

Analysis of the Radiofrequency Dipole and Quadrupole Effects in a QW Resonator

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Abstract

A convenient definition of the multipole coefficients of the accelerating fields produced by any resonator is given: the monopole term gives the transit time factor, the dipole gives a beam deflection, and the quadrupole perturbs the beam focusing; a notable relation holds for a Quarter Wave Resonator. Improving both the spatial resolution and the sensibility of the bead technique permits to determine these coefficients reliably; electronics stability is discussed.

I. INTRODUCTION

The manifest asymmetry of a Quarter Wave Resonator (QWR) [1] around the beam axis z may perturb the beam transport in a heavy ion linac; in this paper, our definition of multipole coefficient is recalled and the effect of the first three coefficients (monopole, dipole and quadrupole) on the beam dynamics is given.

Let x, y, z be the cartesian coordinate, with z the beam axis and x the axis of the QWR central rod (inner conductor in Fig. 2 of Ref. 2); $t = 0$ be the instant when the ion passes the middle plane $z = 0$ of the QWR.

The coefficient computation requires a 3D map of the quasistatic potential $\phi = \Re[\Phi \exp(-i\omega t - i\xi)]$ or of its spatial factor Φ . The bead technique [3] gives a map of the scalar $|E|^2$ easily, since the measuring bead is a sphere, where the spatial factor of the electric field is $\vec{E}(x, y, z) = -\text{grad}\Phi$; it is possible to show that these information are sufficient to deduce E_z and then Φ .

The bead scans the z -values at fixed (x, y) ; to obtain the first three coefficients, only measurements along the line A,B,C,D,F are necessary (see Fig. 1). Recent results with reduced reading error F are here discussed; we use a ogiva tip cavity; the distance AB is $R = 0.6$ and the beam port hole is $a = 1.0$ cm.

II. MULTIPOLE COEFFICIENTS

The quasistatic approximation $\vec{E} = -\text{grad}\Phi + O(\omega^2 b^2/c^2)$, with $b = 9.0$ cm the radius of the QWR outer conductor, allows to compute the field near the central conductor hole and the beam ports, where the beam passes. The Helmholtz's equation reduces then to the Laplace one:

$$\Delta\Phi = 0 + O(\omega^2 b^2/c^2) \quad (1)$$

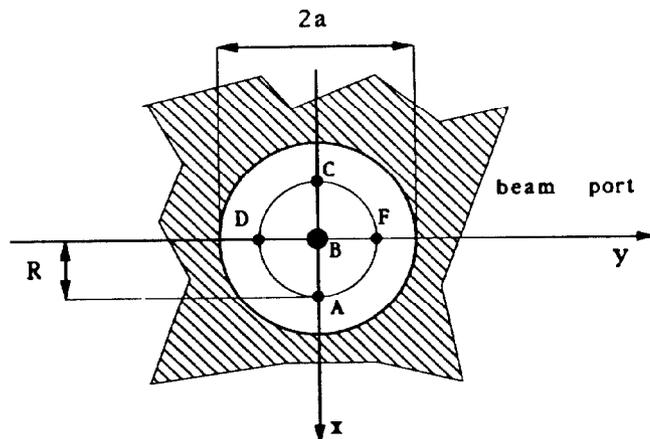


Figure 1. xy -view of the beam port hole, with the scan line intersections A,B,C,D,F.

with the condition $\Phi = V_0$ on the rod tip and $\Phi = 0$ on the other conductors. The QWR has specular symmetry with respect to the zz plane; since there is no conductor in the neighbourhood of the beam $r < a$ the general solution of Eq. (1) is

$$\Phi(r, \vartheta, z) = 2V_0 \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \left\{ c_0(k_z) I_0(|k_z|r) + 2 \sum_{m=1}^{\infty} c_m(k_z) \cos(m\vartheta) I_m(|k_z|r) \right\} \exp[+ik_z z] \quad (2)$$

I_m are the modified Bessel functions.

The variation \vec{K} of the ion momentum \vec{p} through a cavity, named kick, is:

$$\vec{K}(x, y, v_z) = -\Re \left[q e^{-i\xi} \left(\frac{\partial Z}{\partial x}, \frac{\partial Z}{\partial y}, \frac{i\omega Z}{v_z} \right) \right] \quad (3)$$

where $k_v = \omega/v_z$ and the "kick potential" $Z(x, y, k_v)$ is easily written:

$$Z(r, \vartheta, k_v) = \frac{2V_0}{v_z} \left\{ c_0(k_v) I_0(k_v r) + 2 \sum_{m=1}^{\infty} c_m(k_v) \cos(m\vartheta) I_m(k_v r) \right\} \quad (4)$$

Let us neglect terms with $m > 2$ in Eq. (2). Let

$\Phi_A(z)$ be the potential on the scan line A. Solving Eq. (2) we find, within the error $O(k_z R)$:

$$c_0(k_z) = \frac{F(\Phi_A(z) + \Phi_C(z) + 2\Phi_D(z) + 4\Phi_B(z))}{8V_0(I_0(k_z R) + 1)} \quad (5)$$

$$c_1(k_z) = \frac{F(\Phi_A(z) - \Phi_C(z))}{8V_0 I_1(k_z R)} \quad (6)$$

$$c_2(k_z) = \frac{F(\Phi_A(z) + \Phi_C(z) - 2\Phi_D(z))}{16V_0 I_2(k_z R)} \quad (7)$$

here F is the Fourier transform.

A. Dipole zero

Relating c_0 and c_1 is possible only when some correlation between the accelerating field $E_z(0, 0, z)$ and the deflecting field $E_x(0, 0, z)$ is known. Nothing can be said in general, but in the case of a QWR, $\Phi(0, 0, z) = \Phi(0, 0, -z)$, so that E_z has an odd symmetry and E_x has an even symmetry; moreover, the deflecting field E_x , if any, and the accelerating field E_z never vanish inside the acceleration gap. In the separation:

$$E_x(0, 0, z) = \alpha z E_z(0, 0, z) + e_x(z) \quad (8)$$

α can thus be chosen to make the norm $\|e_x\|$ of the second term negligible; we get: $\alpha = \|z E_x E_z\| / \|z E_z\|^2$

Taking the limit $R \rightarrow 0$ in Eq. (6), substituting E_x thanks to Eq. (8) and integrating $E_z = -\partial\Phi/\partial z$ by parts, we find:

$$k_z c_1(k_z) = \alpha \frac{\partial k_z c_0(k_z)}{\partial k_z} + O(e_x/E_z) \quad (9)$$

We note that $k_v c_0(k_v)$ is the transit time factor (TTF) thanks to Eqs. (3) and (4). At the TTF maximum, (optimal ion velocity for acceleration), the dipole deflection c_1 does vanish from Eq. (9), which shows a useful property of QWRs.

III. BEAD TECHNIQUE

A dielectric bead, inserted at the position (x, y, z) in a cavity [3], causes a shift of the resonant frequency f to $f_r = f + pE^2(x, y, z)/U$ where U is the stored energy and the bead sensibility is:

$$p = \frac{3f}{4} \epsilon_0 \frac{\epsilon - 1}{\epsilon + 2} G \quad (10)$$

with ϵ the relative dielectric constant and G the bead volume. The purpose of these measurements is to determine the ratio $|E|/\sqrt{U}$, which is independent from V_0 ; we take $U = 1J$ to simplify the following formulas.

The bead is threaded onto a nylon wire stretched

by a wheel (turned by a stepping motor) and a weight. x, y are constrained by suitable holes in plastic plugs within 0.2 mm , while z increase in steps of $s_z = 21/650 \text{ cm}$ in the present apparatus.

The frequency is very sensitive to temperature (it changes by about 3.0 kHz/K), so that in a quiet and thermalized room still a linear background αz is superimposed onto the bead perturbed frequency:

$$f_r = f + p \frac{E^2}{U} + \alpha z + \text{noise} \quad ; \quad (11)$$

the background may be then subtracted.

A. Fluctuations and electronics

The long term temperature drift was kept within about $0.25K$. We have $\partial \log p / \partial T \cong -2 \cdot 10^{-4} K^{-1}$ for a teflon bead, so that the effect on p of the changing temperature is tiny indeed.

Another delicate part of the experiment is the electronics, (see Fig. 1 in Ref. 3) which actually senses the phase shift θ of the cavity resonator, converts it into a voltage $V \cong k_m \theta$ through a mixer, and, after a suitable amplification, modulates a voltage controlled stabilized master oscillator; the measured frequency shift is therefore:

$$\delta f = C(f_r - f_0) \quad C = \frac{2QBk_m}{f_0 + 2QBk_m} \quad (12)$$

where B is frequency to voltage conversion rate (including amplification) and f_0 the rest frequency of that oscillator. The detection factor C approximates one, and moreover become less sensitive to temperature or whatsoever changes, as much as the gain of the feedback loop increases. Nevertheless the B value can not be increased at will, because the related noise amplification can start selfoscillating in the feedback loop. In our case the electronic amplification was chosen such that $x = (2QBk_m/f_0) \cong 4$ giving $\partial \log C / \partial x = 1/x(1+x) = 1/20$. We can easily estimate a fluctuation $\Delta \log(2QBk_m/f_0) = 0.03$ (two standard deviations) since the mixer sensibility fluctuation dominated. The detection factor thus fluctuates within $C = 0.80(1 \pm 0.006)$.

For scans of the kinds A, B, C , and D we completed respectively 5, 5, 5, 6 measurements successfully. Since 256 data point are needed out of 650 measured, a further average on about 2.5 data is performed with distributed weighting schemes.

The measured δf have (uncorrected) peak heights of about 401,349,382,364 Hz. The contribution of the basic accuracy $F = 1Hz$ to the statistical variance of this measure is fairly small $\sigma_f = F/2\sqrt{3}\sqrt{2n} \cong 0.1Hz$ and is overwhelmed by the C variation, giving $\Delta f = \pm 2.2Hz$ at the peak.

The determination of c_1 and c_2 implies the potential difference between the A and C lines, and

some insight may be gained by the values of $|E|$; we have $p^{1/2}|E|_A = 20.02 \pm 0.06$ versus $p^{1/2}|E|_C = 19.54 \pm 0.06$; the difference $|E|_A - |E|_B = 0.48 \pm 0.08$ is still meaningful, but with large possible errors.

In order to make the detection error comparable to the contribution of the reading error $F = 1Hz$, we need to improve the mixer calibration either to $\Delta k_m/k_m = 0.003$ or the electronic stability, which are both well feasible. In particular, making the measurements faster is of some value: since 2s are needed for each point, we consider to take 325 points in the next experiments, totalling about 6 hours for a set of 32 scans.

B. Analysis of $|E^2|$

A detailed discussion may be found elsewhere [4]; we recall some simple points here. In the regions corresponding to the drift, the field is small, and is significantly affected by σ_f and the variances σ_α and σ_0 in determining the background α and f_0 : the error $\sigma(z)$ on $E_z\sqrt{p}$ is about

$$\sigma(z) \cong \sqrt{\frac{\sigma_f^2 + \sigma_0^2 + \sigma_\alpha^2 z^2}{4pE_z^2(z)} + \frac{\sigma_C^2 p E_z^2}{4C^2}} \quad (13)$$

On the other hand, we know that the field is the sum of the evanescent waves inside the drift tubes. By fitting the peak tails with the $m = 0, 1, 2$ evanescent waves and replacing the measured data with the fit we can: 1) reduce the statical error where it is large, 2) determine a further subtraction of the background $\alpha z + f_0$ more precisely. Point 2) improves σ_α which in turn improves point 1).

IV. DISCUSSION

A plot of the TTF and (on a different scale) of the normalized cavity deflection

$$\alpha_n = \frac{q\omega c_1(\omega/v_z)}{v_z^2 p_z} \propto k_v^3 c_1(k_v) \quad (14)$$

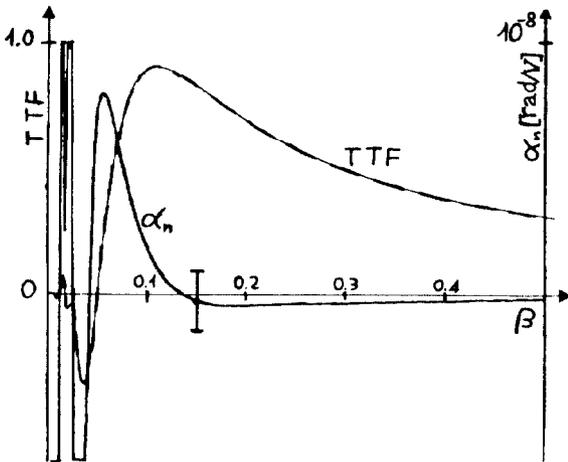


Figure 2. The deflection α_n of Eq. (14) and the TTF.

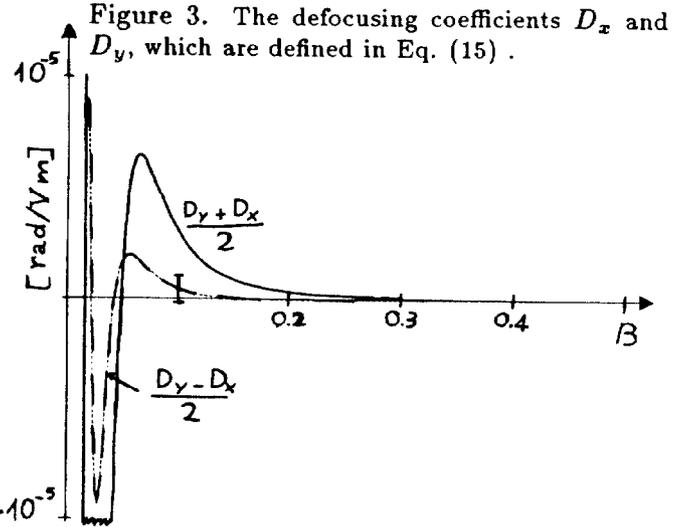


Figure 3. The defocusing coefficients D_x and D_y , which are defined in Eq. (15) .

is shown in Fig. 2; this is related to the actual deflection of a ion with Z charges and nucleon number A as $\alpha_x = -ZV_0 \cos \xi \alpha_n / A$. Notwithstanding the large errors of α_n reflecting the measured c_1 and the approximation of Eq. (9) , the vanishing of the deflection around the maximum point $\beta_{opt} = 0.12$ of the TTF is well satisfied. The actual deflection of a beam through a single cavity is thus small for three circumstances: $\cos \xi \cong -0.3$, $Z/A \ll 1$ and $\beta \cong \beta_{opt}$.

Sum and differences of the defocusing coefficients D_x and D_y :

$$D_x, D_y = \frac{q\omega^2(c_0 \pm c_2)}{v_z^3 p_z} \quad (15)$$

are given in Fig 3; the (de)focusing is equivalent to an inverse focal lens $1/f_y = -ZV_0 \cos \xi D_y / A$ in the y -plane and similarly for f_x . Note that the monopole term — that is the usual RF defocusing — is always dominant. Nothing can be said about the zero of D_x and D_y , due to the large errors in the determination of c_2 . The combination $(D_x + D_y)/2$ has though a negligible error, depending from c_0 only.

In perspective, enough resolution to ascertain the zero of D_x and D_y in the acceleration range $\beta > 0.07$ is foreseen.

V. REFERENCES

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