Abstract

Considerable transverse focusing power can be obtained from the high-gradient RF accelerating structure of a linear collider by making either the aperture or the main cell geometry asymmetric. Such structures, acting as microwave quadrupoles, can be used to stabilize transverse wake fields by creating a spread in the transverse oscillation frequencies of the particles within a bunch. The focusing properties of both slotted-iris structures with circular cells and circular-aperture structures with asymmetric cells have been analysed using the MAFIA computer code and the results compared with the theoretically-determined limiting value obtained for an infinitely thin slit. A suitable geometry for CLIC 30 GHz structures has been established and a design based on the machine-and-braze technique is proposed.

INTRODUCTION

A relativistic beam passing off-centre through the circular aperture of an axially symmetric accelerating structure experiences no focusing at all because of exact cancellation of radial electric and azimuthal magnetic fields. The cancellation is easily removed, however, by giving the structure a suitably asymmetric shape. This creates a radio-frequency quadrupole. It turns out that the combination of high accelerating field and short wavelength of linear colliders makes it easy to obtain substantial quadrupole strength with simple (and technologically realistic) cavity shapes.

The resulting quadrupoles, arranged with alternating gradients at suitable period lengths, can be used to assist (or, in principle, even provide) the necessary transverse focusing and wake-damping [1]. Indeed, the rapid head-to-tail variation of transverse focusing within the bunch offers a powerful mechanism for either BNS damping [2] or autophasing [3]. This is qualitatively discussed below; extensive computational work on wake damping using microwave quadrupoles is reported elsewhere in this conference [4].

Other potential advantages over external magnetic quadrupoles are simultaneous acceleration and focusing (with concomitant saving of length) and the fact that precision-machined copper structures can guarantee the position and stability of the focusing axis with respect to an external reference surface with micrometre precision.

GEOMETRICAL CONFIGURATIONS FOR MICROWAVE FOCUSING

The conceptually simplest configuration is given by a narrow slit forming the beam aperture in a circular (pill-box) cavity. This is also a powerful configuration; it will be used as comparative reference for the more practical solutions described below.

As there can be no electric field parallel to a horizontal (say) slit, the horizontal force on an off-axis particle in the horizontal plane can only be due to the (azimuthal) magnetic field in the cavity. This is given by

\[ B = \frac{x}{2e^2} \sin \phi, \]

where \( x \) is the horizontal displacement and \( E_z \) the peak accelerating field. It follows at once that the effective magnetic focusing gradient \( G_0 \) is given by

\[ G_0 = \frac{\pi E_z}{c \lambda} \sin \phi \]

(in T/m), where \( \phi \) is the RF phase angle measured backwards from the top of the accelerating wave. The situation in this plane is exactly the same as in a plasma lens, the density of axial conduction current being replaced by the displacement current density.

Since an azimuthally closed integral of radial conduction current in the end plate must equal the displacement current terminating within the integration path - and therefore remain the same as in the case of a circular aperture - it can be argued that the vertical electric field at the slit centre is doubled. Therefore, the vertical focusing due to the radial electric field generated in the aperture overcompensates the magnetic gradient of eq. (2) by exactly a factor two. The same result can be predicted by invoking the theorem that a relativistic particle can only experience quadrupole focusing - equal and opposite in both planes - as long as it does not traverse conduction current density or space charge.

COMPUTER ANALYSES

Computational analyses [5] [6] of circular cavity cells with finite width slotted apertures show surprisingly little degradation of performance below eq. (2) even for the practical range of apertures required for acceptable wake fields (\( a_0 < 0.2 \) for CLIC). Results from [5] are summarised below. Following a suggestion by R.B. Palmer [7] RF focusing obtained by combining a circular aperture with a flat or oval cavity has been investigated. This solution has the decisive advantage that the circular aperture which needs careful rounding and a polished surface, can be machined on a lathe.

Basic structure parameters and transverse focusing gradients of various asymmetric cavity geometries at 30 GHz have been calculated using the 3D MAFIA [8] computer program.

Microwave Quadrupole Structures for the CERN Linear Collider

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Two types of asymmetry have been investigated:
(i) circular cavities with rectangular apertures
(ii) rectangular cavities with circular apertures.

For structures with rectangular apertures, three slot heights (3.0, 3.5 and 4.0mm) were analysed. Note that the slots extend over the full width of the iris.

For rectangular cavities with circular apertures four geometries have been investigated. The first three models had sharp cornered rectangular cross-sections with cavity heights of 6.0, 6.4 and 7.0mm. Since in practice however such geometries are difficult to machine a final calculation was made for a radiused rectangular section.

All the calculated results are for operation in the $2\pi/3$ mode and for clarity have been normalised as follows.
Structure parameters are given as a fraction of the normal CLIC accelerating section values \(R' = 114.8\ \text{MHz/m},\ \frac{Q}{\varepsilon} = 4329,\ \varepsilon = 0.074\), and transverse focusing as a fraction of $\pi/c\lambda$.

Circular cavities with rectangular apertures

The results are summarised in Fig.1. It can be seen that the analytic estimate \(\frac{G_0}{E_2} = \frac{\pi}{c\lambda}\) for an infinitely thin slit only over-estimates the focusing gradients of realistic geometries by 10-20%. With the CLIC nominal accelerating field of 80 MV/m maximum transverse gradients of about 70 T/m would be obtained.

For a given stored energy the maximum gradient that can be obtained \(G_0 \approx (G_0/E_2)(R'/Q)^{1/2}\). It is found however that although \(R'/Q^{1/2}\) decreases with decreasing cavity height, the linear increase of \(G_0/E_2\) produces a nett overall gain of \(G_0\) for flatter cavities.

![Fig.1 RF properties of slotted iris cavities](image)

**Rectangular cavities with circular apertures**

The main RF characteristics are summarised in Fig.2.

Comparison of results and practical solutions

Although it is seen from the above that a circular cavity with a 3.5mm slotted iris could produce transverse focusing gradients of 85% of $\pi/c\lambda$, with little or no deterioration in the RF characteristics compared to the normal accelerating sections, it is very difficult to imagine how such a geometry could be machined to have the required radius and surface finish at the aperture for high gradient operation.

The quasi-rectangular cavity with a circular aperture on the other hand is relatively easy to machine. Several precision machined prototype pieces are shown in Fig.3. The fabrication technique is identical to that used for the discs for the normal accelerating sections except that the main body of the cavity is milled.

Transverse focusing gradients of 85% of $\pi/c\lambda$ are obtained with a cavity half height of 3.08mm but at the cost of a 25% reduction in the $R'$ and $R'/Q$ values. Since only about 5% of the linac structure will have asymmetric apertures this reduction is considered acceptable.

![Fig.2 RF properties of rectangular cavities](image)
Wakefields induced near the head of a bunch and acting back on trailing parts of the same bunch tend to produce avalanching self deflections. This instability can be stabilized by creating a gradient of transverse focusing strength along the bunch in such a way that the tail is focused more than the head. The method is named BNS damping after the inventors [2]. A linearized stability criterion is given by

\[ \frac{eN}{\alpha} \frac{\partial}{\partial s} W_\perp \leq \frac{\partial}{\partial s} k^2 \]

(3)

where the longitudinal coordinate \( s \) is measured from the bunch centre, \( eN \) is the bunch charge, \( eU \) the particle energy and \( k \) the transverse wave number.

In a thin-lens FODO system made of superimposed (static) external and microwave quadrupoles of focusing gradients \( G_e \) and \( G_{rf} \), filling fractions \( \eta_e \) and \( \eta_{rf} \) of total linac length, respectively, the wave number \( k \) is given by

\[ 8U \sin \frac{kl}{2} = \alpha \sin^2 \left( \eta_e G_e + \eta_{rf} G_{rf} \right) \]

(4)

where the period length \( L \) is common to both systems. The microwave transverse gradient \( G_{rf} \) may be taken as \( G_0 \) from eq. (2) times a form factor \( \alpha \) (near unity) representing the actual cavity geometry as discussed above. Inserting this into eq. (4), differentiating with respect to \( s \) and noting that \( \partial \phi_0/\partial s = 2\pi/\lambda \), one finds

\[ U \frac{\partial}{\partial s} k^2 = \eta_{rf} \frac{E_z}{\lambda^2} \pi^2 \cos \phi_0 \frac{\mu_0}{\cos \mu_0^2} \]

(5)

where \( \phi_0 \) and \( \mu_0 \) are the RF phase and the transverse phase advance per period \( (k_0L) \) respectively, taken at the bunch centre. This can be readily inserted into the stability criterion [3]. The particle energy cancels out. Inserting typical values (CERN Linear Collider values for example) of \( N = 6 \times 10^9 \), \( \partial W_\perp/\partial s = 1.1 \times 10^{21} \text{V} \text{A}^2 \text{m}^3 \), \( \alpha = 0.85 \), \( E_z = 80 \text{MV/m} \), \( \lambda = 1 \text{cm} \), \( \phi_0 = 0 \) (at the top of the accelerating wave and \( \mu_0 = 90^\circ \) gives \( \eta_{rf} = 0.07 \) for stability - a very satisfactory result considering that the microwave quadrupole sections contribute to acceleration.

Compared with the customary method of satisfying the stability criterion by means of (a large) energy spread within the bunch this method has the advantage of not requiring special manipulations towards the end of the linac in order to reduce the energy spread to the requirement of the final focus. Also, a very large spread would be required to stabilize the CLIC wake fields. The strength of the (static) external focusing system does not explicitly appear in eq. (5). Such a system is likely to be required, however, for gaining flexibility of adjustment and in order to limit the spread of transverse phase advance \( \mu \) within the bunch, given by

\[ \frac{\sin \frac{\mu}{2}}{m + \sin \phi} = \frac{\sin \frac{\mu_0}{2}}{m + \sin \phi_0} \]

(6)

with \( m = \eta_e G_e/\eta_{rf} G_{rf} \).

In practice the situation is complicated by the wake fields' nonlinearity (but also the possibility of nonlinear wake field damping called "autophasing" [3]), by tolerances or misalignments and jitter of quadrupoles and accelerating sections (made severe by the rapid incoherence of oscillations associated with a large spread), and by the small energy spread required by the final focus system. A detailed treatment of these and associated problems with concomitant simulations are presented elsewhere at this conference [4].

REFERENCES