

## Beam Dynamics and RF Evolution in a Multistage Klystron-Like Free-Electron Laser

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### Abstract

Current understandings of beam dynamics and RF evolution in a klystron-like free-electron laser are present. Phase sensitiveness to injection jitters estimated by existing two theories is discussed. BBU suppression due to linear detuning is proposed as an alternative of ever proposed techniques.

### Introduction

A linear multistage klystron-like free-electron laser<sup>1</sup> (MKFEL) has been considered as a possible power source for future TeV-class linear colliders.<sup>2</sup> Recently a gigantic microwave power station in space<sup>3</sup> has also been proposed which employs a circular MKFEL as its pumping heart. Essential issues of those microwave power sources are efficiency and microwave beam's quality such as the uniformity in power and well defined phase.

The efficiency simply depends on the staging number of FELs because its local efficiency is expected to be almost 100%. Therefore, it is primarily important how far a driving electron beam in the MKFEL can be propagated with tolerable loss of beam quality which directly relates to the output microwave beam's quality. Main obstacle for long distant transport is beam breakup(BBU) instabilities<sup>4</sup> including resistive wall instability. Meanwhile, since the first proposal<sup>5</sup> of this kind of microwave sources, serious sensitiveness of phase evolution to injection errors in beam current or energy has been our concern. To clarify those intrinsic natures of a MKFEL, extensive analyses have been done at KEK,LBL,LLNL, and MIT. The studies show how serious they are and what cure can be taken to realize a real machine.

We will briefly summarize the results in the succeeding sections and present our current understanding on the MKFEL. Particularly, we will focus ourselves on theoretical models capable of predicting phase drift caused by injection errors and BBU suppression due to linear detuning of BBU mode.

### RF Phase Drift

Pulse-to-pulse phase drift caused by injection jitters in beam current or energy has been independently considered by Sessler,Whittum,and Wurtele<sup>6</sup> and Takayama<sup>7</sup> in a conceptually similar framework which may be called a macroparticle approach<sup>8</sup>. Introducing three single-period functions determined by numerically integrating the single particle FEL equations of motion and linearizing them with respect to displacement from the designed values, the

former has evaluated a set of linear evolution equations for phase and given a condition for less sensitiveness. One example of single particle FEL simulations which supports the theory has been presented in Ref.6. Using the concept of universal gain function introduced in Ref.8, Takayama has developed a full set of evolution equations for energy, ponderomotive phase, and output RF phase in the recursion form with a mathematically much simpler structure. The theory has been confirmed by comparison with so-called multiparticle simulations.

It is noted that both theories consider this problem on the different specification of FEL. The former assumes a wiggler length of 1.3m in the example which can yield over-rotation of the macroparticle in the phase space(*over-rotation regime*). A substantial fraction of microwave power returns to kinetic beam energy in this regime. A wiggler of length 0.7m in the latter theory pulls out an almost maximum power from beam energy(*maximal power extraction regime*). Takayama has concluded that the sensitiveness of RF phase to current error in the *maximal power extraction regime* for which the maximum amplification per length is obtainable is almost uniquely determined by an amount of input power level. The theory and multiparticle simulations have demonstrated that a size of sensitiveness is acceptable, provided an input power of MW class. If a phase controlled input power source of MW class is not available, we might sacrifice the amplification efficiency per length to resort the *over rotation regime* where the less sensitiveness to the jitter is expected even for a lower power input, according to the former theory.

### BBU Suppression

BBU in the MKFEL has received particular interests because of specific characteristics of the machine itself, that is, a microwave generator where energy of driving beams which is converted to the microwave power is replenished with periodically located induction units. Those characteristics are among (1) large energy spread in a beam slice, (2) sawtooth-like change in energy with propagation, and (3) betatron frequency which is almost uniquely fixed with chosen FEL parameters, close to synchrotron frequency in order of magnitude. Theoretical and simulation studies<sup>9,10</sup> has proved that this large energy spread is not able to yield Landau damping because of rapid oscillation in the energy space as suggested by (3). Characteristics (2) has also been demonstrated no to contribute to BBU suppression, in Ref.10.

Several methods for BBU control have been proposed; (a) Landau damping which results from phase mixing introduced by an effective spread in betatron freq. associated with nonlinear ion channel guiding<sup>11</sup>, (b) BNS damping<sup>12</sup> which results from energy sweep along a beam<sup>10</sup>. Both methods seem to be quite effective for BBU suppression including the resistive wall instability. However their actual feasibility is unknown because detrimental features on the desired FEL performance may be induced with these techniques. For instance, in case (a) we are concerned about a gradual shift in the FEL resonance condition due to beam emittance degradation in nonlinear ion channel guiding. In case (b) the required energy sweep along the pulse  $\delta\gamma$  may be provided by initial sweep and energy replenishment sloped in time. The cumulative phase difference between the pulse head and tail in the *maximal power extraction regime* is roughly estimated by  $\Delta\varphi_s \sim N \cdot \delta\gamma$  where  $N$  is the stage number. For a typical example of  $N = 100$  and  $\delta\gamma/\gamma_0 = 2\%$ ,  $\Delta\varphi_s = 115^\circ$ . This is far beyond an acceptable size in microwave sources for a linear collider unless any compensation were made, because the RF filling time of accelerating cavities is comparable to the pulse length or much longer.

As an alternative<sup>13</sup>, we can consider artificial detuning in the BBU mode frequency for BBU suppression. For simplicity, we consider a continuum model of a beam with current  $I_B$ , relativistic mass factor  $\gamma$  in a focusing system of betatron frequency  $k_\beta$  inside a series of accelerating units of separation  $L$  and gap width  $d$  with the linear detuned BBU mode frequency  $\omega_\lambda(z) = \omega_0(1 + \alpha z)$  where  $\omega_0$  is the BBU mode frequency of the first gap and  $\alpha$  is designated as a detuning parameter. Let  $\xi(\tau, z)$  be the transverse centroid displacement of the beam slice placed at position  $\tau$  in time behind the beam head, at position  $z$  from the injector. The BBU mode is also characterized by quality factor  $Q$  and transverse impedance  $Z_\perp$ . The governing equation for  $\xi(\tau, z)$  is

$$\left(\frac{\partial^2}{\partial z^2} + k_\beta^2\right)\xi = \int_0^\tau W(\tau - \tau', z)\xi(\tau', z)d\tau'\Theta(z) \quad (1)$$

where  $\Theta(z)$  is the step function and  $W$  is the sinusoidal wake function,

$$W(\tau - \tau', z) = \frac{I_B \omega_\lambda^3 Z_\perp \sin(\omega_\lambda(z)(\tau - \tau'))}{I_0 Q d \gamma \omega_\lambda(z)} \quad (2)$$

( $I_0$  : Alfvén Current, 17kA)

Eq.(1) is solved by means of the Laplace transform. Using the WKB approximation and the saddle point calculation, for a long pulse we can obtain an asymptotic solution. This asymptotic solution tells us the BBU continues to grow until a critical distance  $z = z_c$  and beyond that it saturates. Those properties have been confirmed with numerical BBU

simulations, as shown Fig.1. Explicit formulae for  $z_c$  and the saturated BBU amplitude  $|\xi|_{sat}$  are given by

$$z_c = A/(\alpha^2 \tau k_\beta) \quad (3)$$

$$|\xi|_{sat} = \xi_1 \frac{\Gamma(1/3)6^{1/3}}{6\pi} \left[\frac{(\alpha z_c)^2}{\omega_0 \tau}\right]^{1/3} \exp\left[\frac{\pi(\omega_0 \tau)(\alpha z_c)}{4}\right] \quad (4)$$

where  $A = \frac{I_B Z_\perp}{I_0 Q L \gamma}$  and  $\xi_1$  is the off-axial displacement of a cursor head  $0 \leq \tau' \leq \tau_1 \ll \tau$  to create the wake in accelerating gaps. In Figs.2a,b, assuming  $\xi_1 = 5 \times 10^{-3}m$  and  $\tau = 25nsec$ ,  $z_c$  and  $|\xi|_{sat}$  for a typical MKFEL shown in Table are shown as a function of the detuning parameter  $\alpha$  and compared with results of numerical BBU simulations. Both are in good agreement.

The frequency of typical BBU deflection modes such as  $TM_{110,120,130}$  is inversely proportional to the cavity radius of accelerating gaps. Detuning of the BBU mode can be made by controlling this size at the stage of assembling. BBU suppression less than 1mm for  $N = 200$  and a possible initial displacement of several mm can be achieved with detuning  $\alpha \geq 3 \times 10^{-4}m^{-1}$ . This requires about 10% modification in the cavity radius. The size of modification seems to be feasible.

### Conclusion

Current understandings of beam dynamics and RF evolution in the MKFEL have been presented. We conclude that most of principle issues in MKFEL physics have been analyzed and serious obstacles to realizing a machine will be overcome with appropriate cures. However, an interaction between a bunched beam and mode converters, RF extraction septums will induce further detrimental aspects in beam dynamics. Its assessment and cure are left unsolved. Those tasks will be done in a compromise with engineering design of items.

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**Table**

Beam current (kA)	$I_B$	2.0
Gap spacing (m)	$L$	1.5
Betatron wave length (m)	$2\pi/k_\beta$	2.5
BBU mode frequency (MHz)	$\omega_0/2\pi$	800
Quality factor	$Q$	20
Transverse impedance ( $m^{-1}$ )	$\omega_0 Z_\perp$	20.0
Accelerating gap width (cm)	$d$	3.0
Stage number	$N$	200

Fig.2a  $z_c$  vs.  $\alpha$ .

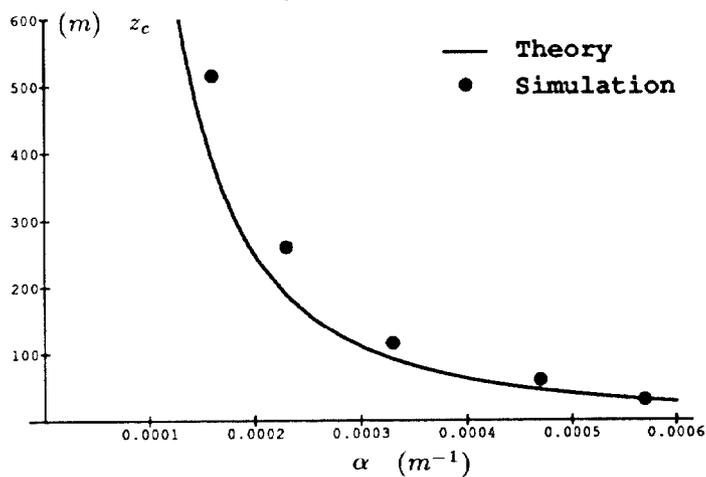


Fig.2b Saturated BBU amplitude  $|\xi|_{sat}$  vs.  $\alpha$ .

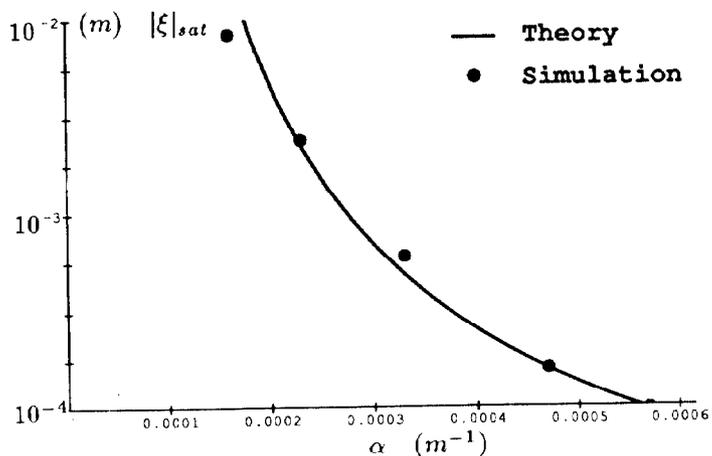


Fig.1 BBU growth for different  $\alpha$  s

