

# Design Considerations for a Fast Modulator in a 'Crossed Undulator'

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## Abstract

The modulator, a short electromagnetic wiggler, can be used to generate a phase shift between the synchrotron radiation of the two undulators in a crossed undulator. The switching frequency between two polarization states can be as high as 10 Hz. This paper discusses some physical requirements for the modulator and a conceptual design of the magnet for a crossed undulator at the Aladdin storage ring (Synchrotron Radiation Center, University of Wisconsin, Madison) as a prototype development for the Advanced Light Source (ALS) and the Advanced Photon Source (APS).\*

## I. INTRODUCTION

Third generation synchrotron light sources, like the ALS at Berkeley and the APS in Argonne, currently under construction, can be powerful sources of X-ray radiation with adjustable polarization state, especially by utilizing the small design emittance of these machines. A device to produce radiation with any desired polarization state is the crossed undulator [1,2] in which the electron beam travels through: first, an undulator; second, a phase shifter; and, third, another undulator. The on-axis radiation from a planar undulator by a zero emittance electron beam observed through an infinitely small pinhole is linearly polarized. The direction of the magnetic field in the first undulator is perpendicular to the field orientation in the second (a schematic drawing of the device is found in [3]). Therefore, the polarization vectors of the radiation from the two undulators are orthogonal. The radiation from the two undulators is mixed in a monochromator to produce the desired polarized radiation. The polarization state can be selected by changing the phase of the radiation from the upstream undulator with respect to the downstream undulator. One way of adjusting the phase is to introduce a drift space of variable length between the two undulators. However, this procedure results in a rather slow (some ten seconds) change of the polarization. Another difficulty is that the drift space between the two undulators enhances the depolarization due to the finite emittance of the electron beam [2]. A more elegant approach without physi-

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cally moving a magnet is to utilize an electromagnetic wiggler as a phase shifter [2]. The magnetic field within the wiggler forces the electrons on a path that is longer than that for the electromagnetic radiation. This phase shift is, in a saturation-free magnet, determined by the current through the wiggler coils. The switching frequency is limited by the eddy current effects in both the wiggler laminations and the walls of the vacuum chamber. Recently, a polarizer of this kind utilizing two hybrid undulators and a dc modulator was installed on the BESSY-1 storage ring [4].

## II. THE MODULATOR

### A. General Description

The modulator is a five pole wiggler with a fixed gap. The magnet is constructed from thin stamped steel laminations that are electrically insulated for each other to suppress eddy current effects in the steel. Figure 1 shows a longitudinal section through the modulator. The center pole and the two side poles can be energized with water-cooled coils. The end poles have no coil and serve as field clamps to reduce the stray field of the modulator and the sextupole coefficient of the field integral.

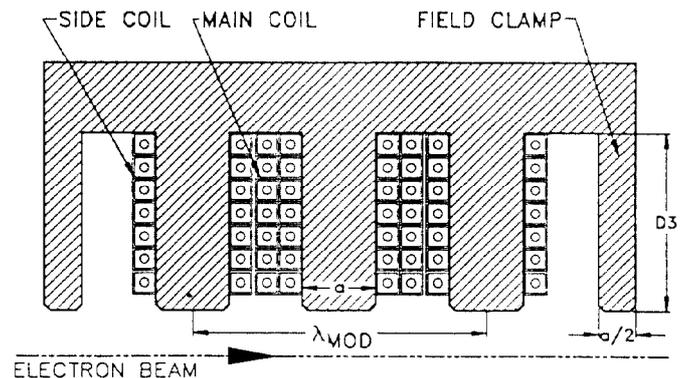


Fig. 1: Longitudinal section through a modulator. The upper half of the magnet is shown. The period length for the SRC magnet is 10 cm, the overall length is 20 cm. The full gap is 3.0 cm,  $a=2.5$  cm and  $D_3=6.1$  cm.

In a saturation-free magnet of this geometry, the first (steering) and second (displacement) field integral of the

wiggler field are approximately zero when the current through the center coil is twice the current through the side coils [5], i.e., the scalar potential of the center pole is  $2V_0$  (side pole  $-V_0$ ). Therefore, if the center coil has twice as many windings as the side coil, the magnet could be driven by one power supply. One condition for proper functioning of the device is to keep steering and displacement as small as possible. Therefore, correction coils may be necessary to dynamically adjust the steering.

### B. Magnetic Design

The modulator shall be able to change the phase of the longest wavelength of interest  $\lambda_R$  by up to  $2\pi$  without perturbing the storage ring. The magnetic design procedure as outlined in [6] consists of two steps. First we determine the necessary current that will produce a phase shift of at least  $2\pi$ . Second, the pole height  $D_3$  [see Fig. 1] is determined. The highest magnetic field in the modulator occurs at the base of the center pole. The magnetic field at the base of the side poles is considerably lower. This could result in steering due to saturation at the base of the center pole. Saturation in the center pole can be minimized by carefully specifying  $D_3$ . In the following paragraphs we describe the design process.

The time of flight difference  $\Delta t$  between the photon and the electron beam is approximately:

$$\Delta t \approx \frac{1}{c} \left[ \frac{L}{2\gamma^2} + s \right] = \frac{m\lambda_R}{c} \quad (1)$$

where  $L$  is the spacing between the two undulators,  $c$  the speed of light,  $\gamma$  the energy of the electrons in units of the rest energy,  $\lambda_R$  the wavelength of the radiation and  $m$  the phase shift in fractions of a wavelength  $\lambda_R$ . The adjustable path difference  $s$  between the electron beam path length and the radiation path length is calculated by:

$$s = \int_{-\infty}^{\infty} (\sqrt{1+x'^2} - 1) dz = \frac{1}{2} \int_{-\infty}^{\infty} x'^2 dz \quad (2)$$

A standard coordinate system is used:  $z$  is measured along the (ideal) average electron beam,  $x$  is the horizontal and  $y$  the vertical distance from  $z$ . The prime symbolizes a derivative with respect to  $z$ . The equation of motion of an electron in a weak wiggler field is in good approximation:

$$x'' = \frac{B(z)}{B\rho} \quad \text{and} \quad x' = \frac{A(z)}{B\rho} \quad (3)$$

where  $B(z)$  is the magnetic field as a function of  $z$ ,  $A(z)$  the vector potential, and  $B\rho$  the rigidity of the electron beam. Combining Eq. 2 and 3 results in:

$$s = \frac{1}{(B\rho)^2} \int_0^{\infty} A^2(z) dz \quad (4)$$

Using a 2D computer program that solves Maxwell's equations (like POISSON [7]), it is straightforward to numerically calculate the integral in Eq. 4 for a given scalar potential  $V_{0,2D}$ . The scalar potential is related to the current by  $3V_{0,2D} = \mu_0 I_{tot,2D}$ , where  $I_{tot,2D}$  are the total Ampereturns about the center pole used in the computer run.

However, we want to invert the problem and calculate the scalar potential  $V_{0,d}$  for a given path difference  $s_d$ . We define the dimensionless quantity  $S$ :

$$S = \frac{\int_0^{\infty} A^2(z) dz}{V_{0,2D}^2 \lambda_{mod}} \quad (5)$$

where  $\lambda_{mod}$  is defined in Fig. 1. Writing  $s = m_d \lambda_R$  and using Eqs. 4 and 5 we obtain:

$$V_{0,d} = \sqrt{\frac{m_d \lambda_R}{S \lambda_{mod}}} B\rho \quad (6)$$

The next step is to calculate the average magnetic field at the base  $B_{base}$  of the center pole. The derivation of Eq. 7 [6] is beyond the scope of this paper. The result is:

$$\overline{B_{base}} = \frac{2V_{0,d}}{p} (3E_B + E_0 + \frac{3D_3}{2h_2}) \quad (7)$$

where  $p$  is the pole thickness in  $z$  direction,  $2h_2 = 0.5\lambda_{mod} - p$ ,  $D_3$  the pole height and  $E_B$  and  $E_0$  are excess flux coefficients. The concept of excess flux coefficients is discussed in [8]. However, we will give a recipe for determining  $E_B$  and  $E_0$ .

$E_B$  can be determined with a POISSON run. A typical field line plot is shown in Fig. 2a.

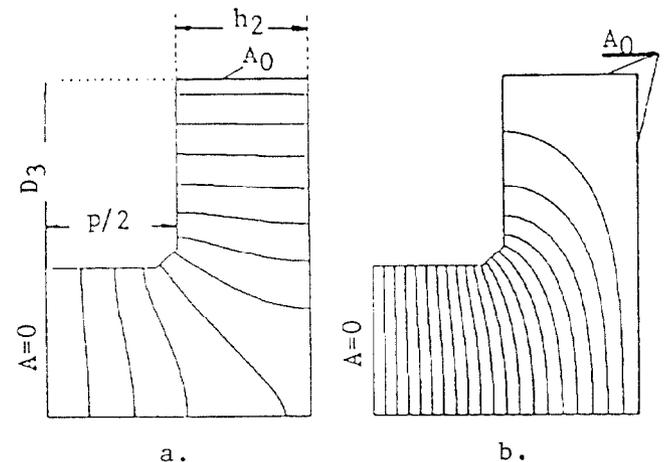


Fig. 2: Field line plots of POISSON runs that are used to determine the excess flux coefficients  $E_B$  (Fig. 2a) and  $E_0$  (Fig. 2b). For details see text.

Only the upper half of a quarter period of the magnet is shown. The bottom boundary is the midplane. A constant vector potential  $A_0$  is assigned to the top boundary.

POISSON calculates the current in each node of the mesh on the top boundary that is necessary to satisfy this boundary condition. These currents are added and multiplied by  $\mu_0$  to get the scalar potential of the pole  $V_{EB}$ . The other boundaries are obvious from the field line plot. The coefficient  $E_B$  is determined with:

$$E_B = \frac{A_0 - D_3}{V_{EB} h_2} \quad (8)$$

$E_0$  is determined with another POISSON run. Now the top and right boundary are set to a fixed vector potential  $A_0$ . The scalar potential of the pole  $V_{E0}$  is again calculated by adding the currents that POISSON places in the nodes on the boundary and multiplying them by  $\mu_0$ . A typical field line plot is shown in Fig. 2b.

$$E_0 = \frac{A_0}{V_{E0}} \quad (9)$$

### III. THE SRC MODULATOR

#### A. Design Parameters

A crossed undulator that will deliver polarized radiation between 8 and 40 eV using an 800 MeV electron beam is being proposed for the Synchrotron Radiation Center (SRC) in Madison-Wisconsin. The horizontal rms electron beam divergence at the location of the device is about 140  $\mu$ rad, whereas the vertical divergence is 15  $\mu$ rad.

The two hybrid undulators will both have five periods. A period length of 10.5 cm results in a  $K_{max}$  of about 3.70 at a magnetic gap of 5.0 cm [9]. The synchrotron radiation opening angle  $1/(\gamma\sqrt{N})$  is 280  $\mu$ rad.

The modulator will have a  $\lambda_{mod}$  of 10 cm and a gap of 3.0 cm. We are designing the wiggler for a maximum phase shift  $m_d$  of 1.5 at a wavelength of 1550 Å. The tolerance on the steering integral of the modulator is completely determined by the storage ring. A steering error of only 3  $\mu$ rad will produce a beam displacement of 10  $\mu$ m (and interfere with experiments) at a bending magnet port [10].

#### B. Results of Calculations

The parameter  $S$  (Eq. 5) for the specified geometry is 1.735. Using Eq. 6, the necessary scalar potential of the pole is calculated to be 3075 G-cm resulting in a total current of 7340 A-turns. The pole height  $D_3$  is determined using Eq. 7 assuming that  $B_{base}$  should not be larger than 1.5 T. The excess flux coefficients were determined to be:  $E_B = 1.328$ ,  $E_0 = 1.437$ . The resulting pole height is 5.64 cm. A slightly taller pole ( $D_3=6.10$  cm) is required to accommodate the coil; the average field at the base of the center pole increases to 1.57 T, which is still acceptable. This result was independently confirmed with a POISSON run using a "real" B-H curve. These results indicate that a slight steering of

about 30  $\mu$ rad is introduced for  $m=1.5$ . It is larger than the allowable steering, but can be compensated by using trim coils. The maximum magnetic field in the midplane is 0.38 T, corresponding to a  $K_{mod}$  of about 3.5.

#### C. Coil Design

The coil [11] can be constructed with a standard square hollow core conductor (min. bore 3.05 mm, effective copper area 31 mm<sup>2</sup>, outer dimensions including insulation 7.95 mm). The maximum current in the conductor is about 370 A; the current density is 11.3 A/mm<sup>2</sup>.

The dissipated power is about 1400 W if we assume that the coils are excited with a dc current of 370 A. An analysis of the cooling requirements [12] showed that a modest water flow (29.2 cm<sup>3</sup>/s at a pressure drop of <5 bar) will cool the magnet efficiently. The temperature rise of the cooling water is less than 4 K and the temperature gradient over the water metal interface is less than 1 K.

Additional heat is dissipated when the modulator is energized with 10 Hz alternating current with an amplitude of 185 A. The power dissipation due to eddy currents in the conductor, the steel laminations and the hysteresis losses is quite small (total <20 W). The total voltage in the ac case is less than 12 V.

### IV. FUTURE RESEARCH

Field perturbations at the location of the electron beam caused by eddy currents in the vacuum chamber will be studied next.

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