Numerical Investigation of High-Current Ion Beam Acceleration and Charge Compensation in Two Accelerating Gaps of Induction Linac

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Abstract
The numerical investigation of the hollow high-current ion beam (HHCIB) dynamics in two magnet-isolated accelerating gaps of induction linac are presented. It has been shown that the applied electric field destroys partially the charge and current compensations, and impairs the brightness of the ion beam when the electron beam energy is not sufficient to overcome the potential difference. The acceleration, the charge and current compensations, and the stability of the high-brightness ion beam can be achieved under the optimum parameters choice.

I INTRODUCTION
Several approach to producing high-current ion beams by means of induction accelerators are now considered for controlled thermonuclear fusion research [1].

One of these methods involves the transport of several beams with source currents of ~ 1A in a vacuum with subsequent current enhancement by raising the energy through combining the beams and bunching in an accelerating pulse [2]. Another way of obtaining a large beam current at low kinetic energy makes use of the collective focusing techniques in which the space-charge forces are balanced by neutralizing the beam ions with electrons, while the electron current is suppressed by the magnet-isolated accelerating gaps. At present kiloampere ion beams are obtained from this type of linear high-current induction accelerator (linac) (see e.g. [3] and Refs. in that). A number of important physical problems discussed in [3] must be studied since the power and brightness requirements for ion beams in the controlled thermonuclear research are very stringent.

The previous study [5] has shown that without the accelerating field i) charge and current compensations of the ion beam by the specially injected electron beam occur; ii) the ion beam is stable for the time greater than the reciprocal Larmor and Langmuir ion frequencies. Here we present the results of our numerical investigation of the electron and ion beams dynamics in a two magnet-isolated accelerating gaps.

II EQUATIONS
The dynamics of a collisionless plasma in both the self-consistent and the external electromagnetic fields in axisymmetric ($\partial / \partial \theta = 0$) geometry, is described by the set of relativistic Vlasov’s equations for the distribution functions of a given type ($s$) of particles $f_s(p, \vec{R}, t)$. Here $\vec{p} = m_s \vec{v}, \vec{v} = \{v_x, v_y, v_z\}, \gamma = [1 - \left(\vec{p} / m_s c^2\right)^2]^{-1/2}, \vec{R} = \{r, z\}$.

The self-consistent electromagnetic fields in Vlasov’s equation are determined by Maxwell’s equations, which in the Lorenz gauge ($\partial / \partial \vec{R} + \frac{1}{c} \partial / \partial t = 0$) take the form of wave equations for the dimensionless scalar $\phi(r, z)$ and vector $A(r, z)$ potentials the right hand of which is defined as

$$\rho = \sum_s \int f_s(p) dp, \vec{J} = \sum_s \int v_s f_s(p) dp$$

We use the dimensionless quantities defined by $[v] = c; [r, z] = c/\omega_{pe}; [t] = \omega_{pe}^{-1}; [n] = n_{0e}; [q] = e; [m] = m_0; [\phi, A] = E_{ch}/e; [E, B] = (4\pi n_{0e} e c^2)^{1/2}; [J] = e n_{0e} c^2; [P_k] = [v] = c^2/\omega_{pe}$, where $\omega_{pe} = (4\pi n_{0e} e^2/m_0)^{1/2}$ is the electron plasma frequency, $E_{ch} = m_0 c^2$ is the rest energy of the electron, $n_{0e}, m_0, e$ are the initial density, rest mass and charge of the electrons respectively, $\gamma$ is the relativistic factor.

The equations of motion, obtained as characteristic equations of Vlasov’s equation have the form:

$$\frac{d{u_x}}{dt} = \frac{1}{\gamma m} \left(\psi \frac{\partial (\gamma A_x)}{\partial r} - \frac{\partial \phi}{\partial r} - u_x \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial r}\right)\right) - \frac{q}{m} \frac{\partial \phi}{\partial r} + \frac{1}{\gamma} \frac{\psi^2}{r^3}$$

$$\frac{d{u_z}}{dt} = \frac{1}{\gamma m} \left(\psi \frac{\partial (\gamma A_z)}{\partial z} - \frac{\partial \phi}{\partial z} + u_x \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial r}\right)\right) - \frac{q}{m} \frac{\partial \phi}{\partial z}$$

where $\vec{u} = \gamma \vec{v}$. $\psi = \gamma^2 \phi = P_B - \frac{q}{m} r A_x$. ($P_B$ is the dimensionless generalized particle momentum), $\gamma = \left[1 + u_x^2 + \left(\psi/r^2\right)^2 + u_z^2\right]^{1/2}$.

The boundary conditions for the potentials are:

$r = 0: \frac{\partial \phi}{\partial r} = 0, \frac{\partial A_x}{\partial r} = A_x, \frac{\partial A_y}{\partial r} = 0; r = r_L: \phi = \phi(z) A_z = A_x, A_y = A_y = 0$;

$$\phi(z) = \begin{cases} 0, & 0 \leq z \leq \Delta_z, \\ (n - 1)\Delta_z + \frac{2\pi}{c} (z - (2n - 1)\Delta_z), & (2n - 1)\Delta_z \leq z \leq 2n\Delta_z, \\ 2n\Delta_z, & 2n\Delta_z \leq z \leq (2n + 1)\Delta_z. \end{cases}$$

$$z = 0: \begin{cases} \frac{\partial A_x}{\partial z} = -\frac{1}{r} \frac{\partial (r A_x)}{\partial r}, \frac{\partial A_y}{\partial z} = \frac{\partial A_y}{\partial z} = 0, \\ \phi|_{z=0} = 0, \phi|_{z=\Delta_z} = \phi_L \end{cases}$$

where $\Delta_{\phi} = (\phi_L - \phi_0)/K$. $\Delta_z = z_L/(2K + 1)$ are the potential difference across the accelerating gap and the length.
Figure 1: Distributions of the total charge density $\rho(r, z)$ (a), scalar potential $\psi(r, z)$ (b), axial current density $j_z(r, z)$ (c), and the distribution functions $f(V)$ (d) of electron (1) and ion (2) beams versus the longitudinal ($V_z$) and transverse ($V_r$) velocities at $t = 280$, $t = 420$ and $t = 720$. 
of that, \( n = 1, \ldots, \mathcal{K} \). \( \mathcal{K} \) is the total number of cusps. The initial conditions for the self-consistent fields are \( \Delta \phi = A_z = A_r = A_\theta = 0 \) (here \( \Delta \) is Laplacian).

The boundary conditions for the distribution functions set the hollow beams injection at \( z = 0 \): \( f_s(\vec{r}, t) = f_s(m_0, \vec{u}, \vec{R}, t) = \delta(u_x) \delta(u_y) \delta(u_z) \) at \( r_{\text{min}} \leq r \leq r_{\text{max}} \) and \( p_z > 0 \), they are equal to zero at \( z = z_L \). Here \( r_{\text{min}} \) and \( r_{\text{max}} \) are the minimum and maximum beams radii respectively, \( u_x = V_x/(1 - V_x^2)^{1/2} \), \( V_x \) is a beams velocity. At \( (r = 0, z = 0) \) set the reflection regime: \( f_s(\vec{r}, \vec{R}, t) = f_s(-r, -r, z, \vec{R}, t) \), \( z \in [0, z_L] \). At the initial time, the distribution functions are equal to zero.

The external magnetic field is defined by the expression \( A_z = -\frac{B_0}{k} J_1(kr) \cos(kz) \) where \( J_1(kr) \) is the first order modified Bessel function, \( B_0 \) is the amplitude of magnetic field, and \( k = \frac{\pi}{L_z} \).

The method and algorithm of the solution of presented equations are described in [5]. The above model was carried out as a 2.5-dimensional cylindrical computer code [4, 5].

III RESULTS AND DISCUSSION

Let a hollow magnetized electron beam with velocity \( V_e \) and a hollow high-current unmagnetized ion beam with velocity \( V_i \) be injected along the z-axis into the external magnetic field. The beam current densities are equal to \( q_a q_0 V_e = q_a q_0 V_i \).

In the calculations we assumed the mass ratio to be \( m_i/m_e = 100, m_e = 20 m_0 \), the number of particles in the cell was \( N_e = 64, N_i = 180 \). The ion beam velocity was supposed \( V_i = 0.285 \). The minimum and maximum beams radii were \( r_{\text{min}} = 30 \) and \( r_{\text{max}} = 32.5 \). The length and radius of the chamber were \( z_L = 157.5 \) and \( r_L = 157.5 \). The amplitude of the external field was \( B_0 = 0.176 \). In all cases two cusps \( \mathcal{K} \) were considered. The number of points and the time step for solving Maxwell's equations were \((64 \times 64) \) and \( \Delta t = 0.025 \). The time step for solving of the equation of the motion was equal to \( \Delta t = 0.05 \).

The potential difference and the electron beam velocity were changed as follows:

<table>
<thead>
<tr>
<th>No. of case</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \phi )</td>
<td>0.8</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>( V_e )</td>
<td>0.85</td>
<td>0.85</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The results of the calculations are shown for case 1 in figure 1. Cases 2 and 3 are not displayed because of the limited scope of paper.

The distributions of \( \rho(r, z) \) (a), \( \phi(r, z) \) (b), \( j_z(r, z) \) (c), presented in figure 1 show that the applied external electric field, which accelerates ions and retards electrons, does not disrupt the electron beam drift through the accelerating gaps. From the functions \( j_z(r, z) \) (fig.1c) it is clearly seen that not only the charge (fig.1a,b) but also the current compensation of the ion beam occur. Figure 1d shows the distribution functions \( f(V) \) of the electron (1) and ion (2) beams versus the longitudinal \( (V_z) \) and transverse \( (V_r) \) velocities at \( t = 280, t = 420, \) and \( t = 720 \) respectively. It is seen that the ion beam generally retains monoenergetic shape, because its spread in \( V_z \) and \( V_r \) does not exceed 10%. The electron beam spread in the velocities is nearly 100%, but this does not prevent the charge compensation of the ion beam by electrons.

In variants 2 and 3 the electron beams energy \( E_{eb} \) was not sufficient to overcome the potential difference in the accelerating gaps. They have demonstrated that the electrons localize mainly in the drift region of the channel in the case 2. In third case the electrons have retarded predominantly by the electric field of the first accelerating gap. Only a slight part of electrons pass to the second gap following the ion beams therefore the ion beam is retarded and the substantial radial spread occurs as the space charge compensation of beam is not quite. The distribution functions has also showed the significant spread both the longitudinal and transverse velocities with the displacement of the distribution function maximum into the positive direction of the transverse velocity about \( \approx 0.1 \).

The above presented results of the computer simulation are correspond to the real model of a high-current linac [3]. The length of the accelerating gap is \( L \approx 5 \) cm, the radius of the chamber is \( R \approx 10 \) cm, the characteristic magnetic field value is \( B_0 \approx 7.5 \) kG, the Larmor radius of electrons is \( r_{L_e} \approx 0.045 \) cm \((r_{L_e} \ll L)\), the Larmor radius of ions is \( r_{L_i} \approx 20 \) cm \((r_{L_i} \gg L)\), the electron beam density \( n_0 = 8 \times 10^{13} \) cm\(^{-3}\). The maximum of the electric potential \( \phi \) (fig 1b) in the drift gap obtained in the computer simulation can be easily rectified by the cold electrons injection to that for the space charge compensation. In the real linac this is also no difficult as the external electric field is not in the drift gap which is sufficiently extended in comparison with the accelerating gap.

Thus the high-current beams can be accelerated in the linac with the substantial space-charge and current compensations without disturbing the stability in deciding on the optimal parameters.

IV REFERENCES


