Synchronization of a Variable Frequency Source with a Fixed Frequency Source Using a Sliding-Mode Controller

L.K. Mestha
SSC Laboratory*, Dallas, Texas 75237

K.S. Yeung
University of Texas, Arlington, Texas 76019

Abstract

One way of synchronizing the SSC Low Energy Booster with the Medium Energy Booster is by matching the longitudinal phase of the designated RF buckets of two machines throughout acceleration to a pre-programmed trajectory. This makes the synchronization predictable in advance. The model associated with the phase-locking is time-varying and model parameters are subjected to disturbance due to errors in the bending magnetic field. Also the disturbance could be due to other feedback loops such as a B-field loop or a beam phase loop in the system. The measured phase error between the two reference waves may not be accurate. Hence in this paper we have shown the design of a Sliding-Mode controller for such an application. In the absence of measurement errors and parameter uncertainties and with no disturbance, the controller reduces to a classical gain feedback. Due to the general approach we have adopted in synthesizing the controller, the techniques can be applied to existing synchronization schemes.

I. INTRODUCTION

For extraction of beam from one accelerator to another, a synchronization loop of the type shown in Figure 1 can be used. This would involve synchronizing the beam frequency or the RF signal of the low-energy machine with an external reference source. The phase difference between the beam and the reference source is used to correct the input frequency of the low energy machine. The reference source could be a separate fixed frequency oscillator driving the high power RF system of the higher energy machine while the synchronization process is under way. Phase synchronization is obtained when the phase error between the reference source and the beam frequency is made equal to zero. A simple design of such a feedback system consists of a state feedback gain k, as shown in Figure 1. Apart from the synchronization loop, it is quite normal to have other feedback loops such as a beam phase loop or a B-field loop providing a small correction function to the variable frequency source. The B-field loop is not able to give full indication of the correction required because of measurement inaccuracy in the field. Hence the field error would act as disturbance to the system. Under those circumstances, the feedback controller, k, will not be able to drive the phase detector output to zero since the state feedback loop cannot handle external disturbance on the system. In this paper we show the synthesis of a sliding-mode controller using Lyapunov Stability Theory. This controller behaves very much like a state feedback controller when the gain associated with robustness is turned off. We also discuss the effects due to Q of the RF cavity when we implement this type of controller.

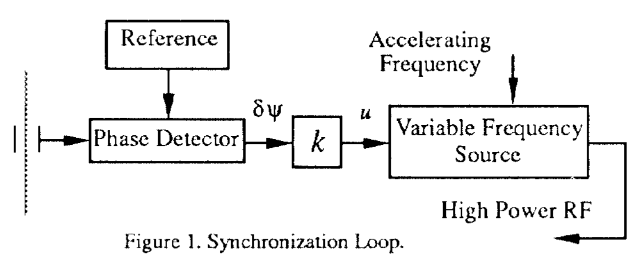

Figure 1. Synchronization Loop.

In our analysis we assume that the synchronization of the low energy machine can be done with the high energy machine throughout the acceleration. However, it can be switched on anytime during acceleration. Although the synchronization scheme in Reference 1 is different in its implementation from the conventional approach, in principle it is similar to Figure 1. Hence the feedback controller can be applied to the conventional phase-lock scheme.

II. FEEDBACK CONTROLLER

In the presence of B-field errors, the phase detector output, $\delta\psi$, can be represented[1] by the following equation with standard notations.

$$\frac{d\delta\psi(t)}{dt} = \frac{R\gamma_T^2}{\gamma_T^2 - \gamma^2}\left(\frac{2\pi\delta f(t)}{h} - \frac{2\pi f}{h}\frac{1}{\gamma_T^2}\frac{\delta B(t)}{B(t)}\right) \quad (1)$$

This equation is derived by ignoring the non-linear terms. For the present analysis the terms associated with $\delta B(t)$ can be regarded as the disturbance to the system. Under no disturbance, a more general way to write the above equation is in state-space form with variables $\{A(t)\}$, $\{b(t)\}$, $\{C(t)\}$ as system matrices, $u(t)$ as the control signal, $y(t)$ the output signal, and $x(t)$ the state variable as follows:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + b(t)u(t) \\ y(t) &= C(t)x(t) \end{aligned} \quad (2)$$

*Operated by Universities Research Association Inc., under Contract with U.S. Department of Energy, Contract No. DE-AC02-89ER40486.

where

$x(t) = \delta\psi(t)$, $A(t) = 0$, $b(t) = \frac{\gamma_T^2}{\gamma_T^2 - \gamma^2} R$,

$C(t) = 1$, and $u(t) = \frac{2\pi}{h} \delta f(t)$.

The time variation of the parameters is represented by (t). For simplicity we do not write the script *(t)* in our discussion. With disturbance, i.e., $\delta B(t) \neq 0$, Equation 2 becomes equal to

$$\dot{x} = b(u + u_d) , \quad (3)$$

where u_d is regarded as the disturbance function and is equal to

$$u_d = -\frac{2\pi f}{h} \frac{1}{\gamma_T^2} \frac{\delta B}{B} . \quad (4)$$

Also, let us assume that the measured state, x_m has an error of x_d , then

$$x_m = x + x_d , \quad (5)$$

where x is the actual state as described by Equation 3. Now the sliding variable is defined with the measured state as follows.

$$S = x_m + \alpha \int_0^t x_m dt . \quad (6)$$

In this equation α is equal to the eigenvalue of the closed loop feedback system. Equation 5 is substituted in Equation 6, and resulting equation is differentiated with respect to time. The terms with $\dot{x}$ are replaced by Equation 3. After simplification, we get

$$b^{-1}\dot{S} = u + u_d + b^{-1}\dot{x}_d + b^{-1}\alpha x_m . \quad (7)$$

The justification for the choice of the stable feedback loop is based on the Lyapunov function candidate. There is however no unique Lyapunov function for this problem. A more suitable one could be as follows:

$$V = \frac{1}{2} b^{-1} S^2 \quad (8)$$

The above function is positive because the system parameter b is positive. Furthermore, from Lyapunov Stability Theory, a system of the type used in Equation 2 is stable when the time derivative of the positive definite Lyapunov function is negative. Hence we will differentiate Equation 8 with respect to time and substitute Equation 7 in place of $b^{-1}\dot{S}$. After simplification we get

$$\dot{V} = S[u + u_d + g\dot{x}_d + g\alpha(x + x_d) - hS]$$

where $g = b^{-1}$ and $h = \frac{1}{2}\frac{\dot{b}}{b^2}$. (9)

Here we can assume that the disturbance signal u_d can be measured. Since the measurement will not be accurate, we can consider this term to have a nominal measurable term and an uncertainty function. When the measurements are not available, the nominal value will be zero. Similarly, parameter uncertainties can be assigned to g and h. Thus we can write:

$$u_d = u_d^\circ + \Delta u_d$$
$$g = g^\circ + \Delta g \quad (10)$$
$$h = h^\circ + \Delta h$$

where the terms u_d°, g° and h° are the nominal quantities, and Δu_d, Δg and Δh are uncertainties in the parameters u_d, g and h , respectively. Now, using Equation 10 into Equation 9, we obtain

$$\dot{V} = S[u + (u_d^\circ + \Delta u_d) + (g^\circ + \Delta g)\alpha x_m - (h^\circ + \Delta h) S + g\dot{x}_d] . \quad (11)$$

The control law, u, is defined in such a way that Equation 11 is always negative. Let it consist of the continuous part u_c and a switching part u_s:

$$u = u_c + u_s$$

where $u_c = -u_d^\circ - g^\circ \alpha x_m + h^\circ S$ and

$$u_s = -(k_x|x_m| + k_s|S| + k_0)\,\mathrm{sgn}S . \quad (12)$$

The function $\mathrm{sgn}S$ in Equation 12 is a *signum* function which has a value of either +1 or -1 when $S \geq 0$ and $S < 0$, respectively. The constants k_x, k_s, and k_0 in Equation 12 are selected so as to make the time derivative of the Lyapunov function negative. With simple algebra we can arrive at the following condition.

$$k_x > sup|\Delta g \alpha|$$
$$k_s > sup|\Delta h| \quad (13)$$
$$k_0 > sup|\Delta u_d + g\dot{x}_d| .$$

In Equation 13, '*sup*' is pronounced as *supremum,* which is the maximum value of the function. The magnitude of the constants depends on the parameter uncertainties, but for stability they must satisfy Equation 13. The continuous part in the control law in Equation 12 holds the phase error zero, while at the same time the switching part introduces the robustness into the loop. Hence the switching part would take care of the disturbance rejection and parameter uncertainties. Equations 6 and 12 form the feedback controller, as shown in Figure 2.

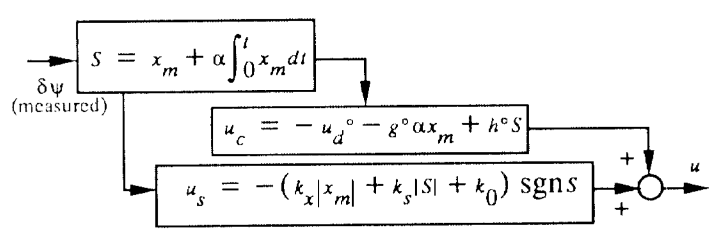

Figure 2. Sliding-mode controller.

Under the condition with no disturbance (u_d=0), no measurement error (x_d=0), and no parameter variation, the gains k_x=k_s=k_0=0. Also, h=0 for a time invariant system, since a state feedback design is applicable to only such systems. Hence the state equation of the closed loop feedback becomes equal to

$$\dot{x} = bu = -\alpha x , \quad (14)$$

where α/b is now equal to the gain, k, shown in Figure 1.

III. ANALYSIS OF THE LOOP PERFORMANCE

The performance of the feedback loop is analysed by considering the phase-locking between the Low Energy Booster and the Medium Energy Booster. The machine parameters shown in Reference 2 are used for the analysis. Figure 3(a) shows a plot of the decay of the phase detector output with respect to time, with the state feedback loop gain of k=2 and 5. The phase error converges to zero as expected. The profile of the phase error is, however, not important but it should be zero at the transfer time (ignoring all the fixed phase associated with the transfer line delays, etc). In an ideal situation, when there is no field error affecting the beam frequency, we would expect the synchronization to be good as shown in Figure 3. The loop performance deteriorates when a step magnitude of u_d=11.6 rad/sec (for $(\delta B)/B = 5 \times 10^{-4}$ for the Low Energy Booster) is introduced to the system and is held high until the extraction time. The time response of the phase error is shown in Figure 3(b) for this disturbance. It is clear from this figure that the system is not robust since the phase error is not held zero or at least to a tolerable value. It can be minimized by making the feedback gain excessively large which may lead to beam oscillations.

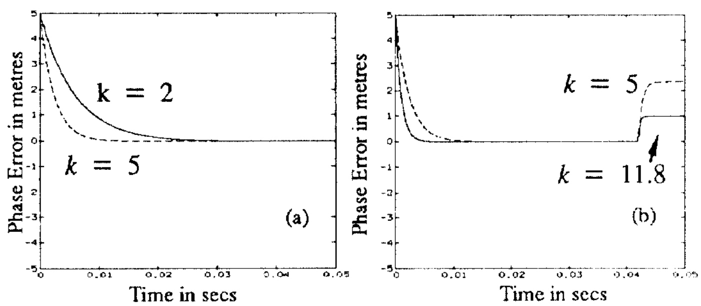

Figure 3. Time response of the phase error
(a) without disturbance; (b) with disturbance (step function at 42ms).

In Figure 4(a) the loop performance is shown with the disturbance function for the sliding-mode controller. In Figure 4(b) the time response of the sliding variable S (Equation 6) is displayed. From these figures it is clear that the product $S\dot{S}$ is always negative. Hence the loop is stable throughout the acceleration. Also, the loop performance is very good under field errors compared to the usual gain feedback. To overcome field errors the feedback controller generates the compensating frequency shift to the oscillator. Since the constant k_0 controls the magnitude of the disturbance rejection, it would be useful to have it set very high. Higher k_0 may result phase oscillations for digital implementation with low sampling rates. However, the

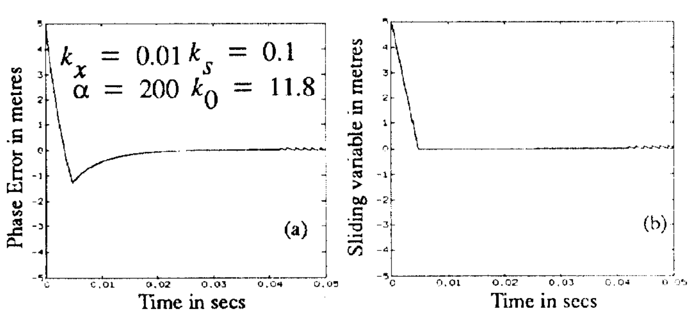

Figure 4. (a) Time response of the phase error (with disturbance); (b) Time response of the sliding variable

analogue loops have no such problems. When there is no parameter variation ($\Delta g \cong 0$ and $\Delta h \cong 0$), the gains k_x and k_s can be negligible. Hence the switching part of the control input to the oscillator is mainly dominated by k_0. Thus it is dominated by the system uncertainties, whereas the continuous part acts on the initial phase error by the same principle by which the state feedback loop works. If the initial phase error is large, then a sudden frequency shift of few khz would introduce beam oscillations. Hence a good solution would be to use the time variation for appropriate gains including the eigenvalue, α. Implementation of such a gain sequence would be easier for a digital synchronization loop.

It is well known that the beam frequency does not change instantaneously when the oscillator is shifted by the control signal, u. The time constant is governed by the Q of the cavity and the amount of detuning caused by the beam current or a separate tuning loop. By assuming that the tuning error is well compensated, the equation between the beam frequency shift and the source frequency shift is given by

$$\dot{u} = -\frac{w_{cav}}{2Q}u + \frac{w_{cav}}{2Q}u_i \qquad (15)$$

where w_{cav} = resonant frequency of the cavity and $u_i = (2\pi\delta f_i)/h$, with δf_i as the oscillator frequency shift. A block diagram representing Equations 2 and 15 is shown in Figure 5. The time response of the phase error is not very differ

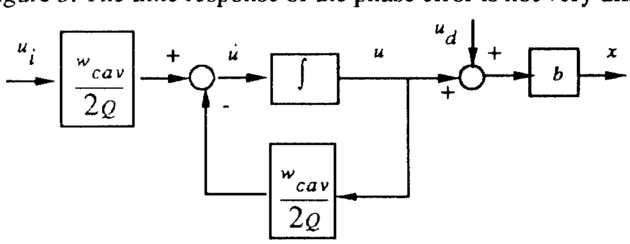

Figure 5.: System model with the Q of the RF cavity

ent from Figures 3 and 4 for the ratio $(2Q)/w_{cav}$ up to 1 millisecond.

IV. CONCLUSIONS

Analytical treatment of the synchronization feedback loop is shown in this paper. Although the gain feedback loop is easier to design and implement, the simulation results show that the controller properties are not useful to handle changes in the synchronization conditions with B-field errors. The sliding-mode controller shows robustness for such applications. For large disturbance rejection, the inherent oscillatory nature of the control signal can be overcome by introducing well-known saturation function.

V. REFERENCES

[1] L.K. Mestha, "Phase-Control Scheme for Synchronous Beam Transfer from the Low Energy Booster to the Medium Energy Booster", SSC Report, SSCL-340, 1990.

[2] Site-Specific Conceptual Design Report of the SSC, SSCL-SR-1056, July 1990.