Ion Clearing by Cyclotron Resonance Shaking

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Abstract

We discuss a new concept in ion clearing for storage rings, that of resonant removal of ions in dipoles by shaking the beam horizontally near the ion cyclotron frequency. This method of beam shaking is similar to the variations on ion bounce shaking developed by Orlov, Alves-Pires and coworkers at CERN, but has advantages in requiring a narrower bandwidth of shaking frequencies and in much higher achievable ion kinetic energies. The results of analytical theory, and computer simulations are discussed.

Introduction

Accumulation of ions has been a limiting factor in the performance of antiproton accumulators. Besides the direct method using clearing electrodes, beam shaking has been proven very effective in further clearing ions[1] and reducing the neutralization level in storage rings. In the so called "resonant shaking"[2] a driving voltage with frequency close to that of the ion oscillatory motion in the beam's electric field (bounce frequency) is applied to the beam, which responds by shaking, generating an oscillating transverse electric field which in turn drives transverse ion motion. Ions traversing different beam sizes, and thus bounce frequencies, through slow longitudinal motion will be locked-on to larger amplitudes; in this way the neutralization effects can be reduced[3]. The same or better results should be achievable through modulation of the driving frequency[4], which is called frequency modulation shaking. This way the process is controllable and does not rely on the ions' longitudinal motion. To distinguish from what we are going to describe here we call this kind of shaking bounce shaking because it is the ion bounce motion that is being driven.

An interesting and important experimental fact is that the bounce shaking described above doesn't work well in horizontal plane. This can be explained by noting that while ions in straight sections are easily removed by the clearing electrodes, ions inside dipole magnets cannot be easily cleared, since there are are often no clearing electrodes inside magnets due to tight space constraints, and thus the only clearing mechanisms are Coulomb heating and $\mathbf{E} \times \mathbf{B}$ longitudinal drift. The former is insignificant while the latter can be greatly reduced by the neutralization itself. As a result the neutralization level inside the magnets is much higher than the rest of the machine and deleterious ion effects are most likely caused mainly by the ions in dipoles. The strong magnetic field inside magnets leads to cyclotron motion of ions in the horizontal plane, which has, in general, a much higher oscillation frequency than that of the bounce motion, and therefore bounce shaking is not effective in the horizontal plane. This explanation leads us consider the possibility of horizontal shaking close to ion cyclotron frequency, which we call cyclotron shaking.

Theory of Cyclotron Shaking

The theoretical analysis for cyclotron shaking is parallel to that of bounce shaking[4], which uses the averaging technique originally developed by Krylov and Bogoliubov[5][6]. For completeness we reproduce analysis here.

Let x be the transverse coordinate of ion in the horizontal plane and z the longitudinal coordinate, then, ignoring the space charge of the ions themselves, the equations of motion for the ion are,

$$\begin{aligned} \frac{d^2x}{dt^2} &= -\omega_c\frac{dz}{dt} + \frac{q}{m}E_x(x - b\cos\omega t) \\ \frac{d^2z}{dt^2} &= -\omega_c\frac{dx}{dt} \end{aligned}$$

where q and m are the charge and mass of ion respectively, ω_c is the cyclotron frequency of the ion, and b is the amplitude of the beam center oscillation driven by the applied voltage. For a round Gaussian beam with rms beam size σ the form of radial electric field is

$$E_r(r) = \frac{2\lambda}{r}(1 - e^{-\frac{r^2}{2\sigma^2}})$$

where λ is the beam line charge density.

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The displacement of the ion from the beam center normalized by the beam rms size σ, $\tilde{x} = \frac{1}{\sigma}[x - b\cos(\omega t)]$, is then described by

$$\frac{d^2\tilde{x}}{dt^2} = A(\omega^2 - \omega_c^2)\cos(\omega t) - \omega_c^2(\tilde{x} - x_0) - \omega_b^2 f(\tilde{x})$$

where $\omega_b = \sqrt{|q\lambda|/m\sigma^2}$ is the maximum ion bounce frequency, $A = b/\sigma$ and $x_0\sigma$ is the horizontal position of the ion's guiding center. Unlike in the case of bounce shaking where the corresponding quantity is always zero, x_0 here is in general not. For simplification we take $f(\tilde{x}) = \frac{2}{\tilde{x}}[1 - \exp(-\frac{1}{2}\frac{\tilde{x}^2}{\sigma^2})]$ which implies that we are only considering ions close to the vertical center of the beam.

We look for the equilibrium solution of the form

$$\begin{aligned} \tilde{x} &= a(t)\cos(\omega t + \theta(t)) + x_0 \\ \frac{d\tilde{x}}{dt} &= -\omega a(t)\sin(\omega t + \theta(t)) \end{aligned}$$

in which $a(t)$ and $\theta(t)$ are slow varying functions relative to the oscillation period. Averaging over a period yields:

$$\begin{aligned} \frac{da}{dt} &= \frac{(\omega_c^2 - \omega^2)}{2\omega} A\sin\theta \\ \frac{d\theta}{dt} &= \frac{(\omega_c^2 - \omega^2)}{2\omega}[1 - \frac{A}{a}\cos\theta] + \frac{\omega_b^2}{2\omega}G(a, x_0) \end{aligned}$$

where $G(a, x_0) = \frac{1}{2\pi}\int_0^{2\pi} f(x_0 + a\cos\phi)\cos\phi d\phi$. The maximum amplitude satisfies $da/dt = d\theta/dt = 0$, therefore we have an implicit relation between the driving frequency and equilibrium ion amplitude:

$$\frac{\omega^2 - \omega_c^2}{\omega_b^2} = \frac{G(a, x_0)}{1 \pm \frac{A}{a}} \qquad (1)$$

Eq. 1 represents hysteresis which is well known for non-linear oscillators. For $x_0 = 0$, $G(a, 0) = \frac{4}{a^2}[1 - e^{(-\frac{a^2}{4})} I_0(\frac{a^2}{4})]$ and Eq. 1 is plotted in Fig. 1. The motion corresponding to the upper half of the left curve is unstable. The beam oscillation amplitude undergoes a jump at a particular driving frequency and it can reach much larger values if the driving frequency is modulated from above that frequency downward, which is the concept frequency modulation shaking based upon.

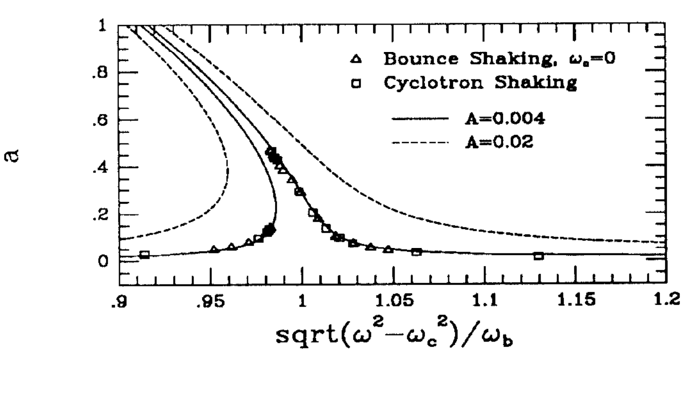

Figure 1: Equilibrium Ion Oscillation Amplitude vs. Shaking Frequency

In general function $G(a, x_0)$ cannot be expressed in closed form. Results by numerical methods are shown in Fig. 2. As x_0 increases G not only drops in magnitude but

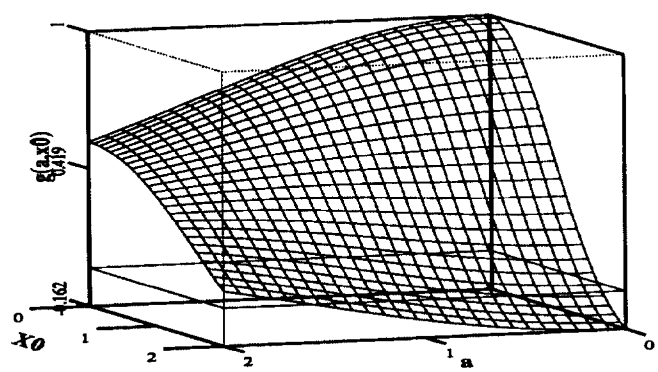

Figure 2: Function $G(a, x_0)$.

also reverses its trend as a function of a. This means not only the resonant frequency decreases, the direction of frequency modulation that increases the ion oscillation amplitude reverses also. Therefore the effect of cyclotron shaking with frequency modulation eventually vanishes. With this discussion in mind, it is clear that cyclotron shaking will only be effective to ions close to beam center. Fortunately, these ions are precisely the ones which pose the greatest threat to beam stability, because they slow the ions' $\mathbf{E} \times \mathbf{B}$ drift inside dipoles by neutralizing the electric field. The drift speed is $\sim 10^3 m/s$ for ions that are 1σ from the center of an unneutralized 100 mA beam in the Fermilab accumulator. This corresponds to a less than 0.2% equilibrium neutralization level inside dipoles, which is much better than the current operation conditions. If the cyclotron shaking can excite the ions created close to beam center to large amplitudes, the drifting motion of the other ions will provide sufficient clearing.

Discussion

Using the Fermilab antiproton accumulator parameters with 200mA beam current, we compared the theoretical predictions on the equilibrium amplitudes with the simulation results of both bounce and cyclotron shaking for protons. As shown in Fig. 1 the data points, which corresponds to $\sigma = 0.22cm, A\sigma = 0.01mm$ and no initial horizontal displacement agree remarkably well with the analytical theory. The modulation of frequency, in both shaking schemes, is also shown by simulation to be very effective, as predicted by theory.

As seen above cyclotron shaking and bounce shaking are very similar in many aspects. The equilibrium amplitudes can be described by the same plot and hence both have the same hysteresis "lock on" effect which is crucial in reducing neutralization effects[3]. However because of the very different frequencies of the cyclotron and bounce motion, the two ways of shaking do have different properties. The cyclotron frequency ω_c, at least in our case, is much higher than the bounce frequency ω_b. The corresponding frequency spread in cyclotron shaking is a factor of $\omega_c/\omega_b = 15$ smaller than that in bounce shaking. This difference affects the frequency modulation process because the beam response to the driving voltage depends strongly on the frequency and that has a big impact on the ion response (see Fig. 1). Obviously the beam response to the driving voltage varies rapidly around betatron sidebands. If in the process of frequency modulation the beam response changes too fast, that could cause ions to loose the lock-on and limit the effectiveness of modulated frequency bounce shaking. The small frequency range of cyclotron shaking helps in stabilizing the beam response in the whole process of frequency change. The factor of 15 in the case of Fermilab accumulator reduces the range to only a small fraction of revolution frequency. Note that if the cyclotron frequency falls very near a betatron side band, a beam-ion instability may be excited. This subject is analyzed in a separate paper[7].

The theory presented earlier only deals with equilibrium responses while frequency modulation inevitably introduces time varying effects. The simulation shows that cyclotron shaking with frequency modulation has a longer lasting transient ion motion which requires a longer modulation period. The long transient in the ion motion in the case of driving cyclotron resonant motion is easily understood by noting that there is a lot more kinetic energy in a cyclotron orbit than in a bounce orbit of the same horizontal amplitude. For the cases where cyclotron shaking are effective, the cyclotron kinetic energy is $E_k = (x_m\omega_c)^2 m/2 \simeq 500$ eV, and the bounce kinetic energy is smaller by a factor of $(\omega_b/\omega_c)^2$, or 2.2 eV. When a bounce-shaken ion has exited the end of the magnets through its $\mathbf{E} \times \mathbf{B}$ drift motion, its removal is still predicated on adequate clearing voltages, which may not always be provided. If the ion is cyclotron-shaken, however, it can by virtue of its large energy easily escape the beam potential well and clear completely. This is indeed verified by simulation. Shown in Fig. 3 are the normalized magnetic field and a sample of ion motion driven by frequency modulated cyclotron shaking. Notice that the tracks shown are actually the alias of ion's oscillation motion. The longitudinal positions that locked-on ions escape the beam and hit the vacuum wall are concentrated in a small range. This fact provides a potential diagnostic to directly measure the effect of the cyclotron shaking, by collecting these energetic ions, and measuring their energy spectra. In addition, inside the magnet, if the cyclotron-shaken ion has an elastic collision with an ion, a neutral, or a beam particle which redirects even 5% of its energy into the vertical plane, it will escape the beam entirely. Preliminary calculations indicate that this beneficial phenomenon may occur at a non-negligible rate in the Fermilab antiproton accumulator.

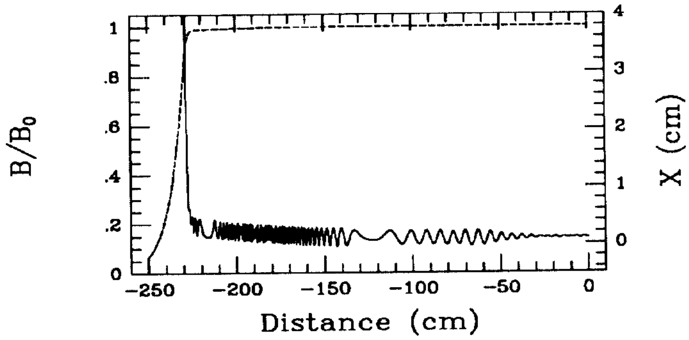

Figure 3: Dipole Field and Ion Motion

In short we conclude that the cyclotron shaking is less susceptible to the change of beam response to external driving voltage and therefore may be more effective in reducing ion neutralization effects. In addition, the large energy associated with exciting cyclotron orbits of radii on the order of the beam size may aid significantly in final removal of the ions from the beam. Experimental study is needed, and is presently being pursued on the Fermilab antiproton accumulator.

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