# A Comparison of Transition Jump Schemes for the Main Injector

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### I. INTRODUCTION

Transition comes when the differential of circulation time with respect to  $\delta = \Delta p/p_0$ , the off momentum parameter, is zero. This happens when the relative rate of change of speed  $\beta$  equals the relative increase in path length - when

$$\frac{1}{\beta} \frac{d\beta}{d\delta} = \frac{1}{\gamma^2} = \frac{1}{\gamma_{\rm I}^2}$$
(1)  
$$= \frac{1}{C_0} \frac{dC}{d\delta} = \frac{1}{C_0} \int_0^{2\pi} \eta \ d\theta = \frac{2\pi}{C_0} \langle \eta \rangle$$

Here C is the circumference of the closed orbit,  $\eta$  is the dispersion function, and  $\theta$  is the accumulated bend angle. Angle brackets <> represent an average over bending dipoles. Significant beam loss and emittance growth are expected in the Main Injector when  $\gamma \approx \gamma_T$ , if nothing is done to ameliorate transition crossing[1]. Here we consider ways in which  $\gamma_T$  can be modified for a short period of time, so that transition crossing is very rapid. Figure 1 illustrates unipolar and bipolar "transition jump" schemes. In order that the passage is graceful, it is necessary that

$$|\gamma - \gamma_{\rm T}| > 0.65$$
 (2)

for as long as possible[2]. Hence the unipolar scheme shown needs a lattice with  $\Delta \gamma_T = -1.3$ , and the bipolar scheme needs two lattices, with  $\Delta \gamma_T = \pm 0.65$ .

A major design issue is, how "gross" are the changes of other single particle properties of the lattice? Another issue is simplicity of design. How strong are the perturbation quadrupoles, how many are there, in how many families? What is the tracking performance? Two prototypical schemes are compared, "matched" and "unmatched", referring to whether or not a large dispersion wave circulates when the perturbation quads are turned on. Results presented from an MI\_15 lattice analysis are valid for the contemporary MI\_17 lattice, with small changes (except where noted).

## II. MATCHED AND UNMATCHED CONFIGURATIONS

A small quadrupole perturbation q shifts the tune by

$$\Delta Q_{\rm H} = \frac{q \,\beta_{\rm quad}}{4\pi} \tag{3}$$

where  $\beta$  is now a beta function. Downstream from the perturbation there is a free horizontal beta wave

$$\frac{\Delta\beta}{\beta} = -q \beta_{\text{quad}} \sin[2(\phi - \phi_{\text{quad}})] \qquad (4)$$

with a phase advancing twice as fast as the betatron phase. There is also a free horizontal dispersion wave, given by

$$\frac{\Delta \eta}{\sqrt{\beta}} = -q \eta_{\text{quad}} \sqrt{\beta_{\text{quad}}} \sin(\phi - \phi_{\text{quad}}) \qquad (5)$$

advancing in step with the betatron phase.

Most of the Main Injector circumference consists of four FODO cell arcs, with a phase advance per cell close to 90 degrees. No betatron wave escapes one of these arcs if the perturbing quads are arranged in pairs, either with identical strengths 90 degrees apart, or with equal and opposite strengths 180 degrees apart. Similarly, no dispersion wave escapes if identical strength pair members are 180 degrees apart, or if equal and opposite members are 360 degrees apart. Neither wave escapes if identical quads are arranged in groups of four, with 90 degrees of phase advance - one FODO cell - between neighbors. This is the matched  $\gamma_T$  jump scheme. There is a total of 48 such quadrupoles at focussing locations in arc cells, where their effect on  $\gamma_{T}$  is the greatest. Essentially, these quadrupoles retune the standard arc FODO cell, changing the matched dispersion function. The change in  $\gamma_{T}$  is first order in the strength of the quadrupoles. Unfortunately, a second family of perturbation quads is required in a dispersion free region, to compensate for the tune shift accumulated through the arc cell retuning. There is no need to avoid dispersion



Figure 1 Main Injector passage through transition with nominal parameters, including unipolar or bioplar jumps.

<sup>\*</sup>Operated by the Universities Research Association, Inc., under contract with the U.S Department of Energy.

waves in the second family, so the quadrupoles are arranged in pairs, 90 degrees apart. In the MI\_15 lattice there is only space for an effective total of 8 quadrupoles. (In MI\_17 there is space for 16 - a significant improvement, as will be seen.) The strength of the second family is about 6 times the strength of the first, and with opposite polarity.

There is only one family of 24 quadrupoles in the unmatched scheme, with opposite polarity members at every other horizontally focussing arc quadrupole[3]. This is similar to the scheme in daily use in the Fermilab Booster [4-6]. There is no global beta wave and no net tune shift, to first order in the perturbation strength. However, there is a large first order global dispersion wave, while the average change in  $\gamma_{T}$  is only of second order. (In a pure FODO lattice with no straights, flipping all perturbation quad polarities results in exactly the same lattice. The "polarity symmetry" is somewhat broken in MI\_15, but the first order term in the variation of  $\Delta\gamma_{T}$  is still negligible.)

Case number	1	2	3	4	5
Matched?		Yes	Yes	Yes	No
$\Delta \gamma_{\rm T}$	0.0	-1.3	-0.65	0.65	-1.3
Population 1		48	48	48	24
Population	2	8	8	8	
Strength 1		055	029	.031	.074
Strength 2		.334	.159	157	_
$\beta_{\text{H min}}(m)$	10.9	1.3	4.68	8.86	8.86
β <sub>H max</sub> (m)	56.7	432.7	81.0	97.5	65.4
$\beta_{V \min}(m)$	10.9	10.3	10.9	10.9	10.7
$\beta_{V \max}(m)$	78.9	83.5	80.1	80.3	82.0
η <sub>min</sub> (m)	-0.12	-1.75	0.47	-0.31	-7.71
η <sub>max</sub> (m)	2.07	4.61	2.64	2.29	9.59

Table 1 Configuration and performance in various schemes.Quad strengths are relative to the regular FODO quad strength.

## III. LINEAR OPTICAL PERFORMANCE

Table 1 compares the 5 cases of interest. It is implicit that the net horizontal and vertical tune shifts are negligibly small, of order  $10^{-3}$ , although they are not exactly zero in any of the cases. The table shows that the assumption that first order perturbation theory is adequate is not completely true for either

scheme. Case 2 (matched, unipolar,  $\Delta \gamma_T = -1.3$ ) shows the largest strength, with a second family strength of one third of the nominal strength of an arc quadrupole. The optical solution is not acceptable, because the maximum horizontal beta function rises to 433 meters. In all cases the vertical beta functions are negligibly disturbed, since the perturbing quads are at horizontally focussing locations. The matched bipolar scheme (cases 3 and 4) causes much more modest optical perturbations - although they are still not negligible - with a maximum horizontal beta of 98 meters. It is expected that this situation would improve dramatically in the MI\_17 lattice, with twice as many quads in the second family, and with none of them placed back to back, as in MI\_15.

In the <u>unmatched</u> scheme (case 5), the minimum and maximum horizontal beta functions are disturbed by less than 20%. However, the dispersion reaches extremes of -7.7 and 9.6 meters, leading to a reduced dynamic aperture for off-momentum particles. This is especially painful at transition, when the momentum width of the beam is at its largest. Large dispersion swings also cause a large variation of  $\gamma_{\rm T}$  with momentum, raising the Johnsen time significantly[7].

## IV. TRACKING RESULTS

The dynamic aperture about a displaced closed orbit is found as a function of  $\delta = \Delta p/p$  using the code TEAPOT. The  $\delta$  range used, 0.0 to 0.01, is about twice the large total momentum spread expected for about 10 milliseconds, or 1000 turns, around transition. Successive particles are launched with decreasing amplitude, in steps of 0.5 centimeters, until a particle survives for 1000 turns at the "stable" amplitude. (Amplitudes are initially identical horizontally and vertically, and are scaled to  $\beta_{max}$  in a FODO cell). While it is clearly incorrect to include synchrotron oscillations, such a static model is flawed by not including ramping and transition in all three spatial dimensions. No appropriate code exist at present.

Only those systematic dipole bend magnetic errors listed in Table 2 are included, according to the expansion

$$B_{\text{vert}} = B_0 \left[ 1 + \sum_n b_n \left( \frac{x}{r_0} \right)^n \right]$$
(6)

The reference radius r0 in Table 2 is 1 inch - 2.54 centimeters. Multipole values come from the Main Injector dipole design calculations[8-10] at a transition momentum of 19.10 Gev/c, and a dipole field of  $B_0 = 0.237$  Tesla. They are consistent with prototype measurements[10]. The sextupole field is dominant, due almost completely to eddy currents. Higher order eddy current effects and remanent fields are negligible.

Results are shown in Figures 2 and 3 for two configurations: a matched  $\gamma_T$  jump with  $\Delta \gamma_T = -0.65$  (case 3 in Table 1), and an unmatched  $\gamma_T$  jump with  $\Delta \gamma_T = -1.3$  (case 5). The net chromaticities are set to zero in both. A straight line is fit to the data, with a slope dx/d $\delta$  analogous to a dispersion if the aperture stop is a "brick wall". The slopes (-2.7 meters in Figure 2 and -7.1 meters in Figure 3)

Source	b2	b4	b <sub>ó</sub>	
	(10 <sup>-4</sup> @ 1 inch)			
Eddy current	3.405	087	028	
Saturation	.215	.184	.046	
	3 620	097	018	

Table 2 Systematic dipole errors used for tracking.



Figure 2 Dynamic aperture versus momentum for a matched bipolar jump, Case 3 in Table 1.



Figure 3 Dynamic aperture versus momentum for an unmatched unipolar jump, Case 5 in Table 1.

are in fair agreement with the corresponding maximum dispersions quoted in Table 1 (2.6 and 9.6 meters). The matched scheme has a definite advantage in improved offmomentum dynamic aperture performance.

### **V. CONCLUSIONS**

While neither the matched nor the unmatched transition jump scheme is entirely satisfactory as presented, either one could be improved and made to work well in the Main Injector.

The unmatched scheme has simpler hardware requirements only one family of perturbation quadrupoles is required. Betatron functions and tunes are negligibly affected (in an ideal lattice without errors). Its disadvantages stem from the large induced dispersion wave, and from the second order dependence of  $\Delta \gamma_T$  on perturbation strength. The 9.6 meter maximum dispersion decreases the dynamic aperture significantly (although not severely) at large momentum offsets.

The matched scheme requires two quadrupole families to keep the betatron tunes unchanged. This leads to a large perturbation strength that, in the worst case of a unipolar  $\Delta \gamma_T = -1.3$  jump, distorts the linear lattice almost to the point of instability. This problem is expected to be greatly ameliorated in the MI\_17 lattice, with a factor of 4 decrease in maximum quadrupole strength. The behavior of the linear lattice in a bipolar jump of strength  $\Delta \gamma_T = \pm 0.65$  is reasonable, even in the MI\_15 lattice.

#### **VI. REFERENCES**

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