Fundamental Mode Detuned Travelling Wave Accelerating Structure

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Abstract

In this paper detuning method for the suppression of the long range wakefields in a linear accelerator structure has been investigated theoretically. A novel travelling wave accelerating structure with its accelerating field phase velocity oscillating around the velocity of light along the structure has been proposed in order to suppress the long range longitudinal accelerating mode wakefield. By detuning also the frequencies of the deflecting modes, the longitudinal and transverse long range wakefields can be suppressed at the same time, which is preferable when a train of bunches is to be accelerated.

I INTRODUCTION

In the design of the next generation TeV e^+e^- Linear Colliders (TLC) the wake field induced instability is one of the main concerns. In this paper we concentrate only on the suppression of the long range wakefields which will influence the following beam bunches. In section 2 aiming at suppressing at the same time longitudinal and transverse wakefields, a novel accelerating structure with the phase velocity of the accelerating field oscillating around the velocity of light along the structure has been proposed. In section 3 some discussions are made concerning practical structure designs. The numerical simulation results demonstrate the feasibility of this novel structure.

II THEORY

To start the discussion we consider an uniform structure (so-called constant impedance structure) and one of its passbands. This passband can be the fundamental or any other higher order one. The dispersion relation of this passband is approximated by

$$\omega^2 = \omega_0^2 (1 - k \cos \phi) \tag{1}$$

where ϕ is the phase shift between adjacent cavities at angular frequency ω , ω_0 is the midband angular frequency corresponding to $\phi = \pi/2$ and k is the coupling coefficient which can be calculated analytically [1] or numerically if the structure dimension is given. Eq. 1 can be simplified as

$$\omega = \omega_0 - \Delta\omega \cos\phi \tag{2}$$

where $\Delta \omega$ is half the angular frequency bandwidth of this passband.

When a particle with charge Q is going through this structure with v = c, the wake potential produced per

period in this passband is [2]

$$W = 2k_w e^{i(\omega_0 - \Delta\omega\cos\phi)\frac{t}{c}} / (1 - \frac{v_g^*}{c}\sin\phi)$$
(3)

where W takes the real or imaginary part of the complex number depending on longitudinal or transverse wake potential with k_w being the loss factor or kick factor correspondingly, and v_g^* is the group velocity at the midband. The physical meaning of eq. 3 is that the structure starts right at the beginning to oscillate at the synchronous frequency $\omega(\phi)$. Now we modulate the frequency ω_0 in eq. 3 over N cells without changing the passband width $\Delta \omega$. ω_0 is replaced by

$$\omega_0 + \delta\omega \cos(\frac{2\pi}{N}m) \tag{4}$$

where m is the mth cell with the same start point as N. The average of W over these N cells is

$$\langle W \rangle = \frac{1}{N} \sum_{m=1}^{N} \frac{2k_w e^{i(\omega_0 + \delta\omega \cos(2\pi m/N) - \Delta\omega \cos\phi)\frac{s}{c}}}{(1 - \frac{v_g^*}{c}\sin\phi)} \quad (5)$$

If N is large and ϕ keeps an almost constant value, eq. 5 can be replaced by an integral as follows:

$$=\frac{\int_{-\pi}^{\pi}2k_{w}e^{i(\omega_{0}+\delta\omega\cos\phi-\Delta\omega\cos\phi)\frac{s}{c}}d\Phi}{2\pi(1-\frac{v_{g}^{*}}{c}\sin\phi)}$$
$$=\frac{1}{(1-\frac{v_{g}^{*}}{c}\sin\phi)}2k_{w}e^{i(\omega_{0}-\Delta\omega\cos\phi)\frac{s}{c}}J_{0}(\delta\omega\frac{s}{c}) \qquad (6)$$

The amplitude of the wake potential is modulated by $J_0(\delta\omega_n s/c)$ and there exist a series of zero amplitude at

$$s_{ni} = u_{0i}c/\delta\omega \qquad (i = 1, 2, 3\cdots) \tag{7}$$

where s_{ni} are the locations where a test particle does not suffer wakefield of the *nth* mode, and u_{0i} is the *ith* root of the zero order Bessel function. Of course, more complicated modulation is possible as shown in ref. [3].

The discussions made above are quite general since the passband can be anyone of those of a travelling wave structure. To suppress the longitudinal wakefield of the fundamental mode, therefore, the frequency at which the wakefield is produced should be detuned along the accelerating structure as shown in Fig. 1.

We assume

$$\omega_w = \omega_{w0} + \delta\omega \sin(Kz) \tag{8}$$

where ω_w is the wakefield angular frequency, $K = 2\pi/L$ and L is the modulation period along the structure. Since



Figure 1: Detuning of the fundamental mode

 ω_w satisfies the dispersion equation and the synchronous condition, one has

$$\omega_w = \omega_0 - \Delta\omega \cos(\beta_w D) \tag{9}$$

$$\frac{\omega_w}{\beta_w} = c \tag{10}$$

where ω_0 is the midband angular frequency, $\Delta \omega$ is half the passband width (assumed to be a constant), and D is the period length of the periodic accelerating structure. We denote

$$\beta_w D = \theta \tag{11}$$

From eq. 8 we can rewrite the dispersion equation as

$$\omega = \omega_{w0} + \delta\omega \sin(Kz) + \Delta\omega\cos\theta - \Delta\omega\cos(\beta D) \quad (12)$$

If we chose our rf source angular frequency to be ω_{rf} which is within the passband, then

$$\omega_{rf} = \omega_{w0} + \delta\omega \sin(Kz) + \Delta\omega \cos\theta - \Delta\omega \cos(\beta_{rf}D) \quad (13)$$

 ω_{rf} is chosen as

$$\omega_{rf} = \omega_{w0} \tag{14}$$

and we define

$$\beta_{rf} = \beta_0 + \Delta\beta \text{ with } \theta_0 = \beta_0 D \tag{15}$$

where $\beta_0 = \omega_{rf}/c$. If $\delta \omega / \Delta \omega \leq \sin \theta_0$, $|\Delta \beta D| \ll 1$ and $v_g/c \ll 1$, it is found from eq. 13 that

$$\Delta\beta = -\frac{\delta\omega sin(Kz)}{\Delta\omega Dsin\theta_0} \tag{16}$$

The group velocity at rf source frequency is

$$v_g = \frac{d\omega_{rf}}{d\beta_{rf}} = \Delta\omega Dsin(\beta_{rf} D)$$
(17)

The average group velocity over the length L is found to be

$$< v_{g} >_{L} = \Delta \omega D sin \theta_{0} J_{0}(\frac{\delta \omega}{\Delta \omega sin \theta_{0}})$$
$$= R_{v} v_{g}|_{z=0}$$
(18)

where

$$R_{v} = J_{0}(\frac{\delta\omega}{\Delta\omega\sin\theta_{0}}) \tag{19}$$

is called group velocity reduction factor. The phase velocity of the accelerating field is

$$v_p = \frac{\omega_{rf}}{\beta_{rf}}$$

$$=\frac{c}{\left(1-\frac{\delta\omega\sin(Kz)}{\Delta\omega\beta_0 D\sin\theta_0}\right)}$$
(20)

The accelerating field travels with its phase velocity oscillating around the velocity of light. It is obvious that the particle is not always synchronized with the rf accelerating field, and therefore, it is important to look carefully to what will happen to the longitudinal and transverse motion of the particles.

The longitudinal accelerating electric field can be written as

$$E_z(r,z,t) = E_0 sin(\omega_{rf}t - \int_0^z \beta_{rf} dz + \phi_0) \qquad (21)$$

where E_0 is assumed to be a constant, and β_{rf} is the wave number at the angular frequency ω_{rf} in the structure. The average accelerating electric field over the length L is

$$< E >_{L} = \frac{1}{L} \int_{0}^{L} E_{0} sin(\omega_{rf} \frac{z}{c} - \int_{0}^{z} \beta_{rf} dz + \phi_{0}) dz$$
$$= E_{0} J_{0} (\frac{\delta \omega}{\Delta \omega DK sin\theta_{0}}) sin(\phi_{0} + \frac{\delta \omega}{\Delta \omega DK sin\theta_{0}})$$
$$= E_{0} Rsin(\phi_{0} + \frac{\delta \omega}{\Delta \omega DK sin\theta_{0}})$$
(22)

where

$$R = J_0(\frac{\delta\omega}{\Delta\omega DK \sin\theta_0}) \tag{23}$$

is called field reduction factor (abbreviated as reduction factor). The transverse momentum change over length L is

$$P = \frac{1}{c} \int_0^L F_r dz \tag{24}$$

It can be proved that when $\beta_z = 1$, P = 0 just like what happens in a conventional accelerating structure. According to the general conclusion expressed in eq. 6, one gets immediately the average fundamental mode longitudinal wake potential over length L as

$$_{L} = \frac{2k_{w}cos(\omega_{w0}\frac{s}{c})J_{0}(\delta\omega\frac{s}{c})}{(1-\frac{v_{s}}{c}sin\theta_{0})}$$
$$= \frac{2k_{w}cos(\omega_{rf}\frac{s}{c})J_{0}(\delta\omega\frac{s}{c})}{(1-\frac{v_{s}}{c}sin\theta_{0})}$$
(25)

We are now in a position to say that the price paid for the fundamental mode wakefield suppression is to lower the accelerating gradient by a factor R compared with the case of a conventional structure.

If the practically accepted minimum R is around 80%, from eq. 23 one has

$$\left|\frac{\delta\omega}{\Delta\omega DKsin\theta_0}\right| = 1 \tag{26}$$

In the following It is assumed that $D = D_0$, where

$$D_0 = \frac{c\theta_0}{\omega_{rf}} = \frac{Q_0}{\beta_0} \tag{27}$$

then

$$|\delta\omega| \le |\frac{2\pi D_0 \Delta\omega \sin\theta_0}{L}| \tag{28}$$

Once the half passband width $\Delta\omega$ (related to the designed average group velocity $\langle v_g \rangle$) and θ_0 are determined, the maximum frequency modulation amplitude $\delta\omega$ can be known from eq. 28. It is natural to assume that the distance between two adjacent bunches is

$$d = \frac{u_{01}c}{|\delta\omega|} \tag{29}$$

where the amplitude of the wake potential of the exciting particle is zero. If the rf pulse length is τ_{rf} the maximum number of acceptable bunches in this rf pulse is

$$N_b = \frac{|\delta\omega|\tau_{rf}}{u_{01}} \tag{30}$$

The larger the absolute value of $\delta \omega$ is, the greater the number of the bunches in a given rf pulse can be.

III DISCUSSION

In the following we will give a concrete example of the fundamental mode detuned structure shown in Fig. 2 (S-band). It is seen that there are two stop-bands at $\pi/4$



Figure 2: Fundamental mode detuned structure and its dispersion curves

mode and $3\pi/4$ mode, respectively. For a given r_0 the group velocity at $2\pi/3$ mode decreases with increasing cavity outer radius modulation dr. For the case of $r_0 = 1.5cm$ and dr = 0.05cm, the group velocity v_g/c at $2\pi/3$ mode is about 0.02. The standing wave electric field distribution of 18 cavities at $2\pi/3$ mode is shown in Fig. 3 calculated by using PRIAM [6]. It is obvious that there is



Figure 3: Standing wave pattern of 18 cavitiess

no attenuation. It is found that the shunt impedance of the synchronous travelling wave is about 30 M Ω /m with Q = 13596 ($r_0 = 1.5cm$ and dr = 0.05cm). Since this example uses electric coupling, the iris radius has to be very

large to avoid the attenuation of the travelling wave, and this large iris radius reduces the shunt impedance greatly. In practice, however, one can increase the shunt impedance by using the backward wave structure where the iris radius can be made smaller. The effect of the fundamental mode detuning is checked by using TBCI, and the result is shown in Fig. 4 where the length of the exciting bunch has been



Figure 4: Fundamental wakefield

chosen very long in order to excite only the fundamental mode. The dipole mode wake potentials (longitudinal, radial and azimuthal potentials in the same picture) of this structure are also calculated and shown in Fig. 5. In Fig. 4



one can see that the first minimum of the wake potential appears at about the 30th wavelength of fundamental accelerating mode after the exciting bunch. The results from PRIAM and TBCI demonstrate the feasibility of this novel accelerating structure.

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