capture efficiency [1]. Testings on similar cavity designs developed at Los Alamos and TRIUMF had found multipacting problems when the cavity voltage went below 20 kV. Recently experiments with the LEB prototype cavity indicate that multipacting starts at voltages below 10 kV. Either way with the present design of 8 to 10 cavities it will be impossible to achieve the low total voltage with all cavities in phase. A solution would be to counterphase the cavities, i.e., to operate a set of cavities in an acceleration mode while operating another set in deceleration mode. Certain constraints are imposed on the cavities to follow the total voltage program. These constraints together with the Robinson stability requirement [2] enforce us to operate the cavities above the minimum power condition. We will show that operating the cavities with a moderate rf feedback would reduce the power requirements considerably as well as the rate of phase sweeping. Moreover we will show that there is a critical feedback gain and below this gain no counterphasing is possible with the available power in our tetrodes.

The paper is divided into three sections. The first section is a short description of the model used in this analysis. The second makes use of the equations described in the first section to estimate the power and phase requirements during the adiabatic capture period. The paper is concluded with a short summary.

II. CAVITIES-BEAM MODEL WITH RF FEEDBACK

We adopt throughout the paper the equivalent circuit model for the cavities-beam system. The model for a single cavity with its rf feedback is shown in fig. (1). The cavity is represented by a parallel RLC circuit with tunable inductor. The beam is modeled as a current generator with a given $I_b(t)$ shape.

![Figure 1](image1)

figure 1. circuit model of LEB cavity with rf feedback

Assuming fast rf feedback and using Kirchhoff's law, the cavity voltage $V$ satisfies the following differential equation

$$\frac{d^2 V}{dt^2} + A \frac{dV}{dt} + BV = D$$

(1)

where

- $A = \omega_f^2/2Q_L - 2/\omega_f \omega_d \frac{d\omega_f}{dt}$
- $B = \omega_f^2 - 1/\omega_f \frac{d\omega_f}{dt}$
- $D = \omega_f^2/2Q_0 (\xi R_s \frac{dV_i}{dt} + R_s \frac{dI_b}{dt})$
\[ \frac{1}{Q_0} \frac{d\omega}{dt} R_c (\xi V_i + I_b) \]

and we define
\[ \omega_0 = \sqrt{1/LC} \]
\[ Q_0^i = \omega_0/(2CR_s) \]
\[ Q_L = Q_0/(1+H) \]
\[ H = \xi \delta R_s \]  

Assuming a solution of the form
\[ V = \bar{V}(t) \exp(-i\int \omega(t) dt) \]  

with the same behavior for \( I_b \) and \( V_i \), and neglecting lower order terms like \( d\omega/dt \ll \omega_0 \omega/Q \) and \( d^2V/dt^2 \ll \omega dV/dt \), we obtain the following equation for the cavity voltage under beam loading and rf feedback:

\[ (2Q_0Z_L/R_S\omega_0)\frac{dV}{dt} + V = Z_L\xi V_i + Z_LI_b \]  

The bar has been omitted from the amplitudes of the voltage, the drive voltage and the beam current.

The complex impedance \( Z_L \) in Eq. (4) is defined by
\[ Z_L = R_S(1+H)/\{ 1 - iQ_L(\omega/\omega_0 - \omega_0/\omega) \} \]  

Eq. (4) is a complex equation which determines the cavity voltage and phase (with respect to the beam) for a given input complex voltage \( V_i \) and current \( I_b \).

In most of the cases the first term in Eq. (4) can be neglected and we obtain an algebraic equation. In the case of two cavities with a given total voltage and phase, the degrees of freedom exceeds the number of equations and there is no unique solution. In this case the problem can be simplified. We assume the following: a) The cavities are operated with the minimum safe voltage to avoid multipacting, equal detuning phase, and b) symmetric counter-phasing where the angles between the input generator voltages and the beam for the two cavities relates by \( 180^\circ \). The phasor diagram for the symmetric counter-phasing is shown in fig. (2).

\[ \text{figure 2. phasor diagram for symmetric counter-phasing} \]

Under these assumptions we end up with four equations for the four unknowns: the two input voltages \( V_{i1} \) and \( V_{i2} \), the detuning angle \( \psi \), and the counter-phase angle \( \theta \). These equations together with the voltage and current programs will be used in the next section to solve the various rf parameters. The Robinson stability criteria for this model is given by

\[ (1 + H)V_i \sin(\theta) / \{ R_S I_b \sin(2\psi) \} > 1 \]  

(see fig. (2) for the various angles).

**III. POWER AND PHASE REQUIREMENTS**

The voltage program expected for the LEB is shown in fig. (3) where we plot the voltage for two cavities out of assumed 8 operating ones. The rf current program is shown in fig. (4) where we consider operation in the test beam mode.

\[ \text{figure 3. the voltage program for two cavities} \]

\[ \text{figure 4. the current program} \]

The small synchrotron oscillations in the current 100 \( \mu \)sec. after injection have been neglected. Using the equations described in section II we draw in fig. (5) the required generator power for the accelerating cavity \( P_a \) and the decelerating cavity \( P_d \) for Pedersen feedback parameter \( H=6.3 \) [3]. It can be seen that the maximum power required from
the accelerating cavity generator does not exceed 20 kW and less than 5 kW is required from the decelerating cavity.

In comparison the required powers for the accelerating and decelerating cavities without rf feedback are 450 kW and 170 kW, respectively, beyond the power delivered by the tetrodes. Fig. (6) describes the required sweep in the counter-phase angle for this feedback level. A maximum rate of 0.3 degrees per micro-second is needed.

In fig. (7) we present the detuning angle vs. time. It can be seen that the rate of detuning is about 0.1 degrees per micro-second. These two sweep rates in the LEB cavities can be achieved without much difficulty. The Robinson stability criteria given in Eq. (6) for H=6.3 is presented in fig. (8). It can be seen that through all the adiabatic capturing phase the two cavity system is in a stable state.

IV. SUMMARY

We have examined the rf counter-phasing during the adiabatic capturing period in the LEB. Our results indicate that counter-phasing with moderate gain of rf feedback is necessary to operate within the available power limits.

V. REFERENCES