Calculations and Model Measurements for the Euterpe Cavity

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Abstract and Introduction

The 400 MeV storage ring Euterpe[1] is under construction. A quarter wave cavity (45 MHz, 50 kV) will accelerate the electrons. Due to space limitations the cavity length should not exceed 0.5 m. Therefore three special geometries are considered which have an 'electrical length' several times the physical length of the cavity. The first employs radial transmission line folding, the second longitudinal folding and the third capacitive loading of the transmission line. Transmission line theory is used to predict the cavity properties. Good agreement is found with SUPERFISH calculations. The capacitive loading option is superior considering its simplicity of construction and high shunt impedance. An LC equivalent circuit is used to model the impedance matching w.r.t. the rf generator. Results are in good agreement with measurements on a scale 1:1 cold model.

I. RADIAL FOLDING

The phase velocity of a voltage wave on a transmission line is $v = 1/\sqrt{LC}$, where L and C are the inductance and capacitance per unit length respectively. To lower the effective wavelength one may increase L or C. Fig. 1a shows a cavity derived from a coaxial line but with C increased substantially by the presence of disks connected to the inner and outer conductors. One cavity cell can be represented by a series inductance ΔL and a shunt capacitance ΔC. The inductance $\Delta L = L_1 + L_2 + L_3$ is simply the contribution of three sections of coaxial transmission line as indicated in Fig. 2a. The capacitance ΔC can be approximated by seven contributions. Two of these are usual coaxial line contributions. The third is the capacitance between two disks. The remaining four are due to the fringing fields at the corners of a disk. Formulas as collected by van Genderen et.al.[2] were used to approximate these fringing capacitances. The total cavity is modelled by a series circuit of N separate cells as depicted in Fig. 2b. There are N possible modes of which the ground mode is the desired accelerating mode. Its frequency is given by $f \approx 1/(4N\sqrt{\Delta L \Delta C})$. Losses are taken into account by the resistance $\Delta\Omega$ which is calculated analytically by integrating the wall material specific resistance over the surface, assuming a constant current in one cell. The shunt impedance and the quality factor can then be expressed as

$$R_{sh} = \frac{V_{gap}^2}{P_{dis}} = \frac{4}{N\Delta\Omega}\frac{\Delta L}{\Delta C}, \qquad Q = \frac{\pi}{2N\Delta\Omega}\sqrt{\frac{\Delta L}{\Delta C}}. \quad (1)$$

As an example we designed a 10 cell structure at 75 MHz, with an overall length of 0.25 m and an outer diameter of 0.25 m. The analytical predictions ($f = 76.4$ MHz, $R_{sh} = 61.2$ kΩ, $Q = 1810$) compare very well with SUPERFISH results ($f = 75.6$ MHz, $R_{sh} = 61.9$ kΩ, $Q = 1880$).

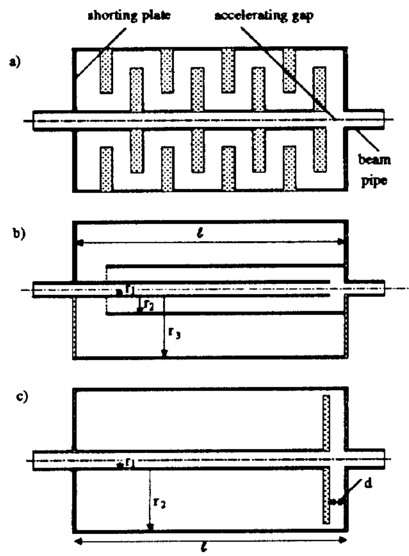

Figure 1: *Three types of $\lambda/4$ transmission line cavities a) employing radial folding, b) employing longitudinal folding, c) employing capacitive loading.*

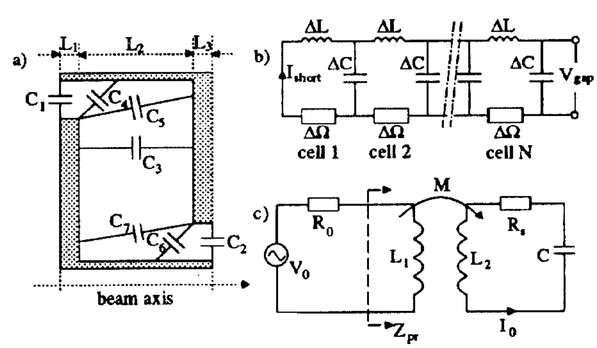

Figure 2: *a) The capacitances and inductances in one cell of a cavity employing radial folding b) equivalent circuit used to model a cavity with radial folding c) equivalent circuit used to model impedance matching.*

II. LONGITUDINAL FOLDING

A considerable improvement in shunt impedance is obtained with the cavity depicted in Fig. 1b. This cavity consists of two coaxial layers connected by a return section. Therefore its physical length will approximately be half the electrical length. Further shortening is possible by adding more coaxial layers. The voltages and currents in two different points on a transmission line are related

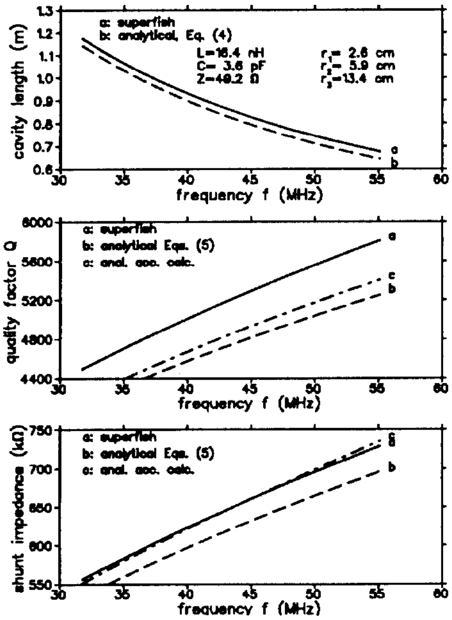

Figure 3: *Comparison between analytical and SUPERFISH results for a cavity employing longitudinal transmission line folding.*

by the matrix equation

$$\begin{pmatrix} V \\ I \end{pmatrix}_2 = \begin{pmatrix} \cos kx & -jZ\sin kx \\ -\frac{j}{Z}\sin kx & \cos kx \end{pmatrix} \begin{pmatrix} V \\ I \end{pmatrix}_1 , \quad (2)$$

where x is the distance between the points, Z is the line impedance and $k = \omega/c$ is the propagation constant. The return section can be modelled by a series inductance L and a shunt capacitance C which are determined similarly as in the previous section. The voltages and currents at the section input and output are related by

$$\begin{pmatrix} V \\ I \end{pmatrix}_{out} = \begin{pmatrix} 1 & -j\omega L \\ -j\omega C & 1-\omega^2 LC \end{pmatrix} \begin{pmatrix} V \\ I \end{pmatrix}_{in} . \quad (3)$$

With Eqs. (2,3) we can transfer the vector (V, I) from the shorting plate to the accelerating gap. The resonance condition is obtained by putting the right/under element of the overall transfer matrix equal to zero. As a special case we consider a cavity consisting of two layers with equal impedances $Z_1 = Z_2 = Z$. In the approximation that $\omega L/Z \ll 1$ and $\omega CZ \ll 1$ we obtain for the cavity length

$$l = \frac{\pi c}{4\omega}\left[1 - \frac{1}{\pi}\left(\frac{\omega L}{Z} + \omega CZ\right)\right] . \quad (4)$$

The shunt impedance R_{sh} and quality factor Q are calculated similarly as in the previous section. For simplicity we ignore the influence of the return section on the current profile in the second layer. Then we find

$$\begin{aligned} R_{sh} &= \frac{(\delta\mu_0/\pi\rho\epsilon_0)\ln^2(r_3/r_2)}{2\ln\frac{r_3}{r_2} + (\frac{1}{r_2}+\frac{1}{r_3})(\frac{1}{2}+\frac{1}{\pi})l + (\frac{1}{r_1}+\frac{1}{r_2})(\frac{1}{2}-\frac{1}{\pi})l} \\ Q &= \frac{kl}{2}\frac{R_{sh}}{Z} , \end{aligned} \quad (5)$$

where δ is the skindepth and ρ is the specific resistance of the wall material. As an example we compare in Fig. 3 the analytical results with SUPERFISH results. The cavity accelerating gap was 3 cm and the return section gap 5 cm. The capacitance C=3.6 pF was calculated with the numerical program RELAX3D. As can be seen, there is good agreement between both results. A further improvement of the analytical results is obtained if the influence of the return section on the current profile in the inner coaxial layer is taken into account (curves c).

III. CAPACITIVE LOADING

The construction of the previous cavity becomes rather complicated because more than two coaxial layers are needed to reduce its length to 0.5 m. A simpler construction is achieved with the cavity depicted in Fig. 1c. This can be seen as a coaxial transmission line terminated with a capacitance. The current and voltage profiles on the coaxial line are determined by the matrix equation Eq. (2). The capacitive loading is taken into account with a matrix as in Eq. (3) but with L put to zero. The resonance condition now becomes

$$\tan kl = \frac{1}{\omega CZ} , \quad (6)$$

where $Z = \sqrt{\mu_0/\epsilon_0}\ln(r_2/r_1)/2\pi$ is the line impedance, l is the length of the coaxial line, r_1 and r_2 are the inner and outer radii of the coaxial line and C is the loading capacitance. In good approximation this capacitance is $C = \pi\epsilon_0 r_2^2/d$, where d is the accelerating gap. For this case the analytical shunt impedance and Q-value are

$$\begin{aligned} R_{sh} &= \frac{4\pi\delta}{\rho}\frac{Z^2\sin^2 kl}{\ln\frac{r_2}{r_1} + \left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{l}{2}+\frac{\sin 2kl}{4k}\right)} , \\ Q &= \frac{2kl+\sin 2kl}{8\sin^2 kl}\frac{R_{sh}}{Z} . \end{aligned} \quad (7)$$

In Fig. 4 the analytical results are compared with SUPERFISH results for a cavity with r_1=2.5 cm, r_2=15 cm and d=1 cm. Once more there is quite good agreement. For the proposed cavity, a shunt impedance of 580 kΩ and a Q-value of 6600 can be achieved with a cavity length of 45 cm and at a frequency of 45 MHz. For a gap voltage of 50 kV, the required rf power would be 4.3 kW.

IV. IMPEDANCE MATCHING

Assuming inductive coupling, the complete rf system can be modelled with the equivalent circuit given in Fig. 2c. The rf source is represented as an ideal rf voltage supply V_0 in series with the characteristic impedance R_0 of the transmission line feeding power to the cavity. The inductive coupling is represented by the mutual inductance $M = k\sqrt{L_1 L}$ where k is the coupling constant and L_1 is the inductance of the coupling loop. The secundary circuit represents the cavity. Its angular resonance frequency and Q-value are given by $\omega_0 = 1/\sqrt{LC}$,

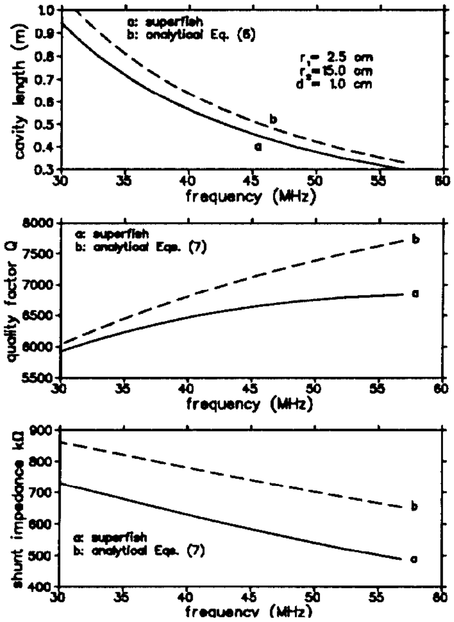

Figure 4: *Comparison between analytical and SUPERFISH results for a cavity employing capacitive loading of the transmission line.*

$Q = (1/R_s)\sqrt{L/C}$. Further, we choose the current I_0 to represent the shorting plate current. Then the shunt impedance is $R_{sh} = 2Z_t^2/R_s$, where $Z_t = V_{gap}/I_0$ is the 'transfer impedance' from the shorting plate to the accelerating gap. The impedance of the cavity as seen from the transmission line is

$$Z_{pr} = j\omega L_1 + \frac{\omega^2 M^2}{R_s(1+2jQ\delta)}\ , \tag{8}$$

with $\delta = (\omega - \omega_0)/\omega_0$. In the complex plane this impedance is a circle with radius $\omega^2 M^2/2R_s$. The voltage reflection coefficient Γ as measured on the transmission line feeding the rf power is given by $\Gamma = (Z_{pr} - R_0)/(Z_{pr} + R_0)$. The requirements for zero reflection can be written as

$$\delta = \frac{\omega L_1}{2QR_0}\ ,\quad M = \frac{\sqrt{R_0 R_s}}{\omega}\left(1 + (\frac{\omega L_1}{R_0})^2\right)^{1/2}\ . \tag{9}$$

The first condition can simply be satisfied if the cavity is detunable. The second condition can be satisfied for example if the loop is rotatable such that the induction M can be varied from zero to a maximum value. This maximum must be larger than the r.h.s. of the second of Eqs. (9). In order to verify the above theoretical predictions, a cold copper model was built of a 2-layer coaxial cavity employing longitudinal transmission line folding. The physical length of this cavity was l=86.3 cm and its inner, middle and outer radii r_1=2.5 cm, r_2=5.4 cm and r_3=12.2 cm. In table 1 we compare the analytical and SUPERFISH predictions of the resonance frequency f_0, quality factor Q and shunt impedance R_{sh} with the measured results. Two different loops were used to couple the rf signal into the cavity. Fig. 5 shows the primary impedance Z_{pr} as measured with a vector impedance meter (Hewlett Packard

method	f_0 (MHz)	Q (-)	R_{sh} (kΩ)
analytical	43.9	4625	556
superfish	44.0	4745	573
measured	43.8	4217	579

Table 1: Calculated and measured properties of a cold cavity model

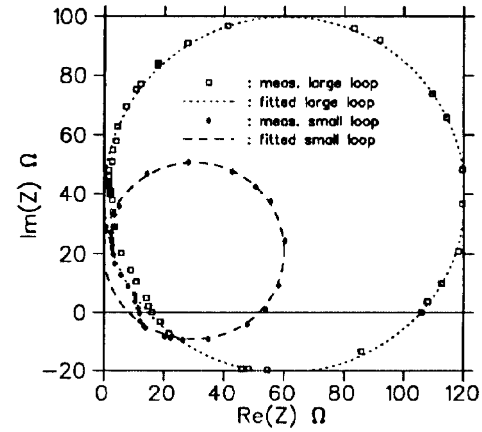

Figure 5: *Cold model cavity impedance as measured on the input transmission line.*

4815A). The smaller circle in Fig. 5 corresponds with the smallest of the two loops (area 11 cm² compared to 15 cm² for the larger loop), which also was made of a thicker wire in order to lower its inductance. For both loops, perfect matching to $R_0 = 50\ \Omega$ is possible but the larger loop needs a rotation over approximately 35 degrees.
The measured shunt impedance in table 1 was deduced from the radius ρ_c of one of the circles in Fig. 5. This radius is related to R_{sh} by

$$\rho_c = \frac{\omega^2 M^2}{4Z_t^2} R_{sh}\ . \tag{10}$$

with $Z_t \approx 48\ \Omega$ for the model cavity. If the area A of the loop is not too large then the mutual induction M can be approximated as

$$M = \frac{\mu_0 A}{2\pi r}\frac{I_c}{I_0}\ , \tag{11}$$

where r is the radial position of the loop and I_c is the circuit current at the coupling position. For the measurement in Fig. 5 the coupling loop was located close to the shorting plate so that $I_c \approx I_0$. From Eqs. (10) and (11) the shunt impedance is easily estimated.

V. REFERENCES

[1] J.I.M. Botman et.al., Nuclear Instruments and Methods **B49**(1990)89-93

[2] W. van Genderen et.al., Nuclear Instruments and Methods **A258**(1987)161-169