# Investigations on Beam Damping Simulations and the Associated Model of CLIC 

G. Guignard, C. Fischer and A. Millich. CERN, 1211 Geneva 23, Switzerland

## Abstract

Controlling the beam stability in the CLIC main linac must be investigated numerically. Strong damping is required to cope with wake field effects and machine imperfections impose careful optics adjustments. The computation of the longitudinal and transverse wake fields has been revisited and the Green functions re-evaluated for the most recent main cavity design. BNS damping and autophasing with magnetic and microwave focusing could then be simulated and the dependence of the results on parameters such as the RF phases and voltage gradients could be studied. Since the emittance is extremely small, beam break-up is sensitive to unavoidable misalignments and algorithms of trajectory corrections have to be investigated. Simulations and beam trackings have been done under specific conditions, using the tools briefly described in this paper. Most relevant examples are presented.

## I DELTA FUNCTIONS OF THE WAKE POTENTIALS

Computation of the wake field Green's functions for the disk-loaded waveguide (DLWG) of CLIC main linac is based on Ref.[1]. The delta functions of the longitudinal and transverse potentials are expressed in terms of the structure's normal modes:

$$
\begin{align*}
& W_{L}^{\delta}(\tau)=2 \sum_{n=1}^{\infty} K_{0 n} \cos \omega_{0 n} \tau \\
& W_{T}^{\delta}(\tau)=2 \sum_{n=1}^{\infty} \frac{K_{1 n} c}{\omega_{1 n} a^{2}} \sin \omega_{1 n} \tau \tag{1}
\end{align*}
$$

where

$$
\mathrm{K}_{\mathrm{n}}=\frac{\omega_{\mathrm{n}}}{4}\left(\frac{\mathrm{R}}{\mathrm{Q}}\right)_{\mathrm{n}} \quad \text { and } \quad \tau=\frac{z-\mathrm{s}}{\mathrm{c}}
$$

Equ. (1) assumes small trajectory amplitudes, measured with respect to the axis of the structure as is the case in CLIC.

Using the code KN7C [2], frequencies $\omega_{\mathrm{on}} / 2 \pi$ and loss factors $\mathrm{K}_{\text {on }}$ of the longitudinal modes synchronous with the ultrarelativistic particles have been computed for CLIC. Since this code requires a simplified input geometry with straight edges DLWG geometry is completely defined by four parameters,
the iris aperture radius the cavity inner radius the cavity gap width the cell length, i.e. DLWG period

$$
a=2.000 \mathrm{~mm}
$$

$$
\mathrm{b}=4.352 \mathrm{~mm}
$$

$$
\mathrm{g}=2.782 \mathrm{~mm}
$$

$$
\mathrm{p}=3.332 \mathrm{~mm}
$$

A total of 204 longitudinal modes have thus been calculated, this number being high enough for a good evaluation of the delta function, except for the very short range part. The optical model (Ref. [1] and refs therein) was included
to evaluate the analytic extension giving a more correct description of the short range wake.

Transverse modes were computed with the code TRANSVRS [3]. The delta function was then evaluated according to (1), using the loss factors and frequencies of 218 modes. Figure 1 shows the longitudinal and transverse delta functions of the wake potentials (scaled with a and $p$ when required) for the edge geometry of the CLIC structure.


Figure 1. Green's functions of the wake fields

## II BEAM STABILITY AND ONE-TO-ONE CORRECTION

Linac and beam model was revisited and the re-evaluated delta functions of the wake potentials used for the simulations. An AG focusing system with FODO lattice is assumed, with a phase advance of $90^{\circ}$ to minimize misalignment sensitivity and $\sqrt{\gamma}$ scaling to maintain a constant stability margin (initial cell length is 5 m ). Microwave quadrupoles can be added and the linac is divided into four sectors, in which the energy is multiplied by 4 going from 5 GeV to 1 TeV . BNS damping [4] and autophasing are optimized by adjustments in each sector of the RF gradient phase $\phi_{\mathrm{RF}}, \mathrm{RF}$ quadrupole phase $\phi_{R F Q}$ and strength. The bunch is longitudinally divided into slices, the dimensions of which are given by the emittances (unequal) and the focusing system. They are populated according to a gaussian and the total charge N is $6 \cdot 10^{9}$. This value, together with a bunch length $\sigma_{\mathrm{z}}=0.17 \mathrm{~mm}$ and $\phi_{\mathrm{RF}}=7-8^{\circ}$, corresponds to an optimum found in studying the compensation of the energy spread $\sigma_{E}$ [5]. Injection off-sets of $4.2 \mu \mathrm{~m}$ and $1 \mu \mathrm{~m}(\mathrm{H}$ and V ) and random misalignments of quadrupoles and cavity-sections ( $1 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$ r.m.s. in the applications) are included.

A one-to-one trajectory correction was implemented in the tracking code MTRACK [6]. Its characteristics are;

1) simulation of average trajectory measurements $\langle x\rangle_{i+1}$, 2) correction of these deviations at ( $i+1$ ) in moving the quadrupole i by $d x ; 3$ ) introduction of random measurement and displacement errors (r.m.s. $\xi_{m}$ and $\xi_{\mathrm{d}}$ ), and of quadrupole misalignments $\delta x_{i+1}$. Using the coefficient $m_{12}$ of the transfer matrix from ito $i+1$ (wake field kicks included) and the focal distance $f$, the displacement requisite to centre the bcam at ( $\mathrm{i}+1$ ) in presence of errors is,

$$
\begin{equation*}
\mathrm{d} x_{\mathrm{i}}=\frac{\langle\mathrm{x}\rangle_{\mathrm{i}+1}+\xi_{\mathrm{m}, \mathrm{i}+1}-\delta \mathrm{x}_{\mathrm{i}+1}}{\mathrm{~m}_{12} / \mathrm{f}}+\xi_{\mathrm{d}, \mathrm{i}} \tag{2}
\end{equation*}
$$

Tracking results presented concern three cases: 1) a structure with conventional focusing and adjustment of $\phi_{R F}$ (optimum per sector at $-38^{\circ},-16^{\circ}-24^{\circ}$ and $-21.5^{\circ}$ ), 2) a structure with microwave and conventional quadrupoles (optimum at $\phi_{\mathrm{RFQ}}=0$ and $\phi_{\mathrm{RF}}=5^{\circ}, 10^{\circ}, 15^{\circ}$ and $23^{\circ}$ ), 3) a structure as in 2) but in one sector with $\phi_{R F}=7^{\circ}$ and $\phi_{\mathrm{RFQ}}=7^{\circ}$ to minimize $\sigma_{\mathrm{E}}$. The cases differ by the way BNS damping and autophasing are achieved and the energy spread ( $5.6,2.7$ and $0.6 \%$ r.m.s. respectively). Blow-up is given in Figs [2] to [4] and case 3 is at the origin of the revised parameters [5]. Since the desired limit of $25 \%$ is not reached, an achromatic correction was investigated.


Figure 2. Blow-up with magnetic focusing only


Figure 3. Blow-up with additional RFQs


Figure 4. Blow-up for minimum $\sigma_{\mathrm{E}}$.

## III ACHROMATIC CORRECTION METHOD

Particles with an energy excursion $\delta=\Delta \mathrm{p} / \mathrm{p}_{\mathrm{o}}$ have a different trajectory and will contribute to bunch dilution. Such dispersive effects can be corrected as well as the trajectory at nominal momentum $p_{0}$. One method proposed [7] leads to a minimization of the expression:

$$
\begin{equation*}
\phi=\sum_{j=1}^{N} \frac{(x j+X j)^{2}}{\sigma_{\xi}^{2}+\sigma_{b}^{2}}+\frac{(\Delta x j+\Delta X j)^{2}}{2 \sigma_{\xi}^{2}} \tag{3}
\end{equation*}
$$

xj and Xj are the measured and calculated deflections at j and for $p_{0}, \sigma_{\xi}$ and $\sigma_{b}$ are the pick-up r.m.s. precision and alignment errors. The first term is the contribution of the nominal trajectory whereas the second one describes the effect of dispersion. The quantities occurring are:

$$
\begin{gather*}
X j=\sum_{i<j} R_{12}(i, j, o) \theta_{i} \\
\Delta x j=x j(\delta)-x j  \tag{4}\\
\Delta X j=X j(\delta)-X j=\sum_{i<j} R_{12}(i, j, \delta) \frac{\theta_{i}}{1+\delta}-X j \tag{5}
\end{gather*}
$$

$\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \delta)$ is the transfer matrix element which transforms a deflection $\theta_{i}$ at $i$ into an excursion at $j$ for given $\delta$; all kicks $i$ preceding $j$ are considered. In Eqs. (4) and (5), $\Delta x_{j}$ and $\left[R_{12}(\mathrm{i}, \mathrm{j}, \delta)-\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \mathrm{o})\right]$ can be developed in $\delta$ with coefficients $a_{n}$ and $c_{n}$ respectively; including the development of $1 /(1+\delta)$ gives [8]:

$$
\begin{equation*}
\Delta x j=\sum_{n} a_{n}^{j} \delta^{n} \tag{6}
\end{equation*}
$$

$\Delta X j=\sum_{i<j} \theta_{i} \sum_{n}\left[\left(\sum_{m=1}^{n}(-1)^{n+m} c_{m}^{i j}\right)+(-1)^{n} R_{12}(i, j, o)\right] \delta^{n}$
To second order in $\delta$ the coefficients $a_{1}$ and $a_{2}$ are determined by measuring $\Delta \mathrm{x}_{\mathrm{j}}$ for two values of $\delta$ and $\mathrm{c}_{1}$ and $\mathrm{c}_{2}$ are extracted from the processing of $\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \delta)$ at these two values and at $\delta=0$.


Figure 5. Variation of $\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \delta)$ with $\delta$


Figure 6. Correction of trajectory


Figure 7. Correction of dispersive effect

## Application to CLJC:

The exercise was tried on the CLIC structure (section II) adapting program MTRACK [6] to process trajectorics and the $\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \mathrm{\delta})$; results presented there after concern the vertical plane: only QDs are considered for kick and pick-up locations.

As ( $\mathrm{j}-\mathrm{i}$ ) increases, nonlinearities appear quickly on the $\mathrm{R}_{12}(\mathrm{i}, \mathrm{j}, \mathrm{\delta})$ - Fig. 5. When ( $\mathrm{j}-\mathrm{i}$ ) $=\pi / 2$ (modulo $\pi$ ) (unbroken lines) a linear and then parabolic approximation fits for the first $2 * 8$ pairs ( $\mathrm{i}-\mathrm{j}$ ) (another set not represented being symmetrical around the x axis) as long as $\delta$ is limited to within $\pm 3.5 \%$; when $(\mathrm{j}-\mathrm{i})=\pi$ (modulo $\pi$ ) (dashed curves) the response is quasi linear up to $2^{*} 6$ pairs in the same $\delta$ domain. If ( j -i) or $\delta$ increase, distortions from the parabolic shape affect both families: hence for CLIC, the second order development holds well on the 26 cells ( 52 quadrupoles) following a kick: this has been successfully tested with a minimization algorithm based on Eqs. (3)-(7). Results in correcting trajectory, dispersion and blow-up are presented in Figs. 6, 7 and 8 , for a 800 m long sector. Trajectory and dispersion are reduced by about an order of magnitude and blow-up kept below $10 \%$, in 2 iterations.

## Conclusions:

The CLIC main linac ( 730 quadrupoles) could then be corrected by about 10 applications of such a second order algorithm on adjacent 55 quadrupole sectors. Investigations are continuing to cover larger regions by adding higher order terms.


Figure 8. Achieved beam blow-up
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