Modified Octupoles for Damping Coherent Instabilities*

M. Cornacchia
Stanford Synchrotron Radiation laboratory, Stanford, CA 94309
and
W. J. Corbett
Stanford Linear Accelerator Center, Stanford, CA 94309
and
K. Halbach
Lawrence Berkeley Laboratory, Berkeley, California 94720

Abstract: The introduction tune spread in circular e^+e^- accelerators with modified octupoles to reduce the loss of dynamic aperture is discussed. The new magnet design features an octupole field component on-axis and a tapered field structure off-axis to minimize loss of dynamic aperture. Tracking studies show that the modified octupoles can produce the desired tune spread in SPEAR without compromising confinement of the beam. The technique for designing such magnets is presented, together with an example of magnets that give the required field distribution.

I. Introduction

The use of octupoles to give amplitude dependent tune spread for Landau damping of transverse collective instabilities is a well known technique. This method is normally used in proton machines when the beam emittance is relatively large. In small emittance accelerators like synchrotron radiation sources, octupoles have found little application. This is because in order to provide appreciable tune spread within the core of the beam, a large octupole strength is required. At large amplitudes the strong octupole field can have a deleterious effect on the dynamic aperture.

In this paper we extend an idea already applied to chromaticity sextupoles [1]. The idea is to design a magnet (modified octupole) whose field behaves like an octupole close to the axis of the magnet (where the core of the beam circulates) and becomes progressively weaker at larger amplitudes in the plane of interest. In this way, the amplitude dependent tune shift required for Landau damping is obtained within the core of the beam, but large amplitude particles can be made to be more stable than with conventional octupoles.

We have carried out tracking studies with modified octupoles in the SPEAR synchrotron radiation source. In Section II, the tune spread required to damp transverse coupled bunch oscillations in SPEAR is estimated, and the field structure of a prototype modified octupole is discussed. In Section III, the results of the tracking simulations are documented. Section IV contains an example of how such magnets can be designed and built.

II. Theory of Modified Octupoles

A. Tune Spread Requirements: In order to test the idea on a real machine, we have considered SPEAR, which is now operating as a synchrotron radiation source. Multibunch operation is therefore common practice at a circulating beam current on the order of 100 mA, and the goal current of 200 mA is anticipated in the near future. We operate well below the expected transverse single bunch threshold, but coupled multibunch instabilities are a potential threat.

Although a discussion of the collective instabilities in SPEAR is not in the scope of this paper, we need to know the tune spread requirement to demonstrate the use of modified octupoles for Landau damping. We have computed (with the program ZAP [2], the growth times of transverse coupled bunch oscillations driven by parasitic modes in the rf cavities. The cavity modes have been computed with URMEL [3]. The ZAP results show that in order to stabilize the fastest growing (dipole) mode at 3 GeV and 200 mA beam current, an amplitude dependent tune spread of 0.0015 within $\sqrt{2}\sigma$ of the betatron amplitude distribution is required in whatever plane the instability develops.

B. Modified Octupoles: In what follows we will use complex notation for the magnetic field $\mathbf{B}$ with $z = x + iy$ and $B^* = B_x - iB_y$. Consider now the following example of a magnetic field structure which behaves like a standard octupole field for small excursions from the beam axis, but is modified for larger off-axis values of z:

$$\mathbf{B}^*(Z) = K_1 z^3 \bullet (1 + K_2 z^2)^6 \tag{1}$$

The constants K_1 and K_2 are used to regulate the octupole field strength and spatial field modification, respectively.

The vertical component of the field (B_y) along the horizontal axis satisfies the condition of behaving like an octupole in the vicinity of the origin, but increases more slowly than a pure octupole field at larger amplitudes when $K_2 < 0$ (see Fig. 1). The opposite is true in the vertical plane (B_x increases with increasing y) as was stated more generally in Ref. [1]. Thus, the field cannot decrease simultaneously in both planes.

However, one can exploit the strong focusing aspect of the accelerator when selecting placement of the magnets, and choose opposite signs for the constant K_2 depending on whether a horizontal or vertical tune spread is required. Then, given sufficient decoupling of the beta functions at the locations of the F- and D-octupoles, the loss of dynamic aperture caused by the strong octupole field can be mitigated by the $(1 + K_2 z^2)^6$ term in Eq. (1).

The amplitude dependent tune shift created by the modified octupole field can be estimated by averaging the angular kick over all possible phases. A similar technique has been used to compute the beam-beam resonances . [4,5] For the magnetic field function given in Eq. (1), the horizontal tune shift is given as a function of the amplitude by the following integral:

$$\Delta\nu_x(a_x) = \frac{K_1 l \bullet \beta_x}{4\pi^2 B\rho \bullet a_x} \bullet \tag{2}$$

$$\int_0^{2\pi} (a_x \cos\phi_x)^3 [1 + K_2 (a_x \cos\phi_x)^2]^6 \cos\phi_x d\phi_x$$

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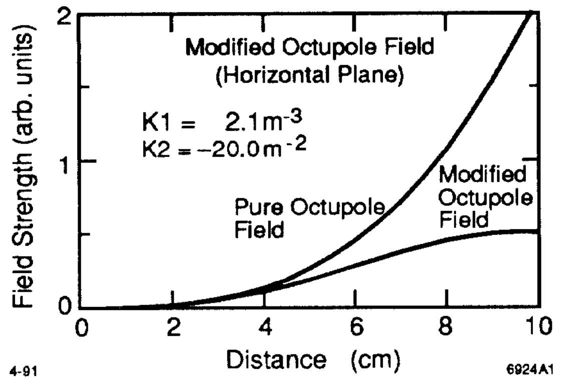

Figure 1. Magnetic field profile in the horizontal plane for a modified octupole.

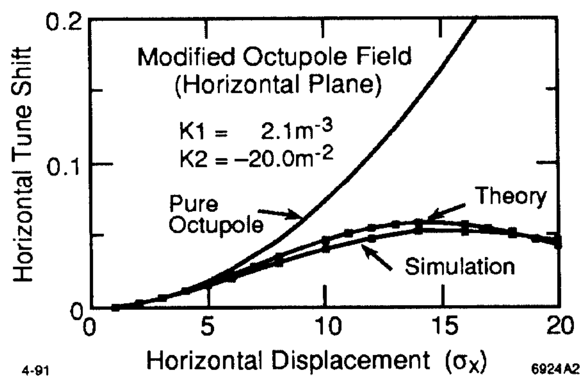

Figure 2. Tune Shift with amplitude in the horizontal plane. Theory and simulation curves refer to modified octupole studies for SPEAR.

where ℓ is the magnet length, β_x is the horizontal β-function at the octupole location (thin lens approximation), a_x is the maximum betatron amplitude at the same location, and the integral is calculated over all possible phases ϕ_x. $B\rho$ is the magnetic rigidity.

For example, Fig. 2 shows the tune shift as a function of amplitude computed with Eq. (2) and as calculated during tracking studies. Considering the sextupole tune shift is not included in Eq. (2), the two results agree quite well. Note that at large amplitudes in betatron space, the amplitude dependent tune shift for the modified octupoles is lower than with conventional octupoles. The tune spread is only required at the core of the beam in order to stabilize the beam against transverse collective instabilities.

III. Tracking Simulations in SPEAR

To simulate the effects of the modified octupoles in SPEAR, tracking studies were made using a collider optics configuration. In this configuration, locations with beta-function ratios β_x/β_y and β_y/β_x of $\approx 6{:}1$ were found in the insertion regions where $\eta_x = \eta_y = 0$ and sufficient space is available to install new magnets. By selecting locations where the betatron coupling is small, we try to confine the amplitude dependent tune shift to a single transverse plane. In terms of RMS beam size, the limiting geometric apertures for SPEAR are $15\sigma_x$ and $13\sigma_y$ for a horizontal emittance of 650 nm-rad with 20% emittance coupling into the vertical plane. In the same units, the dynamic aperture with sextupoles only is about $50\sigma_x$ by $100\sigma_y$ in the principle planes for betatron tunes of $\nu_x = 5.275$, $\nu_x = 5.110$.

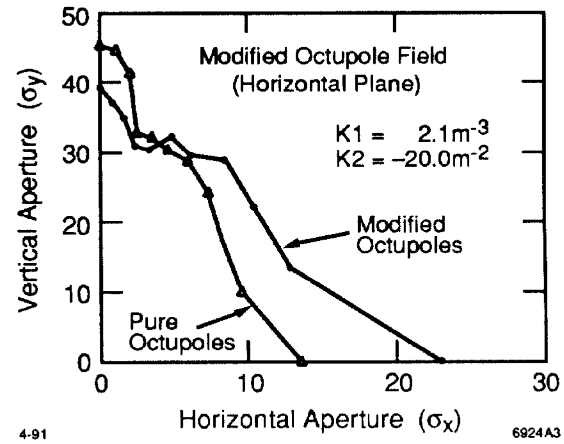

Figure 3. Dynamic aperture increases in the horizontal plane when modified octupoles are used.

The tune shift with amplitude and dynamic aperture are found by tracking with the lattice simulation code GEMINI/FUTAGO [6]. The modified octupole kicks are placed 30 cm from the nearest magnet pole face and represented by a multipole expansion of $B^{\star}(z) = K_1 z^3 (1 + K_2 z^2)^6$. The simulation procedure is to first choose the octupole coefficient for a given transverse plane to produce a tune shift of $\Delta\nu = 0.0015$ at $\sqrt{2}\sigma$ in a given transverse plane. The dynamic aperture for the pure octupole is then found by tracking for up to 20000 turns. No synchrotron oscillations were included. Next, we search for the optimum value of K_2 to increase the aperture in the plane of interest. Generally, K_2 is restricted to values such that $(1 + K_2 z^2) > 0$ is not zero anywhere within the vacuum chamber.

The result of a simulation for a modified F-octupole in SPEAR is shown in Fig. 3. For this case, we have $K_1 = 2.1\ \mathrm{m}^{-3}$, $K_2 = -20m^{-2}$ and the horizontal aperture is increased by almost a factor of two by modifying the field structure. Figure 2 shows the tune shift with amplitude superimposed on the theoretical curve.

In the vertical plane, the F-octupole has less effect because the beta functions are decoupled at the magnet location and the emittance is low. Low vertical emittance, however, complicates the design of a modified D-octupole. In this case, we had to increase the emittance coupling coefficient to value of 30% (limited by insertion device gaps) and reduce the tune shift to < 0.001 (at $\sqrt{2}\sigma_y$) to keep the horizontal dynamic aperture commensurate with the geometric limit.

IV. Design of a Modified Octupole

Figure 4 shows a cross section of the elliptical vacuum chamber (VC), the elliptical good field region (GFR), and a smaller circular GFR. Using the same procedure that was used for the design of modified sextupoles [7] [1,7], Fig. 5 shows the map of the VC and the GFR in the first quadrant of the $x - y$ coordinate system, using the map resulting from $dw/dz = z^3(1 + K_2 z^2)^6$ for $K_2 = -0.002\ \mathrm{cm}^{-2} \bullet (z = x + iy)$.

It should be noted that the map of the y axis appears to fall onto the map of the x axis. However since one has advanced by 360 degrees on the Riemann surface in the w-plane by the time one has moved from the map of the x axis to the map of the y axis, these two maps are separated by a rather large distance in the w-plane, except very close to the origin. The symmetry of the desired field distributions

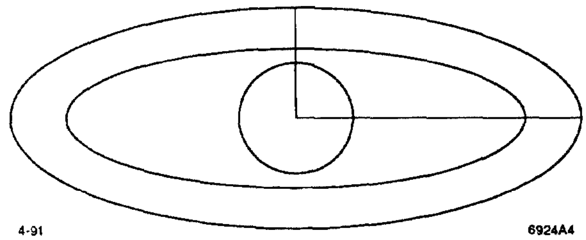

Figure 4. Cross section of a vacuum chamber with half axes 10 an 4 cm, and of good field regions.

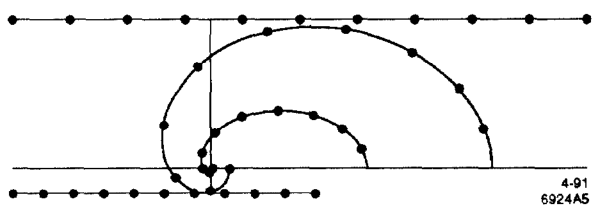

Figure 5. Conformal map of vacuum chamber and good field relations in the first quadrant of the X-Y plane, with flat poles giving a good dipole field in mapped geometry.

obviously demands that the fields on the axes have to be perpendicular to both of them, and their maps.

To get a feeling for the map, the maps of the VC and GFR show the maps of points that are equidistant on the circles in z-geometry before they are "squashed" into ellipses. The map of the circular GFR is so small that one does not see it, not surprising in view of the fact that close to the origin the map is close to $w = 0.25z^4$.

Also shown in Fig. 5 are the proposed flat poles in the w-geometry that should give a good dipole field in w-geometry, thus producing the desired field in z-geometry. Shims at the ends of the polefaces, the rest of the poles, coils, and other details would be incorporated into the design at a later stage.

Figure 6 shows, together with the VC and the GFR, the mapping of the two dipole-poles from the w-geometry to the z-geometry, indicating also the maps of the marked points on the dipole-poles in w-geometry. The excitation of the pole close to the y-axis is $-1/6$ times the excitation of the pole close to the x-axis, reflecting the fact that one pole is closer to the origin than the other, and the properties of the mapping function. This choice is dictated not so much by the desire to keep the excitation down as by the advisability to bring that pole as close as possible to the coordinate origin in order to reduce fringe fields at the ends of the magnet. Slight imperfections and fringe fields that do not break the overall symmetry of the magnet will probably cause a small quadrupole moment for the integrated fields, leading to a (sizeable) breakup of the ideal triple zero field points into three separate zero field points. Special magnetic field measurement techniques that take advantage of this fact to determine sub-harmonics, both in modified octupoles and modified sextupoles, will be described in a forthcoming paper. After measurement of the sub-harmonic strength, a correction is easily made by a small adjustment of the relative excitation of the poles close to the x and y axes.

Figure 7 shows a conceptual cross section of the whole magnet. To generate an integrated octupole field strength of 21Gcm^{-2}, the poles close to the x-axis need to be excited by about 478A-turns if one chooses an effective length of the magnet of about 20 cm.

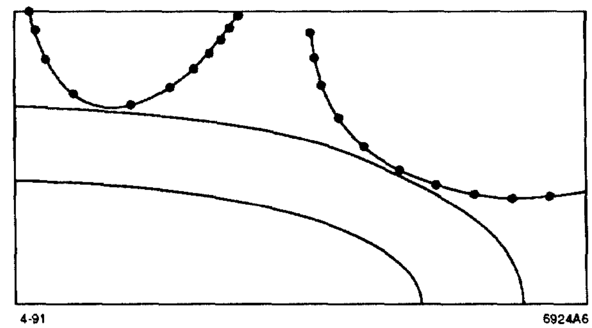

Figure 6. Flat pole faces in W-geometry mapped into Z-geometry.

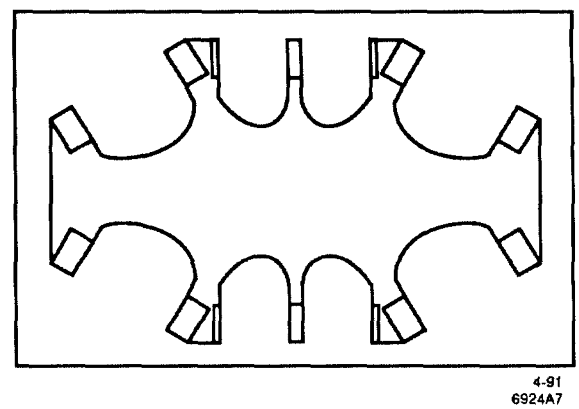

Figure 7. Conceptual design of complete modified octupole.

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